i.19.S APPENDIX: OPTIMUM SWEEP FOR A FIXED NOISE DENSITY

Refer to Section 19.1.4. Figure 19.S.1 is an expanded version of Fig. 19.2. We can see that the slope of the data reduces for higher values of $\sigma_{\varphi 0}$. A second linear approximation, with a shallower slope, is shown for the region where $\sigma_{\varphi 0} > 0.5$. Even this fails at low values of $R_{90\%}$, where the slope becomes even shallower. Also, while results do not seem to change much when ζ exceeds 1 in the initial (low σ) region, the effect of ζ for values as high as 2 is pronounced in the intermediate region. At the highest values of $\sigma_{\varphi 0}$, even $\zeta = 3$ seems to permit higher values of $R_{90\%}$.

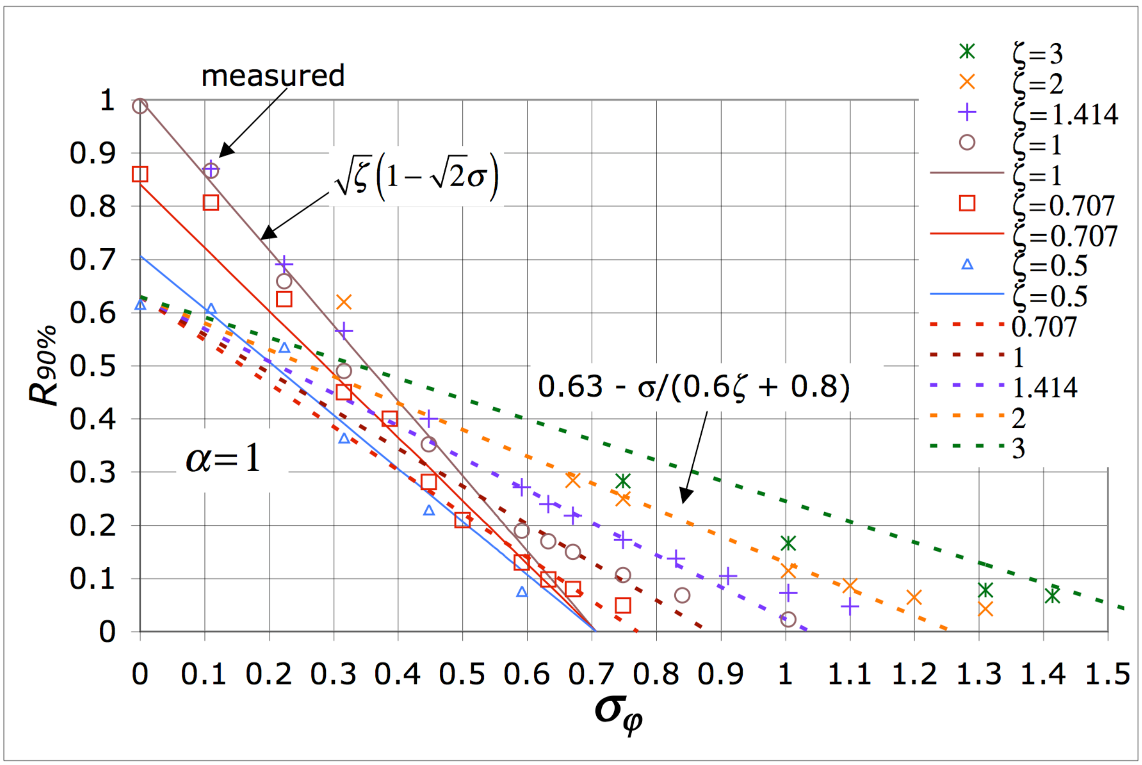

Fig. 19.S.1 $R_{90\%}$ shown over a greater range plus linear approximations.

Figure 19.S.2 shows $R'_{90\%}$, the variable that is normalized to S_{φ}. This is like Fig. 19.6, but higher values of $\sigma_{\varphi 0}$ are shown and the approximation (shown in Fig. 19.S.1) for $\sigma > 0.5$ rad is plotted along with individual points from simulations. Again the data peaks are pushed to the right due to the deviations of the data points in Fig. 19.S.1 from the linear approximations. It appears that the value of R' obtainable for $\zeta = 1.414$ is about as high as that for $\zeta = 2$, but at a higher variance. However, it is uncertain whether continuation of the simulations to even higher values of σ and ζ would produce higher values of $R'_{90\%}$.

