4 Elastic Properties of Dislocations

4.1 Introduction

The atoms in a crystal containing a dislocation are displaced from their perfect lattice sites, and the resulting distortion produces a stress field in the crystal around the dislocation. The dislocation is therefore a source of *internal stress* in the crystal. For example, consider the edge dislocation in Fig. 1.18(b). The region above the slip plane contains the extra half-plane forced between the normal lattice planes, and is in compression: the region below is in tension. The stresses and strains in the bulk of the crystal are sufficiently small for conventional elasticity theory to be applied to obtain them. This approach only ceases to be valid at positions very close to the centre of the dislocation. Although most crystalline solids are elastically *anisotropic*, i.e. their elastic properties are different in different crystallographic directions, it is much simpler to use *isotropic elasticity* theory. This still results in a good approximation in most cases. From a knowledge of the elastic field, the energy of the dislocation, the force it exerts on other dislocations, its energy of interaction with point defects, and other important characteristics, can be obtained. The elastic field produced by a dislocation is not affected by the application of stress from *external* sources: the total stress on an element within the body is the superposition of the internal and external stresses.

4.2 Elements of Elasticity Theory

The *displacement* of a point in a strained body from its position in the unstrained state is represented by the vector

$$\mathbf{u} = [u_x, u_y, u_z] \tag{4.1}$$

The components u_x, u_y, u_z represent projections of $\mathbf{u}$ on the x, y, z axes, as shown in Fig. 4.1. In *linear* elasticity, the nine components of *strain* are given in terms of the first derivatives of the displacement components thus:

$$e_{xx} = \frac{\partial u_x}{\partial x}$$

$$e_{yy} = \frac{\partial u_y}{\partial y} \tag{4.2}$$

$$e_{zz} = \frac{\partial u_z}{\partial z}$$

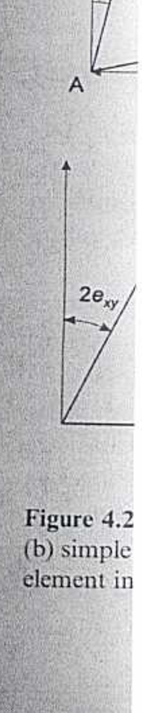

Figure 4.2
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$\frac{u_x}{x}$

$\frac{u_y}{y}$ (4.2)

$\frac{u_z}{z}$

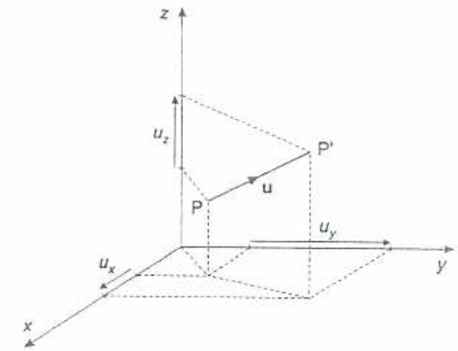

Figure 4.1 Displacement of P to P' by displacement vector **u**.

and

$$e_{yz} = e_{zy} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)$$
$$e_{zx} = e_{xz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) \quad (4.3)$$
$$e_{xy} = e_{yx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

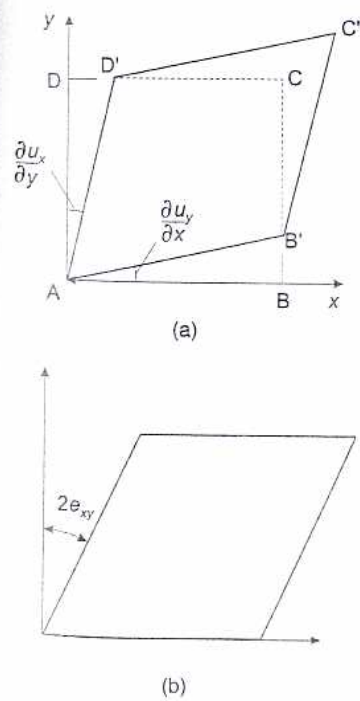

Figure 4.2 (a) Pure shear and (b) simple shear of an area element in the xy plane.

The magnitude of these components is $\ll 1$. Partial differentials are used because in general each displacement component is a function of position (x, y, z). The three strains defined in (4.2) are the *normal* strains. They represent the fractional change in length of elements parallel to the x, y and z axes respectively. The six components defined in (4.3) are the *shear* strains, and they also have simple physical meaning. This is demonstrated by e_{xy} in Fig. 4.2(a), in which a small area element $ABCD$ in the xy plane has been strained to the shape $AB'C'D'$ without change of area. The angle between the sides AB and AD initially parallel to x and y respectively has decreased by $2e_{xy}$. By rotating, but not deforming, the element as in Fig. 4.2(b), it is seen that the element has undergone a simple shear. The simple shear strain often used in engineering practice is $2e_{xy}$, as indicated.

The volume V of a small volume element is changed by strain to $(V + \Delta V) = V(1 + e_{xx})(1 + e_{yy})(1 + e_{zz})$. The fractional change in volume Δ, known as the *dilatation*, is therefore

$$\Delta = \Delta V/V = (e_{xx} + e_{yy} + e_{zz}) \quad (4.4)$$

Δ is independent of the orientation of the axes x, y, z.

In elasticity theory, an element of volume experiences forces via stresses applied to its surface by the surrounding material. *Stress* is the force per unit area of surface. A complete description of the stresses acting therefore requires not only specification of the magnitude and direction of the force but also of the orientation of the surface, for as the

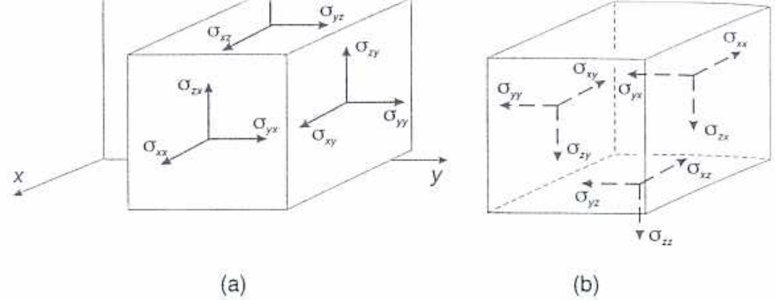

Figure 4.3 Components of stress acting on (a) the top and front faces and (b) the bottom and back faces of an elemental cube.

orientation changes so, in general, does the force. Consequently, nine components must be defined to specify the state of stress. They are shown with reference to an elemental cube aligned with the x, y, z axes in Fig. 4.3(a). The component σ_{ij}, where i and j can be x, y or z, is defined as the force per unit area exerted in the $+i$ direction on a face with outward normal in the $+j$ direction by the material *outside* upon the material *inside*. For a face with outward normal in the $-j$ direction, i.e. the bottom and back faces shown in Fig. 4.3(b), σ_{ij} is the force per unit area exerted in the $-i$ direction. For example, σ_{yz} acts in the positive y direction on the top face and the negative y direction on the bottom face.

The six components with $i \neq j$ are the *shear* stresses. (As explained in section 3.1, it is customary in dislocation studies to represent the shear stress acting on the slip plane in the slip direction of a crystal by the symbol τ.) By considering moments of forces taken about x, y and z axes placed through the centre of the cube, it can be shown that rotational equilibrium of the element, i.e. net couple = 0, requires

$$\sigma_{yz} = \sigma_{zy} \quad \sigma_{zx} = \sigma_{xz} \quad \sigma_{xy} = \sigma_{yx} \tag{4.5}$$

Thus, the order in which subscripts i and j is written is immaterial. The three remaining components σ_{xx}, σ_{yy}, σ_{zz} are the *normal* components. From the definition given above, a *positive* normal stress results in *tension* and a *negative* one in *compression*. The effective pressure acting on a volume element is therefore

$$p = -\frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) \tag{4.6}$$

For some problems, it is more convenient to use cylindrical polar coordinates (r, θ, z). The stresses are still defined as above, and are shown in Fig. 4.4. The notation is easier to follow if the second subscript j is considered as referring to the face of the element having a constant value of the coordinate j.

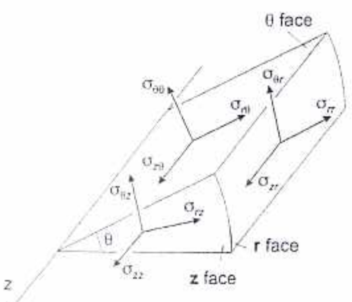

Figure 4.4 Components of stress in cylindrical polar coordinates.

