Mathematics Specialist Units 1 and 2 Formula Sheet

Trigonometry

$\sec\theta = \frac{1}{\cos\theta}, \ \cos\theta \neq 0$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}, \ \sin\theta \neq 0$

$\cot\theta = \frac{1}{\tan\theta}, \ \tan\theta \neq 0$

$\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin 2A = 2 \sin A \cos A$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A \cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A \sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

In any triangle ABC

Sine rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

Vectors

Magnitude

$|\mathbf{a}| = |(a_1, a_2)| = \sqrt{a_1^2 + a_2^2}$

Unit vector

$\hat{\mathbf{a}} = \frac{1}{|\mathbf{a}|} \cdot \mathbf{a}$

Scalar product

$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \cos\theta = a_1 b_1 + a_2 b_2$

Vector projection of $\mathbf{a}$ on $\mathbf{b}$

$\mathbf{p} = p \cdot \hat{\mathbf{b}}$ where $p = \mathbf{a} \bullet \hat{\mathbf{b}}$

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Number

Natural

$\mathbb{N} = \{\ 1,\ 2,\ 3,\ ...\ \}$

Integer

$\mathbb{Z} = \{\ ...,\ -2,\ -1,\ 0,\ 1,\ 2,\ ...\ \}$

Rational

$\mathbb{Q} = \{\ x:\ x = \frac{a}{b},$ where $a, b \in \mathbb{Z},\ b \neq 0\ \}$

Real

The set of real numbers $\mathbb{R}$ consists of the set of all rational and irrational numbers.

Complex

$\mathbb{C} = \{\ z:\ z = a + bi,$ where $a, b \in \mathbb{R},\ i^2 = -1\ \}$

Combinatorics

There are $n(n-1)(n-2)\times ... \times 3 \times 2 \times 1 = n!$ ways to arrange n different objects in an ordered list.

Permutations

$$^nP_r = n(n-1)(n-2)...(n-r+1) = \frac{n!}{(n-r)!}$$

Combinations

$$^nC_r = \binom{n}{r} = \frac{n(n-1)(n-2)...(n-r+1)}{r!} = \frac{n!}{r!(n-r)!}$$

Inclusion-exclusion principle

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Matrices

Determinant

If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then det $\mathbf{A} = ad - bc$

Inverse

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Dilation

$$\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Reflection

$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

Function

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$