

## Motion Notes

### Straight Line Motion

When describing the motion of an object in one dimension there are some important terms we need to know.

1. Distance and displacement
2. Speed and velocity
3. Acceleration

The distance travelled by an object is just how far the object moved regardless of the direction the movement. For example a person walks 10m north then 25m south the distance the person travelled is  $10 + 25 = 35\text{m}$

The displacement (s) on the other hand **does** depend on the direction of the motion. If we use the above example then the displacement of the person is  $10 + -25 = -15\text{m}$

1. a) Find the distance travelled if a person walks from home 350m west to post a letter then stops at a friend's house 215m east of the post box.

$$565\text{m}$$

- b) How far is the person from home?

$$350 - 215 = 135\text{m}$$

Speed is the measure of the rate of change in distance. This means that speed indicates how quickly you are moving regardless of the direction travelled. Using the example from distance, if the person took 10 s to walk the 35m the speed he travelled was 3.5 m/s

Velocity ( $v$ ) is the rate of change in the displacement of an object. This means that like speed, it indicates how quickly you are moving, but it has a direction involved. If you use the displacement example from above, then if it takes 10s to travel the journey, the velocity is:

$$v = \frac{s}{t}$$

$$v = \frac{-15}{10}$$

$$v = -1.5 \text{ ms}^{-1}$$

2. If it took the person 1 min to make the journey from question 1 find:

a) The average speed

$$v = \frac{s}{t} = \frac{565}{60} = 9.416 \text{ ms}^{-1}$$

b) The average velocity.

$$v = \frac{s}{t} = \frac{135}{60} = 2.25 \text{ ms}^{-1}$$

Note that I used average speed and velocity for the above question. This is due to the fact that the speed/velocity is measured over time and therefore must be the average for that time. We sometimes use what is known as 'instantaneous' speed/velocity but this is still an average but over a very short time.

Acceleration ( $a$ ) is a measure of the rate of change in velocity. It too is a vector as it has a direction and a magnitude, negative accelerations mean a decrease in velocity. It can be found by  $a = \frac{v-u}{t}$  where  $u$  is the initial velocity and  $v$  is the final velocity. For example we want to find the acceleration of a car that travels from 0 to  $30 \text{ ms}^{-1}$  in 3 s.

$$a = \frac{v-u}{t}$$

$$a = \frac{30-0}{3}$$

$$a = 10 \text{ ms}^{-2}$$

Note that the unit for acceleration is meters per second per second ( $\text{ms}^{-2}$ ).

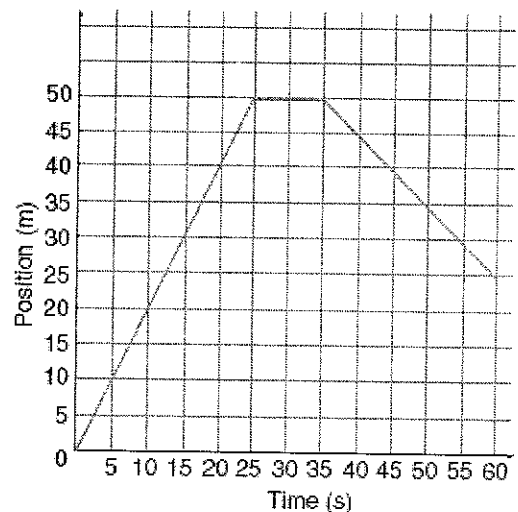
3. A cheetah running at  $30 \text{ m s}^{-1}$  slows down as it approaches a stream. Within 3 s, its speed has reduced to  $9 \text{ m s}^{-1}$ . Calculate the average acceleration of the cheetah.

$$\begin{aligned} a &= \frac{v-u}{t} \\ &= \frac{9-30}{3} \\ &= -7 \text{ ms}^{-2} \end{aligned}$$

### Graphing Motion

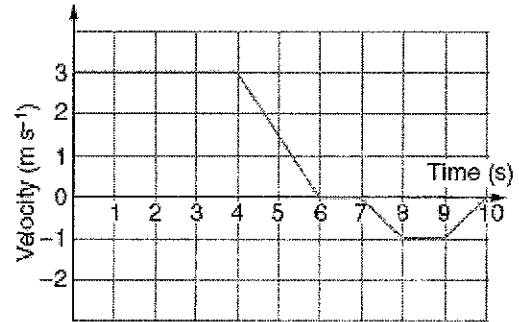
At times, the motion of an object travelling even in a straight line can be complicated. The object may travel forwards or backwards, speed up or slow down, or even stop. Where the motion remains in one dimension, the information can be presented in graphical form.