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are required:

$$\sigma_{xx} = 2Ge_{xx} + \lambda(e_{xx}$$
$$\sigma_{yy} = 2Ge_{yy} + \lambda(e_{xx}$$
$$\sigma_{zz} = 2Ge_{zz} + \lambda(e_{xx}$$
$$\sigma_{xy} = 2Ge_{xy} \quad \sigma_{yz} =$$

λ and G are the Lamé c
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$$E = 2G(1 + \nu) \quad \nu =$$

Typical values of E and
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component. Thus, for ar

$$dE_{el} = \frac{1}{2}dV \sum_{i=x,y,z} \sum_{j=x,}$$

and similarly for polar

4.3 Stress Field of a Straight Dislocation

Screw Dislocation

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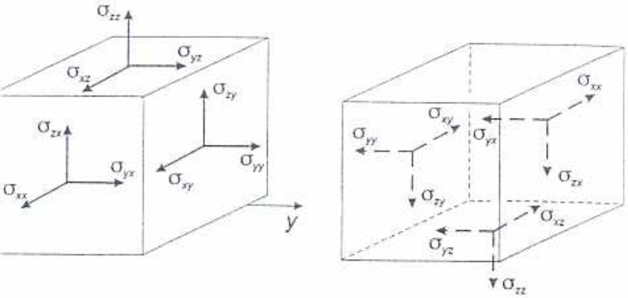
$$u_x = u_y = 0$$



(a) (b)

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θ, z). The stresses are still defined as above, and are
4. The notation is easier to follow if the second subscript
is referring to the face of the element having a constant
ordinate j.

The relationship between stress and strain in linear elasticity is *Hooke's Law*, in which each stress component is linearly proportional to each strain. For isotropic solids, only two proportionality constants are required:

$$\begin{aligned} \sigma_{xx} &= 2Ge_{xx} + \lambda(e_{xx} + e_{yy} + e_{zz}) \\ \sigma_{yy} &= 2Ge_{yy} + \lambda(e_{xx} + e_{yy} + e_{zz}) \\ \sigma_{zz} &= 2Ge_{zz} + \lambda(e_{xx} + e_{yy} + e_{zz}) \\ \sigma_{xy} &= 2Ge_{xy} \quad \sigma_{yz} = 2Ge_{yz} \quad \sigma_{zx} = 2Ge_{zx} \end{aligned} \tag{4.7}$$

λ and G are the Lamé constants, but G is more commonly known as the *shear modulus*. Other elastic constants are frequently used, the most useful being *Young's modulus*, E, *Poisson's ratio*, ν, and *bulk modulus*, K. Under uniaxial, normal loading in the longitudinal direction, E is the ratio of longitudinal stress to longitudinal strain and ν is minus the ratio of lateral strain to longitudinal strain. K is defined to be $-p/\Delta$. Since only two material parameters are required in Hooke's law, these constants are interrelated. For example,

$$E = 2G(1+\nu) \quad \nu = \lambda/2(\lambda+G) \quad K = E/3(1-2\nu) \tag{4.8}$$

Typical values of E and ν for metallic and ceramic solids lie in the ranges 40–600 GN m^{-2} and 0.2–0.45 respectively.

The internal energy of a body is increased by strain. The *strain energy* per unit volume is one-half the product of stress times strain for each component. Thus, for an element of volume dV, the elastic strain energy is

$$dE_{el} = \frac{1}{2} dV \sum_{i=x,y,z} \sum_{j=x,y,z} \sigma_{ij} e_{ij} \tag{4.9}$$

and similarly for polar coordinates.

4.3 Stress Field of a Straight Dislocation

Screw Dislocation

The elastic distortion around an infinitely-long, straight dislocation can be represented in terms of a cylinder of elastic material. Consider the screw dislocation AB shown in Fig. 4.5(a); the elastic cylinder in Fig. 4.5(b) has been deformed to produce a similar distortion. A radial slit $LMNO$ was cut in the cylinder parallel to the z-axis and the free surfaces displaced rigidly with respect to each other by the distance b, the magnitude of the Burgers vector of the screw dislocation, in the z-direction.

The elastic field in the dislocated cylinder can be found by direct inspection. First, it is noted that there are no displacements in the x and y directions:

$$u_x = u_y = 0 \tag{4.10}$$

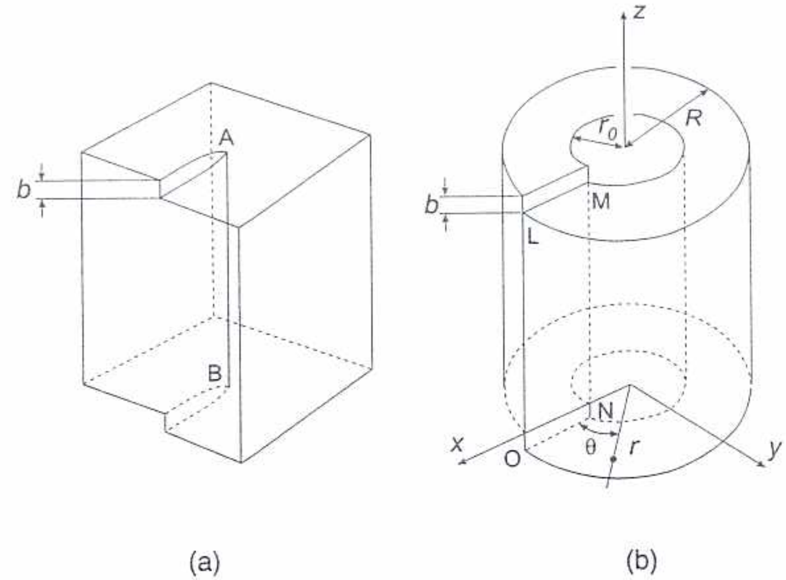


Figure 4.5 (a) Screw dislocation AB formed in a crystal. (b) Elastic distortion of a cylindrical ring simulating the distortion produced by the screw dislocation in (a).

Secondly, the displacement in the z-direction increases uniformly from zero to b as θ increases from 0 to 2π:

$$u_z = \frac{b\theta}{2\pi} = \frac{b}{2\pi}\tan^{-1}(y/x) \tag{4.11}$$

It is then readily found from equations (4.2) and (4.3) that

$$\begin{aligned} e_{xx} &= e_{yy} = e_{zz} = e_{xy} = e_{yx} = 0 \\ e_{xz} &= e_{zx} = -\frac{b}{4\pi}\frac{y}{(x^2+y^2)} = -\frac{b}{4\pi}\frac{\sin\theta}{r} \\ e_{yz} &= e_{zy} = \frac{b}{4\pi}\frac{x}{(x^2+y^2)} = \frac{b}{4\pi}\frac{\cos\theta}{r} \end{aligned} \tag{4.12}$$

From equations (4.7) and (4.12), the components of stress are

$$\begin{aligned} \sigma_{xx} &= \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yx} = 0 \\ \sigma_{xz} &= \sigma_{zx} = -\frac{Gb}{2\pi}\frac{y}{(x^2+y^2)} = -\frac{Gb}{2\pi}\frac{\sin\theta}{r} \\ \sigma_{yz} &= \sigma_{zy} = \frac{Gb}{2\pi}\frac{x}{(x^2+y^2)} = \frac{Gb}{2\pi}\frac{\cos\theta}{r} \end{aligned} \tag{4.13}$$

The components in cylindric pler form. Using the relation:

$$\sigma_{rz} = \sigma_{xz}\cos\theta + \sigma_{yz}\sin\theta$$
$$\sigma_{\theta z} = -\sigma_{xz}\sin\theta + \sigma_{yz}\cos\theta$$

and similarly for the shear s found to be

$$e_{\theta z} = e_{z\theta} = \frac{b}{4\pi r}$$
$$\sigma_{\theta z} = \sigma_{z\theta} = \frac{Gb}{2\pi r}$$

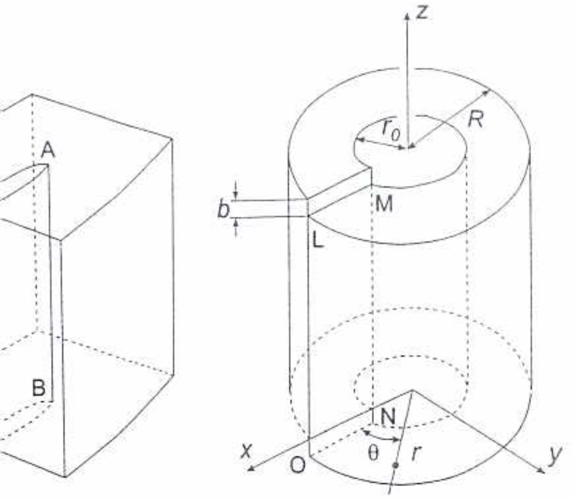
The elastic distortion contai and consists of pure shear: σ_z of constant θ and $\sigma_{\theta z}$ acts in to the axis (Fig. 4.4). The fie the cut $LMNO$ can be mad a dislocation of *opposite* sigr the field components are *reve*

The stresses and strains are to infinity as $r \to 0$. Solids c this reason the cylinder in F radius r_0. Real crystals are n of a dislocation in a crystal is valid and a non-linear, atomi The region within which the l the *core* of the dislocation. Fr reaches the theoretical limit about 10% when $r \approx b$. A rea r_0 therefore lies in the range

