Measurement Errors

Precision of an experiment is a measure of the reliability of the experiment (how well it can be reproduced.

Accuracy refers to the agreement between a measurement and the true or correct value

I measured time to a precision of 0.1s. The time I measured was 0.2s. What is the relative uncertainty?

$$0.1/0.2=0.5$$

If you need the % then

$$0.5x100=50\%$$

Rule 1:

If a measured quantity is multiplied or divided by a constant then the absolute uncertainty is multiplied or divided by the same constant. (In other words the relative uncertainty stays the same.)

- Suppose that you want to find the average thickness of a page of a book. We might find that 100 pages of the book have a total thickness of 9mm. If this measurement is made using an instrument having a precision of 0·1mm, we can write

thickness of 100 pages, $T = 9{\cdot}0\text{mm} \pm 0{\cdot}1\text{mm}$

and, the average thickness of one page, t, is obviously given by
$t = T/100$

therefore our result can be stated as $t = 9/100\text{mm} \pm 0{\cdot}1/100\text{mm}$
or $t = 0{\cdot}090\text{mm} \pm 0{\cdot}001\text{mm}$

Rule 2: If two measured quantities are added or subtracted then their absolute uncertainties are added.

- To find a change in temperature, DT, we find an initial temperature, T1, a final temperature, T2, and then use DT = T2 - T1
- If T1 is found to be 20°C and if T2 is found to be 40°C then DT= 20°C.
- But if the temperatures were measured to a precision of ±1°C then we must remember that
- 19°C < T1 < 21°C and 39°C < T2 < 41°C
- The smallest difference between the two temperatures is therefore (39 - 21) = 18°C and the biggest difference between them is (41 - 19) = 22°C
- This means that
- 18°C < DT < 22°C
- In other words
- DT = 20°C ± 2°C

Rule 3: If two (or more) measured quantities are multiplied or divided then their relative uncertainties are added.

- To measure a surface area, S, we measure two dimensions, say, x and y, and then use
- $S = xy$
- Using a ruler marked in mm, we measure $x = 50mm \pm 1mm$ and $y = 80mm \pm 1mm$
- This means that the area could be anywhere between
- $(49 \times 79)mm^2$ and $(51 \times 81)mm^2$
- that is
- $3871mm^2 < S < 4131mm^2$
- To state our answer we now choose the number half-way between these two extremes and for the indeterminacy we take half of the difference between them.
- Therefore, we have
- so .... $S = 4000mm^2 \pm 130mm^2$
- (well…actually $4001mm^2$ but the "1" is irrelevant when the uncertainty is $130mm^2$).
- Now, let's look at the relative uncertainties.
- Relative uncertainty in x is 1/50 or 0·02mm.
- Relative uncertainty in y is 1/80 or 0·0125mm. So, if the theory is correct, the relative uncertainty in the final result should be (0·02 + 0·0125) = 0·0325.
- Check
- Relative uncertainty in final result for S is 130/4000 = 0·0325

Rule 4: If a measured quantity is raised to a power then the relative uncertainty is multiplied by that power. (If you think about this rule, you will realise that it is just a special case of rule 3.)

- To find the volume of a sphere. We then use the formula: V = (4/3)pr3
- Suppose that the diameter of a sphere is measured (using an instrument having a precision of ±0×1mm) and found to be 50mm.
- Diameter = 50·0mm ± 0·1mm
- so, ... r = 25·0mm ± 0·05mm
- This means that V could be between
- (4/3)p(24·95)3 and (4/3)p(25·05)3
- so ...... 65058mm3 < V < 65843mm3
- As in the previous example we now state the final result as
- V = 65451mm3 ± 393mm3
- Check
- Relative uncertainty in r is 0×05/25 = 0·002
- Relative uncertainty in V is 393/65451 = 0·006 so, again the theory is verified