A position-time graph indicates the position of an object at any time for motion that occurs over an extended time. These graphs can also provide additional information. The slope of the graph shows the velocity of the object.



The steeper the slope the faster the object is moving. If the object is accelerating then the graph will curve from this we can get the acceleration.

A graph of *velocity against time* shows how the velocity of an object changes with time. This type of graph is useful for analysing the motion of an object moving in a complex manner, for example a ball bouncing up and down. The slope of a  $v-t$  graph shows the acceleration of the object. The area under the graph shows the displacement of the object in the given time.



### Motion under constant acceleration

Often the motion of an object is complex but there is one type of 1d motion and that is when the motion is under constant acceleration. The main version of this is when the object is under free fall. The acceleration used is the one due to gravity which equals  $9.8 \text{ ms}^{-1}$ .

There are three formulas for this. The first is the simplest and is used when we have the only speeds, acceleration, and time. It starts with  $a = \frac{v-u}{t}$  and then it can be rearranged to be written as  $v = u + at$ .

4. Find the final velocity of a ball falling for 5 second if it falls of a building.

$$\begin{aligned}
 v &= u + at \\
 &= 0 + 9.8 \times 5 \\
 &= 49 \text{ ms}^{-1}
 \end{aligned}$$

The second one is used for problems when we have to find the distance travelled and no final velocity. It is written as  $s = ut + \frac{1}{2}at^2$ .

5. Find the height of the building from question 4.

$$\begin{aligned} & \sqrt{2ut + \frac{1}{2}at^2} \\ & = 0 + \frac{1}{2} \times 9.8 \times 5^2 \\ & = 122.5 \text{ m} \end{aligned}$$

The third one is used when we don't have the time taken but we have information about the other variables. This is written as  $v^2 = u^2 + 2as$ .

6. How tall is a cliff if a person starts at rest and lands at  $23\text{ms}^{-1}$ .

$$\begin{aligned} v^2 &= u^2 + 2as \\ 23^2 &= 0^2 + 2 \times 9.8 \times s \\ 23^2 &= 19.6s \\ s &= \frac{23^2}{19.6} \\ &= 29.99 \text{ m} \\ &= 30 \text{ m} \end{aligned}$$

### Newton's Laws

Sir Isaac Newton is often known as the father of classical physics (physics pre 20<sup>th</sup> century). His three laws of motion are the basis for a wide variety of physics today.

Newton's first law is called the law of inertia. It states that:

A body will either remain at rest or continue with constant speed in a straight line (i.e. constant velocity) unless it is acted on by a net force.

This means that an object will keep doing what it's doing until acted upon by an external force.

7. A rocket is flying in deep space at  $35\text{ms}^{-1}$ . Three weeks later it starts entering the gravitational field of a nearby galaxy and changes direction. What is the velocity of the rocket 1 week before it enters the galaxy. Why?

$35\text{ms}^{-1}$  because there is no external force acting on the rocket

Newton's second law is the law where most of the work in physics classes comes from. It states that;

The acceleration of a body,  $a$ , is directly proportional to the net force acting on it,  $\Sigma F$ , and indirectly proportional to its mass,  $m$ .

This is more commonly known as  $\Sigma F = ma$

This means that the net force on an object is equal to the mass of that object times the acceleration that force is causing the velocity of the object to change.