Edge Dislocation

The stress field is more co represented in an isotropic c edge dislocation in Fig. 4.6 produced in the cylinder by a by a distance b in the x-dire strains in the z-direction are strain. Derivation of the fiel present treatment, however.

$$\sigma_{xx} = -Dy\frac{(3x^2+y^2)}{(x^2+y^2)^2}$$
$$\sigma_{yy} = Dy\frac{(x^2-y^2)}{(x^2+y^2)^2}$$



(a) (b)

rew dislocation AB formed in a crystal. (b) Elastic distortion
g simulating the distortion produced by the screw dislocation

splacement in the z-direction increases uniformly from
creases from 0 to 2π:

$$\text{an}^{-1}(y/x) \tag{4.11}$$

found from equations (4.2) and (4.3) that

$$_{zz} = e_{xy} = e_{yx} = 0$$

$$\frac{b}{4\pi}\frac{y}{(x^2+y^2)} = -\frac{b}{4\pi}\frac{\sin\theta}{r} \tag{4.12}$$

$$\frac{x}{\pi(x^2+y^2)} = \frac{b}{4\pi}\frac{\cos\theta}{r}$$

(4.7) and (4.12), the components of stress are

$$\sigma_{zz} = \sigma_{xy} = \sigma_{yx} = 0$$

$$-\frac{Gb}{2\pi}\frac{y}{(x^2+y^2)} = -\frac{Gb}{2\pi}\frac{\sin\theta}{r} \tag{4.13}$$

$$\frac{Gb}{2\pi}\frac{x}{(x^2+y^2)} = \frac{Gb}{2\pi}\frac{\cos\theta}{r}$$

The components in cylindrical polar coordinates (Fig. 4.4) take a simpler form. Using the relations

$$\begin{aligned}\sigma_{rz} &= \sigma_{xz}\cos\theta + \sigma_{yz}\sin\theta\\ \sigma_{\theta z} &= -\sigma_{xz}\sin\theta + \sigma_{yz}\cos\theta\end{aligned} \tag{4.14}$$

and similarly for the shear strains, the only non-zero components are found to be

$$\begin{aligned}e_{\theta z} &= e_{z\theta} = \frac{b}{4\pi r}\\ \sigma_{\theta z} &= \sigma_{z\theta} = \frac{Gb}{2\pi r}\end{aligned} \tag{4.15}$$

The elastic distortion contains no tensile or compressive components and consists of pure shear: $\sigma_{z\theta}$ acts parallel to the z-axis in radial planes of constant θ and $\sigma_{\theta z}$ acts in the fashion of a torque on planes normal to the axis (Fig. 4.4). The field exhibits complete radial symmetry and the cut $LMNO$ can be made on any radial plane θ = constant. For a dislocation of *opposite* sign, i.e. a left-handed screw, the signs of all the field components are *reversed*.

The stresses and strains are proportional to $1/r$ and therefore diverge to infinity as $r \to 0$. Solids cannot withstand infinite stresses, and for this reason the cylinder in Fig. 4.5 is shown as hollow with a hole of radius r_0. Real crystals are not hollow, of course, and so as the centre of a dislocation in a crystal is approached, elasticity theory ceases to be valid and a non-linear, atomistic model must be used (see section 10.3). The region within which the linear-elastic solution breaks down is called the *core* of the dislocation. From equation (4.15) it is seen that the stress reaches the theoretical limit (equation (1.5)) and the strain exceeds about 10% when $r \approx b$. A reasonable value for the *dislocation core radius* r_0 therefore lies in the range b to $4b$, i.e. $r_0 \lesssim 1$ nm in most cases.

Edge Dislocation

The stress field is more complex than that of a screw but can be represented in an isotropic cylinder in a similar way. Considering the edge dislocation in Fig. 4.6(a), the same elastic strain field can be produced in the cylinder by a rigid displacement of the faces of the slit by a distance b in the x-direction (Fig. 4.6(b)). The displacement and strains in the z-direction are zero and the deformation is called plane strain. Derivation of the field components is beyond the scope of the present treatment, however. The stresses are found to be

$$\sigma_{xx} = -Dy\frac{(3x^2+y^2)}{(x^2+y^2)^2}$$

$$\sigma_{yy} = Dy\frac{(x^2-y^2)}{(x^2+y^2)^2}$$

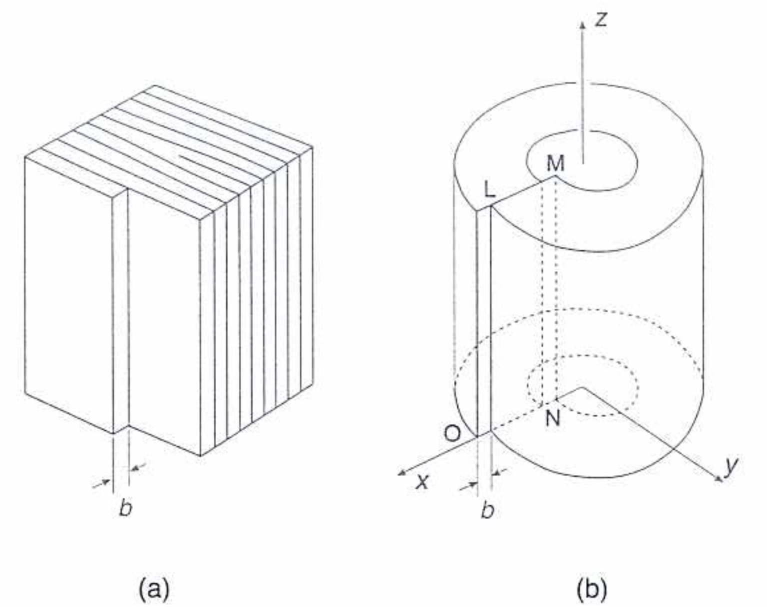


Figure 4.6 (a) Edge dislocation formed in a crystal. (b) Elastic distortion of a cylindrical ring simulating the distortion produced by the edge dislocation in (a).

$$\sigma_{xy} = \sigma_{yx} = Dx\frac{(x^2 - y^2)}{(x^2 + y^2)^2}$$
$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$$
$$\sigma_{xz} = \sigma_{zx} = \sigma_{yz} = \sigma_{zy} = 0 \qquad (4.16)$$

where

$$D = \frac{Gb}{2\pi(1-\nu)}$$

The stress field has, therefore, both dilational and shear components. The largest normal stress is σ_{xx} which acts parallel to the slip vector. Since the slip plane can be defined as $y = 0$, the maximum compressive stress (σ_{xx} negative) acts immediately above the slip plane and the maximum tensile stress (σ_{xx} positive) acts immediately below the slip plane. The effective pressure (equation (4.6)) on a volume element is

$$p = \frac{2}{3}(1+\nu)D\frac{y}{(x^2+y^2)} \qquad (4.17)$$

It is compressive above the slip plane and tensile below. These observations are implied qualitatively by the type of distortion illustrated in Figs 1.18 and 4.6(a).

As in the case of the screw, the signs of the components are reversed

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edge and screw characte
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4.4 Strain Energy of a Dislocation

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$$E_{total} = E_{core} + E_{elastic}$$

The elastic part, stored c
tion of the energy of ea
calculation for the screv
appropriate volume elem
ness dr. From equation (
unit length of dislocation

$$dE_{el}(\text{screw}) = \frac{1}{2}2\pi r\, dr$$
$$= 4\pi r\, dr\, G$$

Thus, from equation (4.1
(Fig. 4.5) per unit length

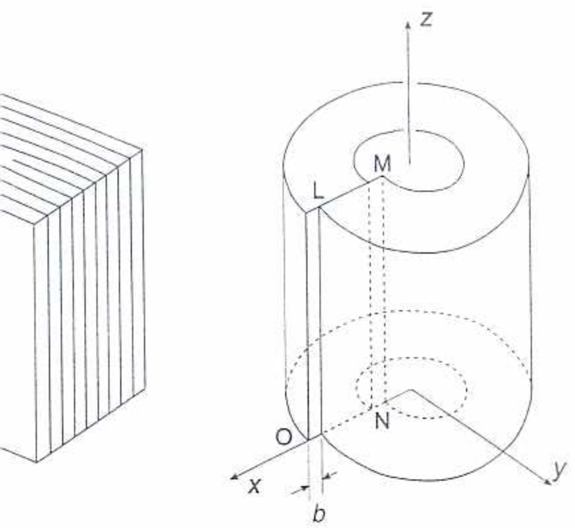
$$E_{el}(\text{screw}) = \frac{Gb^2}{4\pi}\int_{r_0}^{R}\frac{d}{r}$$

where R is the outer radi
The above approach is
having less symmetric fiel
work done in displacing t
4.6) against the resisting in
area dA of $LMNO$, the w

$$dE_{el}(\text{screw}) = \frac{1}{2}\sigma_{zy} b\, dA$$

$$dE_{el}(\text{edge}) = \frac{1}{2}\sigma_{xy} b\, dA$$

with the stresses evaluate



ge dislocation formed in a crystal. (b) Elastic distortion of a ulating the distortion produced by the edge dislocation in (a).

$$x\frac{(x^2-y^2)}{(x^2+y^2)^2}$$

$$\sigma_{yy})$$

$$_{z} = \sigma_{zy} = 0 \qquad (4.16)$$

$$\nu)$$

nas, therefore, both dilational and shear components. nal stress is σ_{xx} which acts parallel to the slip vector. ne can be defined as $y = 0$, the maximum compressive tive) acts immediately above the slip plane and the stress (σ_{xx} positive) acts immediately below the slip ive pressure (equation (4.6)) on a volume element is

$$\frac{y}{(x^2+y^2)} \qquad (4.17)$$

above the slip plane and tensile below. These observa- qualitatively by the type of distortion illustrated in Figs

of the screw, the signs of the components are reversed of opposite sign, i.e. a negative edge dislocation with along the negative y-axis. Again, the elastic solution has

an inverse dependence on distance from the line axis and breaks down when x and y tend to zero. It is valid only outside a core of radius r_0.