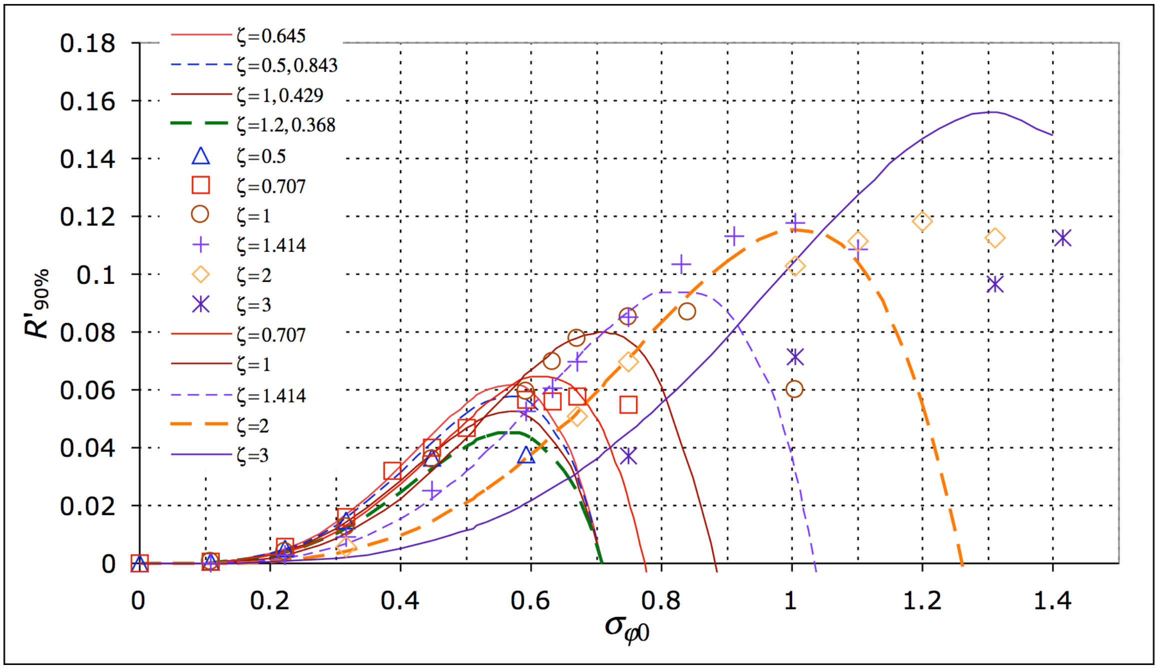

Fig. 19.S.2 $R'_{90\%}$ shown for intermediate values of $\sigma_{\varphi 0}$.

Example 19.S.1 Sweep Speed Attainable in a Given Noise Density

How fast can we sweep to acquire lock when $N_0/P_c = 0.01\ \mathrm{rad}^2$ / Hz?

Based on Fig. 19.S.2, we choose $\zeta = 1.414$ and $\sigma = 1$ as near optimum. Then the noise bandwidth would be 100 Hz and $R'_{90\%}$ would be about 1.2. The sweep rate would be

$$\dot{f} = \frac{R'}{S^2} = \frac{0.12}{10^{-4}/\mathrm{Hz}^2} \frac{2\pi\,\mathrm{rad}}{\mathrm{cycle}} = 7540\,\mathrm{Hz/sec}\,.$$

The natural frequency is obtained from

$$B_n = \frac{\omega_n}{4}\left(2\zeta + 0.5/\zeta\right) \Rightarrow 100\,\mathrm{Hz} = \frac{\omega_n}{4}\left(2.828 + 1/2.828\right)$$

as 125.7 Hz so

$$R = \frac{|\dot{\Omega}|}{\omega_n^2} = \frac{7540\,\mathrm{Hz/sec}}{1.58 \times 10^4\ \mathrm{Hz}^2} \frac{\mathrm{cycle}}{2\pi\,\mathrm{rad}} = 0.076\,.$$

If we should choose instead to use $\zeta = 1$ and $\sigma = 0.3$, for example, the theoretical value of *R,* from Fig. 19.S.1, is about 0.58. The noise bandwidth would be smaller by the variance ratio, 0.3^2, giving 0.09×100 Hz = 9 Hz. The corresponding value of ω_n is

$$\omega_n = 9\,\text{Hz}\left(2\pi\,\text{rad/cycle}\right)\left(4\right)/\left(2\times 1 + 0.5/1\right) = 90.5\,\text{rad/sec}\,,$$

and the sweep speed is

$$\dot{f} = R\omega_n^2 = 0.58\left(90.5\,\text{rad/sec}\right)^2\left(\text{cycle}/2\pi\,\text{rad}\right) = 756\,\text{Hz/sec}\,,$$

which is 10 times slower. This factor can also be seen in Fig. 19.S.2.