8. If a 80 kg person is in free fall, find the size of the force of gravity acting on that person.

$$\begin{aligned} F &= ma \\ &= 80 \times 9.8 \\ &= 784 \text{ N} \end{aligned}$$

The first two laws of motion talk about changes in an objects motion. They mention 'net force' in them. A book sitting on a desk is not moving therefore is at rest. However this does not mean that there are no forces acting on it. Gravity is still acting on the book but the book isn't moving. This is because the desk is pushing up on the book with the same amount of force as gravity. This means that the net force is 0 and there is no movement.

9. With how much force does the ground need to push on a 1000 kg shipping container?

$$\begin{aligned}
 f &= ma \\
 &= 1000 \times 9.8 \\
 &= 9800 \text{ N}
 \end{aligned}$$

Newton realised that all forces exist in pairs, and that each force in the pair acts on a different body. This forms the basis of his third law of motion which states that;

For every action force (object A on B), there is an equal and opposite reaction force (object B on A):

$$F(\text{A on B}) = -F(\text{B on A})$$

This is commonly known as 'for every action there is an equal and opposite reaction.' This means that if you push on a wall with a certain force, the wall pushes back on you with the same force.

10. A person wearing roller skates pushes on a wall with a force of 560 N. If the person weighs 60kg, find the acceleration of the person.

$$\begin{aligned}
 f_{\text{wall}} &= f_{\text{person}} \\
 \cancel{560} &= ma \\
 560 &= 60a \\
 \frac{560}{60} &= a \\
 a &= 9.3 \text{ m s}^{-2}
 \end{aligned}$$

### Momentum

Consider a collision between two footballers on the football field. From

Newton's second law, each force can be expressed as:

$$F_{net} = ma$$

and using the relationship for acceleration:

$$a = \frac{v-u}{\Delta t}, F = \frac{m(v-u)}{\Delta t}$$

We can simplify this to say that  $F\Delta t = ma$

This formula relates to two important ideas

- One is that  $Ft$  is known as impulse which is related to collisions and large forces over small time.
- And the idea of momentum ( $\rho$ ) which is the product of the mass of an object and it's velocity  $\rho = mv$

Momentum can be thought of as the tendency of an object to keep on moving with the same speed in the same direction. It is a property of any moving object and its size depends solely on that object.

Momentum has one very important property. In collisions it needs to be conserved, that is the total momentum of the objects involved (system) needs to be the same as the final momentum of the system. This is known as The Law of Conservation of Momentum and states that;

In any collision or interaction between two or more objects in an isolated system, the total momentum of the system will remain constant; that is, the total initial momentum will equal the total final momentum:

$$\Sigma \rho_i = \Sigma \rho_f$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

11. Find the momentum of a ball that is 2kg in mass and travelling at 3.6 ms<sup>-1</sup>

$$\begin{aligned} \rho &= mv \\ &= 2 \times 3.6 \\ &= 7.2 \text{ kg ms}^{-1} \end{aligned}$$



12. A 1500 kg car collides with a 1200 kg car so that they are travelling  $3 \text{ ms}^{-1}$  in the direction of the larger car. Find the initial velocity of the small car if the large car was moving at  $15 \text{ ms}^{-1}$  before the collision.

$P = mv$      $P_{\text{before}} = P_{\text{after}}$

$$m_L v_L + m_S v_S = (m_L + m_S) v_F$$

$$1500 v_L + 1200 \times 15 = (1500 + 1200) \times 3$$

$$1500 v_L + -18000 = 8100$$

$$1500 v_L = 26100 \quad v_L = 17.4 \text{ ms}^{-1}$$

$m_L = 1500$   
 $m_S = 1200$   
 $v_L = ?$   
 $v_S = 15 \text{ ms}^{-1}$   
 $v_F = 3 \text{ ms}^{-1}$

### Work

The word work has many meanings in everyday usage. In physics the word work means a force acting on an object to make it move over a distance. Work is a scalar quantity even though both force and displacement are vectors. In reality the work done is a measure of the energy used to carry out the work. It can be written as:

$$W = Fs$$

Where  $W$  = Work done (J),  $F$  = force (N), and  $s$  = displacement (m)

The SI unit for work is called the Joule (J):

$$1 \text{ J} = 1 \text{ N} \cdot \text{m}$$

This formula is used only when the force acting is in the same direction as the displacement of the object.