The elastic field produced by a *mixed* dislocation (Fig. 3.8(b)) having *edge and screw character* is obtained from the above equations by adding the fields of the edge and screw constituents, which have Burgers vectors given by equation (3.2). The two sets are independent of each other in isotropic elasticity.

4.4 Strain Energy of a Dislocation

The existence of distortion around a dislocation implies that a crystal containing a dislocation is not in its lowest energy state. The extra energy is the *strain energy*. The total strain energy may be divided into two parts

$$E_{\text{total}} = E_{\text{core}} + E_{\text{elastic strain}} \qquad (4.18)$$

The elastic part, stored outside the core, may be determined by integration of the energy of each small element of volume. This is a simple calculation for the screw dislocation, because from the symmetry the appropriate volume element is a cylindrical shell of radius r and thickness dr. From equation (4.9), the elastic energy stored in this volume *per unit length* of dislocation is

$$\begin{aligned} \mathrm{d}E_{\text{el}}(\text{screw}) &= \frac{1}{2}2\pi r\,\mathrm{d}r(\sigma_{\theta z}e_{\theta z} + \sigma_{z\theta}e_{z\theta}) \\ &= 4\pi r\,\mathrm{d}r\,Ge_{\theta z}^2 \end{aligned} \qquad (4.19)$$

Thus, from equation (4.15), the total elastic energy stored in the cylinder (Fig. 4.5) per unit length of dislocation is

$$E_{\text{el}}(\text{screw}) = \frac{Gb^2}{4\pi}\int_{r_0}^{R}\frac{\mathrm{d}r}{r} = \frac{Gb^2}{4\pi}\ln\left(\frac{R}{r_0}\right) \qquad (4.20)$$

where R is the outer radius.

The above approach is much more complicated for other dislocations having less symmetric fields. It is generally easier to consider E_{el} as the work done in displacing the faces of the cut $LMNO$ by b (Figs 4.5 and 4.6) against the resisting internal stresses. For an infinitesimal element of area dA of $LMNO$, the work done is

$$\begin{aligned} \mathrm{d}E_{\text{el}}(\text{screw}) &= \frac{1}{2}\sigma_{zy}b\,\mathrm{d}A \\ \mathrm{d}E_{\text{el}}(\text{edge}) &= \frac{1}{2}\sigma_{xy}b\,\mathrm{d}A \end{aligned} \qquad (4.21)$$

with the stresses evaluated on $y = 0$. The factor $\frac{1}{2}$ enters because the stresses build up from zero to the final values given by equations (4.13) and (4.16) during the displacement process. The element of area is a strip length of dislocation is

$$E_{el}(\text{screw}) = \frac{Gb^2}{4\pi}\int_{r_0}^{R}\frac{dx}{x} = \frac{Gb^2}{4\pi}\ln\left(\frac{R}{r_0}\right)$$
$$E_{el}(\text{edge}) = \frac{Gb^2}{4\pi(1-\nu)}\int_{r_0}^{R}\frac{dx}{x} = \frac{Gb^2}{4\pi(1-\nu)}\ln\left(\frac{R}{r_0}\right) \qquad (4.22)$$

The screw result is the same as equation (4.20). Strictly, equations (4.22) neglect small contributions from the work done on the core surface $r = r_0$ of the cylinder, but they are adequate for most requirements.

Equations (4.22) demonstrate that E_{el} depends on the core radius r_0 and the crystal radius R, but only logarithmically. E_{el} (edge) is greater than E_{el} (screw) by $1/(1-\nu) \approx 3/2$. Taking $R = 1$ mm, $r_0 = 1$ nm, $G = 40\,\text{GN m}^{-2}$ and $b = 0.25$ nm, the elastic strain energy of an edge dislocation will be about $4\,\text{nJ m}^{-1}$ or about 1 aJ (6 eV) for each atom plane threaded by the dislocation. In crystals containing many dislocations, the dislocations tend to form in configurations in which the superimposed long-range elastic fields cancel. The energy per dislocation is thereby reduced and an appropriate value of R is approximately half the average spacing of the dislocations arranged at random.

Estimates of the energy of the core of the dislocation are necessarily very approximate. However, the estimates that have been made suggest that the core energy will be of the order of 1 eV for each atom plane threaded by the dislocation, and is thus only a small fraction of the elastic energy. However, in contrast to the elastic energy, the energy of the core will vary as the dislocation moves through the crystal and this gives rise to the lattice resistance to dislocation motion discussed in section 10.3.

The validity of elasticity theory for treating dislocation energy outside a core region has been demonstrated by computer simulation (section 2.7). Figure 4.7 shows data for an atomic model of alpha iron containing a straight edge dislocation with Burgers vector $\frac{1}{2}[111]$ and line direction $[11\bar{2}]$. E_{total} is the strain energy within a cylinder of radius R with the dislocation along its axis. The energy varies logarithmically with R, as predicted by equation (4.22), outside a core of radius 0.7 nm, which is about $2.6b$. The core energy is about $7\,\text{eV nm}^{-1}$.

It was mentioned in the preceding section that the elastic field of a *mixed* dislocation (see Fig. 3.8(b)) is the superposition of the fields of its edge and screw parts. As there is no interaction between them, the total elastic energy is simply the sum of the edge and screw energies with b replaced by $b\sin\theta$ and $b\cos\theta$ respectively:

$$E_{el}(\text{mixed}) = \left[\frac{Gb^2\sin^2\theta}{4\pi(1-\nu)} + \frac{Gb^2\cos^2\theta}{4\pi}\right]\ln\left(\frac{R}{r_0}\right)$$
$$= \frac{Gb^2(1-\nu\cos^2\theta)}{4\pi(1-\nu)}\ln\left(\frac{R}{r_0}\right) \qquad (4.23)$$

Figure 4.7 The strain en
straight edge dislocation a
simulation for a model of

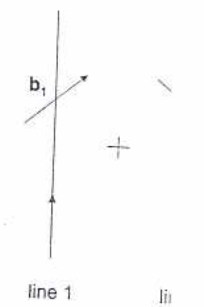


Figure 4.8 Reaction of tw

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the dislocation and also t
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$E_{el} = \alpha Gb^2$

where $\alpha \approx 0.5-1.0$. Thi
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equation (4.24), the elasti
proportional to b_1^2, b_2^2 an

el to the z-axis, and so the total strain energy per unit on is

$$\ldots \frac{b^2}{\pi} \int_{r_0}^{R} \frac{dx}{x} = \frac{Gb^2}{4\pi} \ln\left(\frac{R}{r_0}\right)$$

$$\ldots \frac{Gb^2}{(1-\nu)} \int_{r_0}^{R} \frac{dx}{x} = \frac{Gb^2}{4\pi(1-\nu)} \ln\left(\frac{R}{r_0}\right) \tag{4.22}$$

; the same as equation (4.20). Strictly, equations (4.22) tributions from the work done on the core surface der, but they are adequate for most requirements.

2) demonstrate that E_{el} depends on the core radius r_0 dius R, but only logarithmically. E_{el} (edge) is greater by $1/(1-\nu) \approx 3/2$. Taking $R = 1$ mm, $r_0 = 1$ nm, nd $b = 0.25$ nm, the elastic strain energy of an edge e about $4\,\mathrm{nJ\,m^{-1}}$ or about 1 aJ (6 eV) for each atom the dislocation. In crystals containing many disloca- ions tend to form in configurations in which the super- ge elastic fields cancel. The energy per dislocation is nd an appropriate value of R is approximately half the of the dislocations arranged at random.

