MATHEMATICS

UNITS 3C AND 3D

FORMULA SHEET
2015

Number and algebra

Index laws: For $a, b > 0$ and m, n real,

$a^m b^m = (a\,b)^m$ $\quad a^m a^n = a^{m+n}$ $\quad (a^m)^n = a^{mn}$

$a^{-m} = \dfrac{1}{a^m}$ $\quad \dfrac{a^m}{a^n} = a^{m-n}$ $\quad a^0 = 1$

For $a > 0$ and m an integer and n a positive integer, $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Differentiation: If $f(x) = y$ then $f'(x) = \dfrac{dy}{dx}$

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

If $f(x) = e^x$ then $f'(x) = e^x$

Product rule: If $y = f(x)\,g(x)$ then $y' = f'(x)\,g(x) + f(x)\,g'(x)$ or If $y = uv$ then $\dfrac{dy}{dx} = \dfrac{du}{dx}v + u\dfrac{dv}{dx}$

Quotient rule: If $y = \dfrac{f(x)}{g(x)}$ then $y' = \dfrac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$ or If $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \dfrac{\dfrac{du}{dx}v - u\dfrac{dv}{dx}}{v^2}$

Chain rule: If $y = f(g(x))$ then $y' = f'(g(x))\,g'(x)$ or If $y = f(u)$ and $u = g(x)$ then $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

Integration:

Powers: $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c,\ n \neq -1$

Exponentials: $\int e^x dx = e^x + c$

Fundamental Theorem of Calculus:

$\dfrac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$ and $\int_a^b f'(x)\,dx = f(b) - f(a)$

Incremental formula: $\delta y \simeq \dfrac{dy}{dx}\delta x$

Exponential growth and decay:

If $\dfrac{dy}{dt} = ky$, then $y = Ae^{kt}$

Space and measurement

Circle: $C = 2\pi r = \pi D$, where C is the circumference, r is the radius and D is the diameter
$A = \pi r^2$, where A is the area

Triangle: $A = \frac{1}{2}bh$, where b is the base and h is the perpendicular height

Parallelogram: $A = bh$

Trapezium: $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides

Prism: $V = Ah$, where V is the volume and A is the area of the base

Pyramid: $V = \frac{1}{3}Ah$

Cylinder: $S = 2\pi rh + 2\pi r^2$, where S is the total surface area
$V = \pi r^2 h$

Cone: $S = \pi rs + \pi r^2$, where s is the slant height
$V = \frac{1}{3}\pi r^2 h$

Sphere: $S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Volume of solids of revolution:

$V = \int \pi y^2 \, dx$ rotated about the x-axis

$V = \int \pi x^2 \, dy$ rotated about the y-axis

Chance and data

Probability: For any event A and its complement $\bar{A}$, and event B

$P(A) + P(\bar{A}) = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)\, P(B|A) = P(B)\, P(A|B)$

In a binomial distribution:

Mean: $\mu = np$ and standard deviation: $\sigma = \sqrt{np(1-p)}$

A confidence interval for the mean of a population is:

$$\bar{x} - z\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z\frac{\sigma}{\sqrt{n}}$$

where μ is the population mean,
σ is the population standard deviation,
$\bar{x}$ is the sample mean,
n is the sample size and
z is the cut-off value on the standard normal distribution corresponding to the confidence level.

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.