An example of this is if you push a box with a force of 10 N over a distance of 1 m over a frictionless surface. The work done is:

$$\begin{aligned}
 W &= Fs \\
 &= 10 \times 1 \\
 &= 10 \text{ J of work}
 \end{aligned}$$

13. What is the work done by a force pushing a 30kg box along a frictionless surface with an acceleration of  $3 \text{ ms}^{-2}$  and over a distance of 435cm?

$$W = Fs$$

$$F = ma$$

$$\begin{aligned} W &= mas \\ &= 30 \times 3 \times 4.35 \\ &= 39.15 \text{ J} \end{aligned}$$

14. What is the work done by a 60kg person falling for 15m?

$$W = mgs$$

$$= 60 \times 9.8 \times 15$$

$$= 8820 \text{ J}$$

### Energy

Energy is a concept, which has no concrete basis. Energy can be defined as the ability to do work. Energy is a quantity measured in Joules and is a scalar quantity.

Physics has many laws known as conservation laws. Energy is one thing that has its own conservation law. The conservation of energy states that energy cannot be created or destroyed but only transformed from one form to another. We can describe all changes in the physical world by tracing the flow of energy as it changes from one form to another, or from one place to another. This law is studied in a large range of subjects, from motorcar collisions to engines to quantum physics.

One of the most commonly experienced forms of energy is the energy of moving objects or Kinetic Energy. The faster an object moves the more kinetic energy the object has. It has been found experimentally that the amount of kinetic energy is found by:

$$E_k = \frac{1}{2}mv^2$$

Where  $E_k$  is the kinetic energy (J),  $m$  is the mass (kg), and  $v$  is velocity ( $\text{ms}^{-1}$ ).

15. a) What is the kinetic energy of the person falling in question 6 at the start of the persons fall.

0 because  $v=0$

b) What is the kinetic energy of the person at the end of their fall?

Same as the work done  
8820J

As stated earlier, energy cannot be created or destroyed only transformed from one form to another. There are times when it is difficult to see where the energy is. To place an object on a high shelf requires energy, however where does that energy go when the object's at rest. The energy given to the object gets turned into Potential energy. In this case the potential energy is due to gravity, therefore it is known as gravitational potential energy. The amount of gravitational potential energy an object has can be found by:

$$E_{gp} = mgh$$

Where  $E_{gp}$  is the gravitational potential energy (J),  $m$  is mass (kg),  $g$  is acceleration due to gravity ( $\text{ms}^{-2}$ ), and  $h$  is height (m).

16. Calculate the gravitational potential energy of:

a) A skier with a mass of 70 kg at the top of a chair lift. The vertical height of the lift is 80 m.

$$\begin{aligned} E_p &= mgh \\ &= 70 \times 9.8 \times 80 \\ &= 54880 \text{ J} \\ &= 5.488 \times 10^4 \text{ J} \end{aligned}$$

b) Rosa, 50 kg, on the 1 m diving springboard

$$\begin{aligned} E_p &= mgh \\ &= 50 \times 9.8 \times 1 \\ &= 490 \text{ J} \\ &= 4.9 \times 10^2 \text{ J} \end{aligned}$$

c) Rosa on the 10m diving tower

$$\begin{aligned} E_p &= mgh \\ &= 50 \times 9.8 \times 10 \\ &= 4900 \text{ J} \\ &= 4.9 \times 10^3 \text{ J} \end{aligned}$$