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$$\ldots \left[\frac{Gb^2 \sin^2\theta}{4\pi(1-\nu)} + \frac{Gb^2\cos^2\theta}{4\pi}\right] \ln\left(\frac{R}{r_0}\right)$$

$$\ldots \frac{Gb^2(1-\nu\cos^2\theta)}{4\pi(1-\nu)} \ln\left(\frac{R}{r_0}\right) \tag{4.23}$$

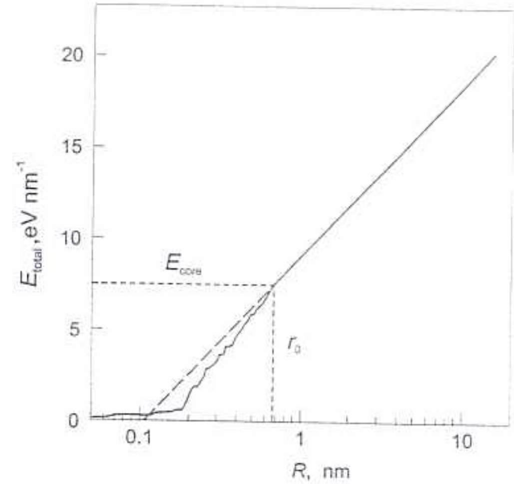

Figure 4.7 The strain energy within a cylinder of radius R that contains a straight edge dislocation along its axis. The data was obtained by computer simulation for a model of iron. (Courtesy Yu. N. Osetsky.)

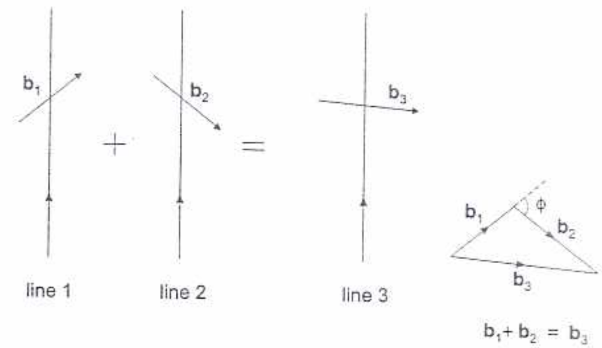

Figure 4.8 Reaction of two dislocations to form a third.

which falls between the energy of an edge and a screw dislocation.

From the expressions for edge, screw and mixed dislocations it is clear that the energy per unit length is relatively insensitive to the character of the dislocation and also to the values of R and r_0. Taking realistic values for R and r_0 all the equations can be written approximately as

$$E_{el} = \alpha G b^2 \tag{4.24}$$

where $\alpha \approx 0.5$–1.0. This leads to a very simple rule (*Frank's rule*) for determining whether or not it is energetically feasible for two dislocations to react and combine to form another. Consider the two dislocations in Fig. 4.8 with Burgers vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ given by the Burgers circuit construction (section 1.4). Allow them to combine to form a new dislocation with Burgers vector $\mathbf{b}_3$ as indicated. From equation (4.24), the elastic energy per unit length of the dislocations is proportional to b_1^2, b_2^2 and b_3^2 respectively. Thus, if $(b_1^2 + b_2^2) > b_3^2$, the reaction is favourable for it results in a reduction in energy. If $(b_1^2 + b_2^2) < b_3^2$, the reaction is unfavourable and the dislocation with Burgers vector b_3 is liable to dissociate into the other two. If $(b_1^2 + b_2^2) = b_3^2$, there is no energy change. These three conditions correspond to the angle ϕ in Fig. 4.8 satisfying $\pi/2 < \phi \leq \pi$, $0 \leq \phi < \pi/2$ and $\phi = \pi/2$ respectively. In this argument the assumption is made that there is no additional interaction energy involved, i.e. that before and after the reaction the reacting dislocations are separated sufficiently so that the interaction energy is small. If this is not so, the reactions are still favourable and unfavourable, but the energy changes are smaller than implied above. Frank's rule is used to consider the feasibility of various dislocation reactions in Chapters 5 and 6.

4.5 Forces on Dislocations

When a sufficiently high stress is applied to a crystal containing dislocations, the dislocations move and produce plastic deformation either by slip as described in section 3.3 or, at sufficiently high temperatures, by climb (section 3.6). The load producing the applied stress therefore does work on the crystal when a dislocation moves, and so the dislocation responds to the stress as though it experiences a force equal to the work done divided by the distance it moves. The force defined in this way is a virtual, rather than real, force, but the force concept is useful for treating the mechanics of dislocation behaviour. The *glide* force is considered in this section and the climb force in section 4.7.

Consider a dislocation moving in a slip plane under the influence of a uniform resolved shear stress τ (Fig. 4.9). When an element dl of the dislocation line of Burgers vector **b** moves forward a distance ds the crystal planes above and below the slip plane will be displaced relative to each other by b. The average shear displacement of the crystal surface produced by glide of dl is

$$\left(\frac{ds\,dl}{A}\right)b \qquad (4.25)$$

where A is the area of the slip plane. The external force due to τ acting over this area is $A\tau$, so that the work done when the element of slip occurs is

$$dW = A\tau\left(\frac{ds\,dl}{A}\right)b \qquad (4.26)$$

The glide force F on a unit length of dislocation is defined as the work done when unit length of dislocation moves unit distance. Therefore

$$F = \frac{dW}{ds\,dl} = \frac{dW}{dA} = \tau b \qquad (4.27)$$

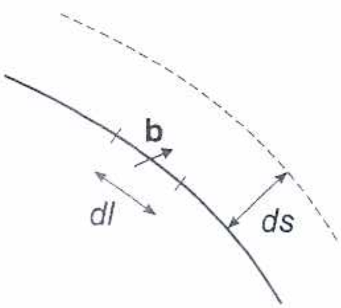

Figure 4.9 The displacement ds

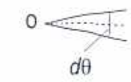

Figure 4.10 Curved elem

along its length, irrespe
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In addition to the for
tion has a *line tension* wh
bubble or a liquid. Th
section, the strain energ
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$T = \alpha G b^2$

Consider the curved
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there is a shear stress whic
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curvature R is found in t
centre of curvature is $d\theta$
along OA due to the ap
dislocation is $\tau_0 b\,dl$ from
along OA due to the li
$2T\sin(d\theta/2)$, which is eq
be in equilibrium in this c

$T d\theta = \tau_0 b\,dl$

i.e. $\tau_0 = \frac{T}{bR}$

Substituting for T from e

$\tau_0 = \frac{\alpha G b}{R}$

able for it results in a reduction in energy. If reaction is unfavourable and the dislocation with is liable to dissociate into the other two. If e is no energy change. These three conditions corre- le ϕ in Fig. 4.8 satisfying $\pi/2 < \phi \leq \pi$, $0 \leq \phi <$ spectively. In this argument the assumption is made litional interaction energy involved, i.e. that before on the reacting dislocations are separated sufficiently on energy is small. If this is not so, the reactions are l unfavourable, but the energy changes are smaller . Frank's rule is used to consider the feasibility of reactions in Chapters 5 and 6.

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(4.25)

of the slip plane. The external force due to τ acting τ, so that the work done when the element of slip

b (4.26)

a unit length of dislocation is defined as the work done dislocation moves unit distance. Therefore

$= \tau b$ (4.27)

ear stress in the glide plane resolved in the direction of ce F acts normal to the dislocation at every point

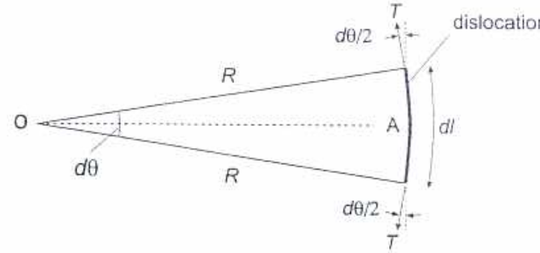

Figure 4.10 Curved element of dislocation under line tension forces T.

along its length, irrespective of the line direction. The positive sense of the force is given by the physical reasoning of section 3.3.

In addition to the force due to an externally applied stress, a dislocation has a *line tension* which is analogous to the surface tension of a soap bubble or a liquid. This arises because, as outlined in the previous section, the strain energy of a dislocation is proportional to its length and an increase in length results in an increase in energy. The line tension has units of energy per unit length. From the approximation used in equation (4.24), the line tension, which may be defined as *the increase in energy per unit increase in the length of a dislocation line*, will be

$$T = \alpha G b^2 \tag{4.28}$$

Consider the curved dislocation in Fig. 4.10. The line tension will produce forces tending to straighten the line and so reduce the total energy. The direction of the net force is perpendicular to the dislocation and towards the centre of curvature. The line will only remain curved if there is a shear stress which produces a force on the dislocation line in the opposite sense. The shear stress τ_0 needed to maintain a radius of curvature R is found in the following way. The angle subtended at the centre of curvature is $\mathrm{d}\theta = \mathrm{d}l/R$, assumed to be $\ll 1$. The outward force along OA due to the applied stress acting on the elementary piece of dislocation is $\tau_0 b \mathrm{d}l$ from equation (4.27), and the opposing inward force along OA due to the line tension T at the ends of the element is $2T\sin(\mathrm{d}\theta/2)$, which is equal to $T\mathrm{d}\theta$ for small values of $\mathrm{d}\theta$. The line will be in equilibrium in this curved position when

$$T\mathrm{d}\theta = \tau_0 b\, \mathrm{d}l$$
$$\text{i.e.} \quad \tau_0 = \frac{T}{bR} \tag{4.29}$$

Substituting for T from equation (4.28)

$$\tau_0 = \frac{\alpha G b}{R} \tag{4.30}$$

This gives an expression for the stress required to bend a dislocation to a radius R and is used many times in subsequent chapters. A particularly direct application is in the understanding of the Frank–Read dislocation multiplication source described in Chapter 8.

Equation (4.30) assumes from equation (4.24) that edge, screw and mixed segments have the same energy per unit length, and the curved dislocation of Fig. 4.10 is therefore the arc of a circle. This is only strictly valid if Poisson's ratio ν equals zero. In all other cases, the line experiences a torque tending to rotate it towards the screw orientation where its energy per unit length is lower. The true line tension of a mixed segment is

$$T = E_{el}(\theta) + \frac{d^2 E_{el}(\theta)}{d\theta^2} \tag{4.31}$$

where $E_{el}(\theta)$ is given by equation (4.23). T for a screw segment is four times that of an edge when $\nu = 1/3$. Thus, for a line bowing under a uniform stress, the radius of curvature at any point is still given by equation (4.29), but the overall line shape is approximately elliptical with major axis parallel to the Burgers vector; the axial ratio is approximately $1/(1-\nu)$. For most calculations, however, equation (4.30) is an adequate approximation.

4.6 Forces between Dislocations

A simple semi-qualitative argument will illustrate the significance of the concept of a force between dislocations. Consider two parallel edge dislocations lying in the same slip plane. They can either have the same sign as in Fig. 4.11(a) or opposite sign as in Fig. 4.11(b). When the dislocations are separated by a large distance the total elastic energy per unit length of the dislocations in both situations will be, from equation (4.24)

$$\alpha G b^2 + \alpha G b^2 \tag{4.32}$$

When the dislocations in Fig. 4.11(a) are very close together the arrangement can be considered approximately as a single dislocation with a Burgers vector magnitude $2b$ and the elastic energy will be given by

$$\alpha G (2b)^2 \tag{4.33}$$

which is twice the energy of the dislocations when they are separated by a large distance. Thus the dislocations will tend to repel each other to reduce their total elastic energy. When dislocations of opposite sign (Fig. 4.11(b)) are close together, the effective magnitude of their Burgers vectors will be zero, and the corresponding long-range elastic energy zero also. Thus dislocations of opposite sign will attract each other to reduce their total elastic energy. The positive and negative edge dislocations in Fig. 4.11(b) will combine and annihilate each other. These conclusions regarding repulsion and attraction also follow for dislocations of mixed orientation from Frank's rule by putting $\phi = 0$ or π in Fig. 4.8. Similar effects occur when the two dislocations do not lie in the same slip plane (Fig. 4.11(c)), but the conditions for attraction and repulsion are usually more complicated, as discussed below.

Figure 4.11 Arrang
lying in parallel slip
(b) unlike dislocation
slip planes separated

The basis of the
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first. Consider two
The total energy of
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energy between I a
displacing the face
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components of the
to either the x or
length of II are

$$E_{int} = + \int_x^\infty (b_x$$

$$E_{int} = - \int_y^\infty (b_x$$

where the stress co
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in the understanding of the Frank–Read dislocation
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tio ν equals zero. In all other cases, the line experi-
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h is lower. The true line tension of a mixed segment is

$$ {}_1(\theta) \quad (4.31) $$

n by equation (4.23). T for a screw segment is four
lge when $\nu = 1/3$. Thus, for a line bowing under
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tative argument will illustrate the significance of the
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$$ (4.32) $$

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idered approximately as a single dislocation with
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$$ (4.33) $$

nergy of the dislocations when they are separated by
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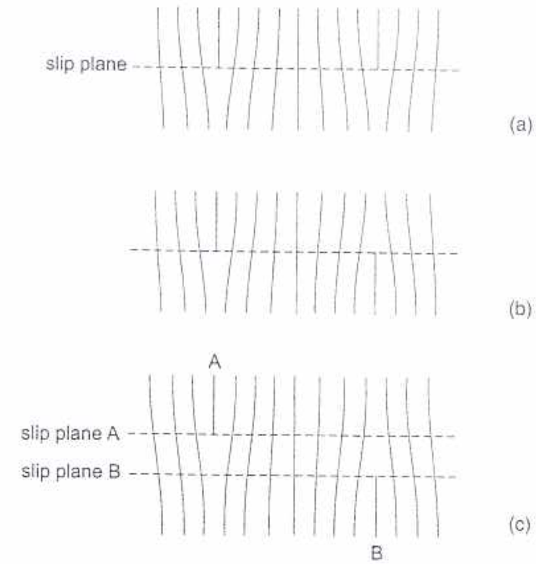


Figure 4.11 Arrangement of edge dislocations with parallel Burgers vectors lying in parallel slip planes. (a) Like dislocations on the same slip plane, (b) unlike dislocations on the same slip plane, and (c) unlike dislocations on slip planes separated by a few atomic spacings.

The basis of the method used to obtain the force between two dislocations is the determination of the additional work done in introducing the second dislocation into a crystal which already contains the first. Consider two dislocations lying parallel to the z-axis (Fig. 4.12). The total energy of the system consists of (a) the self-energy of dislocation I, (b) the self-energy of dislocation II, and (c) the elastic interaction energy between I and II. The *interaction energy* E_{int} is the work done in displacing the faces of the cut which creates II in the presence of the stress field of I. The displacements across the cut are b_x, b_y, b_z, the components of the Burgers vector **b** of II. By visualising the cut parallel to either the x or y axes, two alternative expressions for E_{int} per unit length of II are

$$
\begin{aligned}
E_{int} &= +\int_x^{\infty} (b_x\sigma_{xy} + b_y\sigma_{yy} + b_z\sigma_{zy})\mathrm{d}x \\
E_{int} &= -\int_y^{\infty} (b_x\sigma_{xx} + b_y\sigma_{yx} + b_z\sigma_{zx})\mathrm{d}y
\end{aligned}
\qquad (4.34)
$$

where the stress components are those due to I. (The signs of the right-hand side of these equations arise because if the displacements of **b** are taken to occur on the face of a cut with outward normal in the positive y and x directions, respectively, they are in the direction of positive x, y, z for the first case (x-axis cut) and negative x, y, z for the second (y-axis cut).)

The interaction force on II is obtained simply by differentiation of these expressions, i.e. $F_x = -\partial E_{\text{int}}/\partial x$ and $F_y = -\partial E_{\text{int}}/\partial y$. For the two parallel edge dislocations with parallel Burgers vectors shown in Fig. 4.12, $b_y = b_z = 0$ and $b_x = b$, and the components of the force per unit length acting on II are therefore

$$F_x = \sigma_{xy} b \quad F_y = -\sigma_{xx} b \tag{4.35}$$

where σ_{xy} and σ_{xx} are the stresses of I evaluated at position (x, y) of II. The forces are reversed if II is a negative edge i.e. the dislocations have opposite sign. Equal and opposite forces act on I. F_x is the force in the glide direction and F_y the force perpendicular to the glide plane. Substituting from equation (4.16) gives

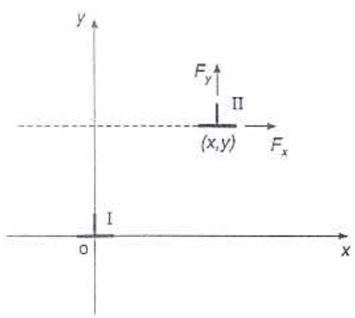


Figure 4.12 Forces considered for the interaction between two edge dislocations.

$$\begin{aligned} F_x &= \frac{Gb^2}{2\pi(1-\nu)} \frac{x(x^2 - y^2)}{(x^2 + y^2)^2} \\ F_y &= \frac{Gb^2}{2\pi(1-\nu)} \frac{y(3x^2 + y^2)}{(x^2 + y^2)^2} \end{aligned} \tag{4.36}$$

Since an edge dislocation can move by slip only in the plane contained by the dislocation line and its Burgers vector, the component of force which is most important in determining the behaviour of the dislocations in Fig. 4.12 is F_x. For dislocations of the same sign, inspection of the variation of F_x with x reveals the following:

F_x	*nature*	*x range*
negative	repulsive	$-\infty < x < -y$
positive	attractive	$-y < x < 0$
negative	attractive	$0 < x < y$
positive	repulsive	$y < x < \infty$

The sign and nature of F_x is reversed if I and II are edge dislocations of opposite sign. F_x is plotted against x, expressed in units of y, in Fig. 4.13. It is zero when $x = 0, \pm y, \pm\infty$, but of these, the positions of stable equilibrium are seen to be $x = 0, \pm\infty$ for edges of the same sign and $\pm y$ if they have the opposite sign.