Consider a person weighing 50kg jumping off the ground. The height above the ground this person was able to achieve was 50cm and he left the ground at a speed of:

$$v^2 = u^2 + 2as$$

$$0 = u^2 + 2 \times -9.8 \times .5$$

$$9.8 = u^2$$

$$u = 3.13 \text{ ms}^{-1}$$

The amount of kinetic energy the person has when he leaves the ground can be found by:

$$E_k = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times 50 \times 9.8$$

$$= 245 \text{ J}$$

The amount of gravitational potential energy can be found when the person is at the top of their jump:

$$E_{gp} = mgh$$

$$= 50 \times 9.8 \times 0.5$$

$$= 245 \text{ J}$$

You will notice that the kinetic energy that person has at the start is equal to the amount of gravitational potential energy the person has at the top of their jump. This means that the amount of kinetic energy that the person loses is equal to the gravitational potential energy he gains. It is also possible to show that the total amount of energy in the system (the person jumping) is the same at all time during the event (see attached diagram). Due to this it is possible to calculate the energy of the system at all times and the amount of potential and kinetic energy at any time.

17. a) Find the amount of energy a 1400kg car has at the top of a 10-story building. 1 story is about 4m high.

$$E_p = mgh$$

$$E_p = 1400 \times 9.8 \times 40$$

$$h = 10 \times 4$$

$$= 40 \text{ m}$$

$$= 548800 \text{ J}$$

$$= 5.488 \times 10^5 \text{ J}$$

b) What is the amount of energy the car has just before it hits the ground after falling from the building.

$$E_T = E_p \text{ at top} \\ = 8.488 \times 10^5 \text{ J}$$

c) At what height is the car off the ground when the car is going  $19 \text{ ms}^{-1}$ ?

$$E_T = E_K + E_p \\ 8.488 \times 10^5 = \frac{1}{2}mv^2 + mgh$$

$$296100 = 13720h \\ h = 21.6 \text{ m}$$

$$8.488 \times 10^5 = 700 \times 19^2 + 1400 \times 9.8h \\ 8.488 \times 10^5 = 25700 + 1400 \times 9.8h$$

Power is a term used all the time to talk about many things used everyday. Power can be defined as the rate of doing work. It can also be defined as the rate at which energy is used. The formulas for power are:

$$P = \frac{W}{t}$$

Where P is power, W is work (J), and t is time in seconds, and;

$$P = \frac{\Delta E}{\Delta t}$$

Where  $\Delta E$  is the change in energy (J), and  $\Delta t$  is the change in time (s).

The SI unit for power is J/s or the watt (W).  $1 \text{ W} = 1 \text{ J/s}$ .

Power has many other units that are not standard. The most common is the horsepower which is defined as  $1 \text{ hp} = 746 \text{ W}$ .

It is possible to find the amount of force applied to an object when you are only given the power applied to the object. An example is:

A car engine can produce 200 kW of power. If the car is travelling at an average velocity of  $20 \text{ ms}^{-1}$ , what is the force applied by the engine.

$$P = \Delta E / \Delta t$$

$$P = Fs/t$$

$$P = Fv$$

$$200000 = F \times 20$$

$$200000/20 = F$$

$$F = 10000 \text{ N}$$

$$= 10 \text{ kN}$$

18. Calculate the power you develop when you;

a) Do 5.00 kJ of work in moving a desk across your room in 12.0 seconds?

$$P = \frac{W}{t}$$

$$= \frac{5000}{12}$$

$$P = 416.6 \text{ W}$$

b) Lift a 2.20 kg dumbbell from the ground to 2.30 m into the air above your head in 0.8 s?

$$P = \frac{Fs}{t}$$

$$= \frac{mgs}{t}$$

$$= \frac{2.2 \times 9.8 \times 2.3}{0.8}$$

$$P = 61.985 \text{ W}$$





19. If the engine is in the previous example about the car is only 15% efficient, what is the actual force being applied by the engine.

$$\text{eff} = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100$$

$$P_{\text{in}} = 200 \text{ kW}^?$$

$$\text{eff} = 15$$

$$P_{\text{out}} = ?$$

$$15 = \frac{P_{\text{out}}}{200 \times 10^3}$$

$$P_{\text{out}} = 15 \times 200 \times 10^3$$

$$= 30000 \text{ W}$$

$$V = 20 \text{ m s}^{-1}$$

$$P = FV$$

$$F = \frac{P}{V}$$

$$= \frac{30000}{20}$$

$$= 1500 \text{ N.}$$