It follows that an array of edge dislocations of the same sign is most stable when the dislocations lie vertically above one another as in Fig. 4.14(a). This is the arrangement of dislocations in a small angle pure tilt boundary described in Chapter 9. Furthermore, edge dislocations of opposite sign gliding past each other on parallel slip planes tend to form stable *dipole* pairs as in Fig. 4.14(b) at low applied stresses (section 10.8).

Comparison of the glide force F_x in equation (4.35) with F in equation (4.27) shows that since σ_{xy} is the shear stress in the glide plane of

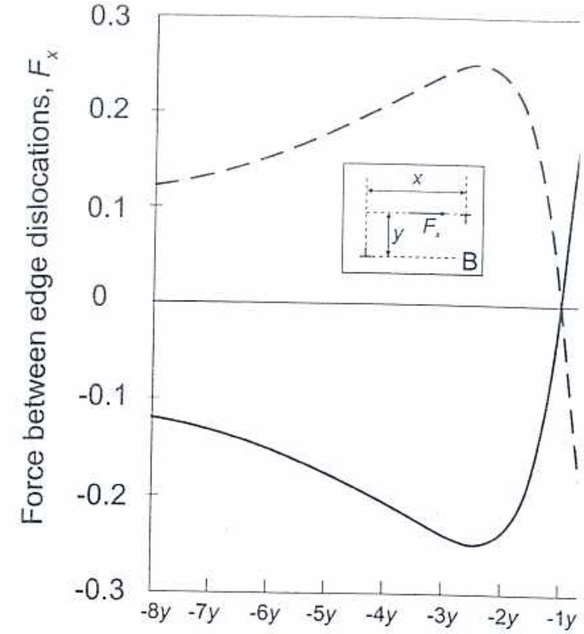
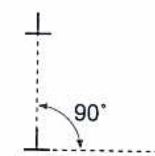


Figure 4.13 Glide force per unit length between parallel ed
equation (4.36). Unit of force F_x is $Gb^2/2\pi(1-\nu)y$. The full c
unlike dislocations.



(a)

Figure 4.14 Stable posi
(b) opposite sign.

dislocation II acting in
holds for both externa
Consider two paral
The radial and tangen

$F_r = \sigma_{z\theta} b \quad F_\theta = \sigma_{zr}$

and substituting from

$F_r = Gb^2/2\pi r \quad F_\theta =$

The force is much sim
tions because of the ra

[illegible] is obtained simply by differentiation of [illegible] $\partial E_{int}/\partial x$ and $F_y = -\partial E_{int}/\partial y$. For the two [illegible] parallel Burgers vectors shown in Fig. [illegible] and the components of the force per unit [illegible]

[illegible] (4.35)

[illegible] stresses of I evaluated at position (x, y) of II. [illegible] is a negative edge i.e. the dislocations have [illegible] opposite forces act on I. F_x is the force in the [illegible] force perpendicular to the glide plane. Sub- [illegible] (4.16) gives

[illegible] (4.36)

[illegible] can move by slip only in the plane contained [illegible] and its Burgers vector, the component of force [illegible] in determining the behaviour of the disloca- [illegible] F_x. For dislocations of the same sign, inspection of [illegible] with x reveals the following:

x range
[illegible]
[illegible]
[illegible]
[illegible]

[illegible] of F_x is reversed if I and II are edge dislocations of [illegible] plotted against x expressed in units of y, in Fig. 4.13. [illegible] 0, ±y, ±x, but of these, the positions of stable [illegible] to be x = 0, ±x for edges of the same sign and ±y [illegible] opposite sign.

[illegible] array of edge dislocations of the same sign is most [illegible] dislocations lie vertically above one another as in Fig. [illegible] arrangement of dislocations in a small angle pure [illegible] in Chapter 9. Furthermore, edge dislocations [illegible] past each other on parallel slip planes tend to [illegible] pairs as in Fig. 4.14(b) at low applied stresses

[illegible] glide force F_x in equation (4.35) with F in equation [illegible] is the shear stress in the glide plane of

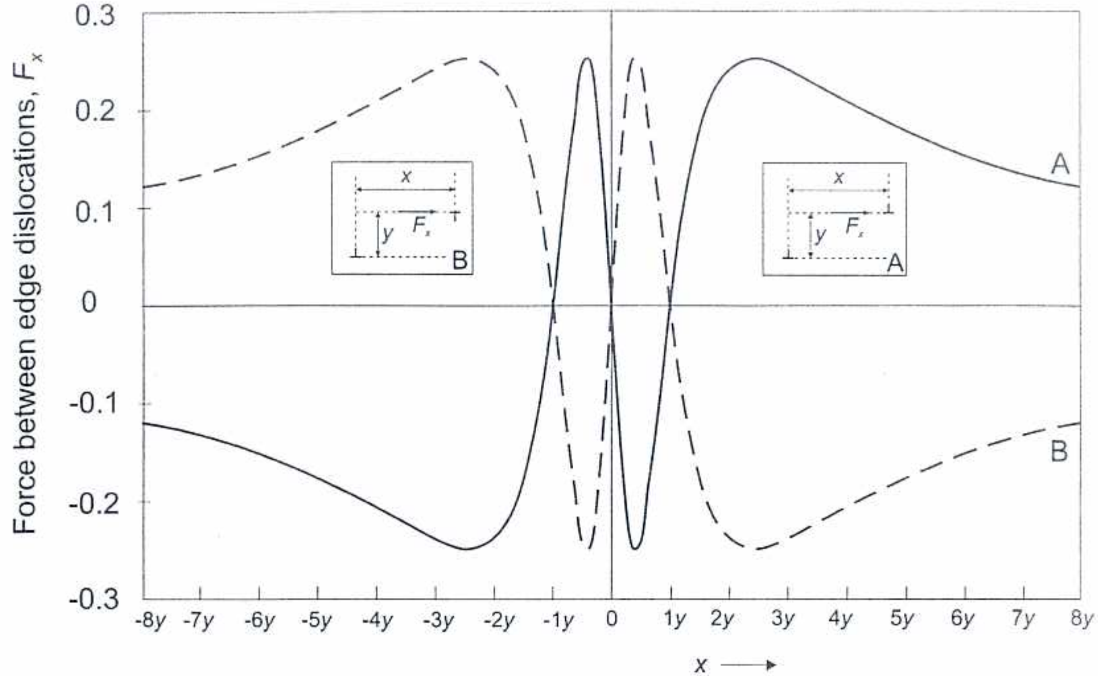


Figure 4.13 Glide force per unit length between parallel edge dislocations with parallel Burgers vectors from equation (4.36). Unit of force F_x is $Gb^2/2\pi(1-\nu)y$. The full curve A is for like dislocations and the broken curve B for unlike dislocations.

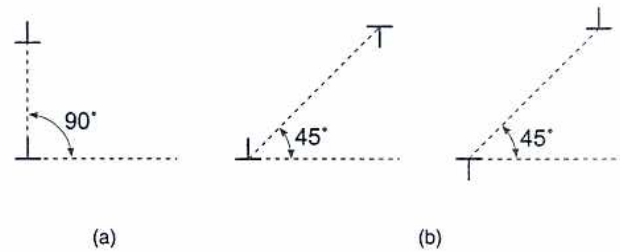


Figure 4.14 Stable positions for two edge dislocations of (a) the same sign and (b) opposite sign.

dislocation II acting in the direction of its Burgers vector, *equation* (4.27) *holds for both external and internal sources of stress.*

Consider two parallel screw dislocations, one lying along the z-axis. The radial and tangential components of force on the other are

$$F_r = \sigma_{z\theta} b \quad F_\theta = \sigma_{zr} b \tag{4.37}$$

and substituting from equations (4.15)

$$F_r = Gb^2/2\pi r \quad F_\theta = 0 \tag{4.38}$$

The force is much simpler in form than that between two edge dislocations because of the radial symmetry of the screw field. F_r is repulsive for screws of the same sign and attractive for screws of opposite sign. It is readily shown from either equations (4.35) or equations (4.37) that no forces act between a pair of parallel dislocations consisting of a pure edge and a pure screw, as expected from the lack of mixing of their stress fields (see section 4.3).

4.7 Climb Forces

The force component F_y in equation (4.35) is a *climb force* per unit length resulting from the normal stress σ_{xx} of dislocation I attempting to squeeze the extra half-plane of II from the crystal. This can only occur physically if intrinsic point defects can be emitted or absorbed at the dislocation core of II (see section 3.6). As in the case of glide forces, climb forces can arise from external and internal sources of stress. The former are important in *creep*, and the latter provided the example of conservative climb in section 3.8. Line tension can also produce climb forces, but in this case the force acts to reduce the line length in the extra half-plane: shrinkage of prismatic loops as in Fig. 3.19 is an example. However, since the creation and annihilation of point defects are involved in climb, *chemical forces* due to defect concentration changes must be taken into account in addition to these *mechanical forces*.

It was seen in section 3.6 that when an element **l** of dislocation is displaced through **s**, the local volume change is $\mathbf{b} \times \mathbf{l} \cdot \mathbf{s}$. Consider a segment length l of a positive edge dislocation climbing upwards through distance s in response to a mechanical climb force F per unit length. The work done is Fls and the number of vacancies absorbed is bls/Ω, where Ω is the volume per atom. The vacancy formation energy is therefore changed by $F\Omega/b$. As a result of this chemical potential, the equilibrium vacancy concentration at temperature T in the presence of the dislocation is reduced to

$$\begin{aligned} c &= \exp[-(E_f^v + F\Omega/b)/kT] \\ &= c_0 \exp(-F\Omega/bkT) \end{aligned} \tag{4.39}$$

where c_0 is the equilibrium concentration in a stress-free crystal (equation (1.3)). For negative climb involving vacancy emission ($F < 0$) the sign of the chemical potential is changed so that $c > c_0$. Thus, the vacancy concentration deviates from c_0, building up a chemical force per unit length on the line

$$f = \frac{bkT}{\Omega} \ln(c/c_0) \tag{4.40}$$

until f balances F in equilibrium. Conversely, in the presence of a supersaturation c/c_0 of vacancies, the dislocation climbs up under the chemical force f until compensated by, say, external stresses or line tension. The latter is used in the analysis for a dislocation climb source in section 8.7. The nature of these forces is illustrated schematically in Fig. 4.15. By substituting reasonable values of T and Ω in equation (4.40), it is easy

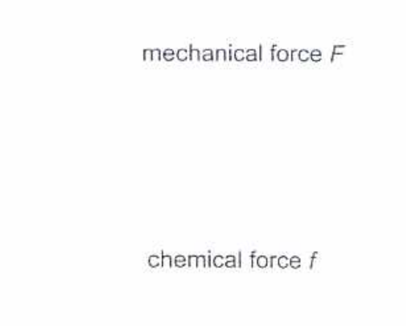


Figure 4.15 Mechanical and che
Vacancies (shown as □) have a l
equilibrium concentration c_0 in a

to show that even moderate s
forces much greater than thos

The rate of climb of a di
direction and magnitude of th
(b) the mobility of jogs (sect
vacancies through the lattice

4.8 Image Forces

A dislocation near a surface
bulk of a crystal. The disloc
because the material is effecti
tion energy is lower: conversel
treat this mathematically, extr
stress components given in se
conditions are satisfied. When
tions (4.35) and (4.37), they
straight dislocation lines parall

Consider screw and edge di
a surface $x = 0$ (Fig. 4.16): th
the x direction. For a *free* sur
zero on the plane $x = 0$. Co
these boundary conditions a
result is modified by adding
dislocation of opposite sign
solution for the stress in the

$$\sigma_{zx} = \frac{-Ay}{(x_-^2 + y^2)} + \frac{Ay}{(x_+^2 + y^2 ...}$$

$$\sigma_{zy} = \frac{Ax_-}{(x_-^2 + y^2)} - \frac{Ax_+}{(x_+^2 + y^2 ...}$$

sign and attractive for screws of opposite sign. It is
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$$/bkT)$$

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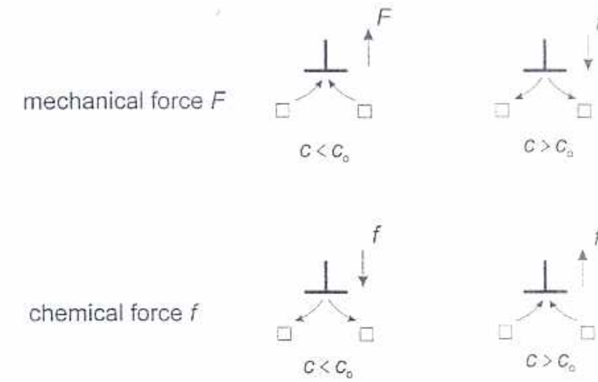

Figure 4.15 Mechanical and chemical forces for climb of an edge dislocation. Vacancies (shown as □) have a local concentration c in comparison with the equilibrium concentration c_0 in a dislocation-free crystal.

to show that even moderate supersaturations of vacancies can produce forces much greater than those arising from external stresses.

The rate of climb of a dislocation in practice depends on (a) the direction and magnitude of the mechanical and chemical forces, F and f, (b) the mobility of jogs (section 3.6) and (c) the rate of migration of vacancies through the lattice to or from the dislocation.

4.8 Image Forces

A dislocation near a surface experiences forces not encountered in the bulk of a crystal. The dislocation is attracted towards a free surface because the material is effectively more compliant there and the dislocation energy is lower: conversely, it is repelled by a rigid surface layer. To treat this mathematically, extra terms must be added to the infinite-body stress components given in section 4.3 in order that the required surface conditions are satisfied. When evaluated at the dislocation line, as in equations (4.35) and (4.37), they result in a force. The analysis for infinite, straight dislocation lines parallel to the surface is relatively straight-forward.

Consider screw and edge dislocations parallel to, and distance d from, a surface $x = 0$ (Fig. 4.16): the edge dislocation has Burgers vector **b** in the x direction. For a *free* surface, the tractions σ_{xx}, σ_{yx} and σ_{zx} must be zero on the plane $x = 0$. Consideration of equation (4.13) shows that these boundary conditions are met for the screw if the infinite-body result is modified by adding to it the stress field of an imaginary screw dislocation of opposite sign at $x = -d$ (Fig. 4.16(a)). The required solution for the stress in the body ($x > 0$) is therefore

$$\sigma_{zx} = \frac{-Ay}{(x_-^2 + y^2)} + \frac{Ay}{(x_+^2 + y^2)} \qquad (4.41)$$

$$\sigma_{zy} = \frac{Ax_-}{(x_-^2 + y^2)} - \frac{Ax_+}{(x_+^2 + y^2)} \qquad (4.42)$$

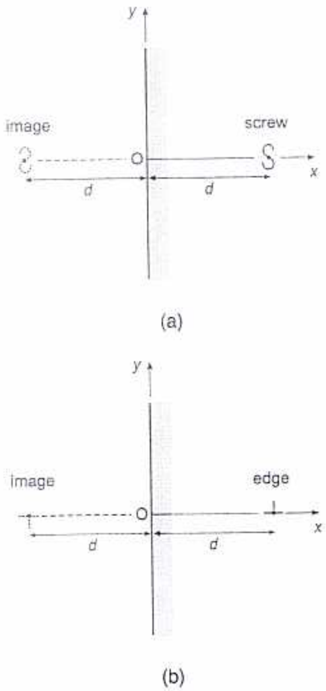

Figure 4.16 (a) Screw and (b) edge dislocations a distance d from a surface $x = 0$. The image dislocations are in space a distance d from the surface.

where $x_- = (x - d)$, $x_+ = (x + d)$ and $A = Gb/2\pi$. The force per unit length in the x-direction $F_x(=\sigma_{zy}b)$ induced by the surface is obtained from the second term in σ_{zy} evaluated at $x = d$, $y = 0$. It is

$$F_x = -Gb^2/4\pi d \tag{4.43}$$

and is simply the force due to the *image* dislocation at $x = -d$. For the edge dislocation (Fig. 4.16(b)), superposing the field of an imaginary edge dislocation of opposite sign at $x = -d$ annuls the stress σ_{xx} on $x = 0$, but not σ_{yx}. When the extra terms are included to fully match the boundary conditions, the shear stress in the body is found to be

$$\sigma_{yx} = \frac{Dx_-(x_-^2 - y^2)}{(x_-^2 + y^2)^2} - \frac{Dx_+(x_+^2 - y^2)}{(x_+^2 + y^2)^2} - \frac{2Dd[x_- x_+^3 - 6xx_+y^2 + y^4]}{(x_+^2 + y^2)^3} \tag{4.44}$$

where $D = A/(1 - \nu)$. The first term is the stress in the absence of the surface, the second is the stress appropriate to an image dislocation at $x = -d$, and the third is that required to make $\sigma_{yx} = 0$ when $x = 0$. The force per unit length $F_x(=\sigma_{yx}b)$ arising from the surface is given by putting $x = d$, $y = 0$ in the second and third terms. The latter contributes zero, so that the force is

$$F_x = -Gb^2/4\pi(1 - \nu)d \tag{4.45}$$

and is again equivalent to the force due to the image dislocation.

The *image forces* decrease slowly with increasing d and are capable of removing dislocations from near-surface regions. They are important, for example, in specimens for transmission electron microscopy (section 2.4) when the slip planes are orientated at large angles ($\simeq 90°$) to the surface. It should be noted that a second dislocation near the surface would experience a force due to its own image *and* the surface terms in the field of the first. The interaction of dipoles, loops and curved dislocations with surfaces is therefore complicated, and only given approximately by images.

Further Reading

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