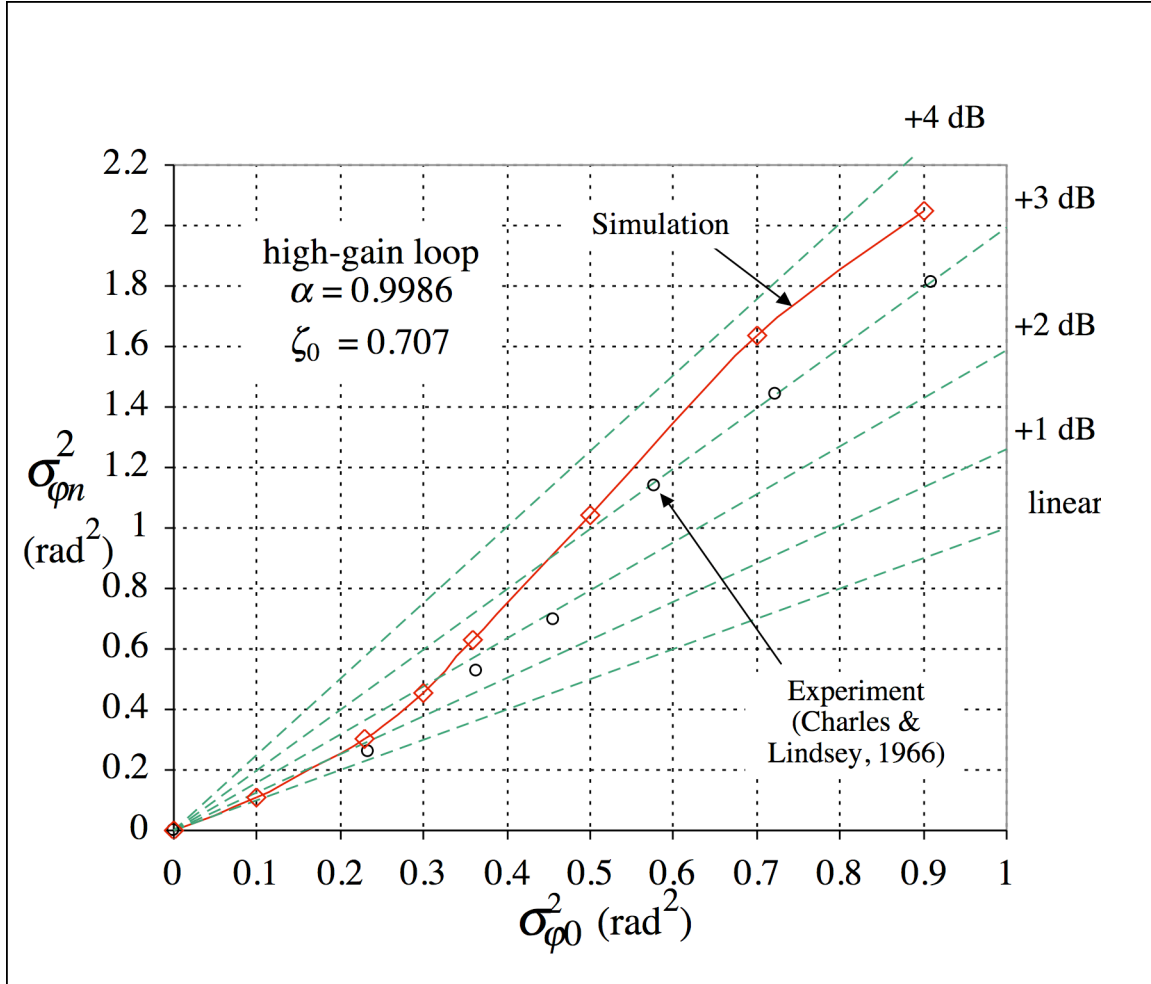


## 17.D APPENDIX: ADDITIONAL DATA

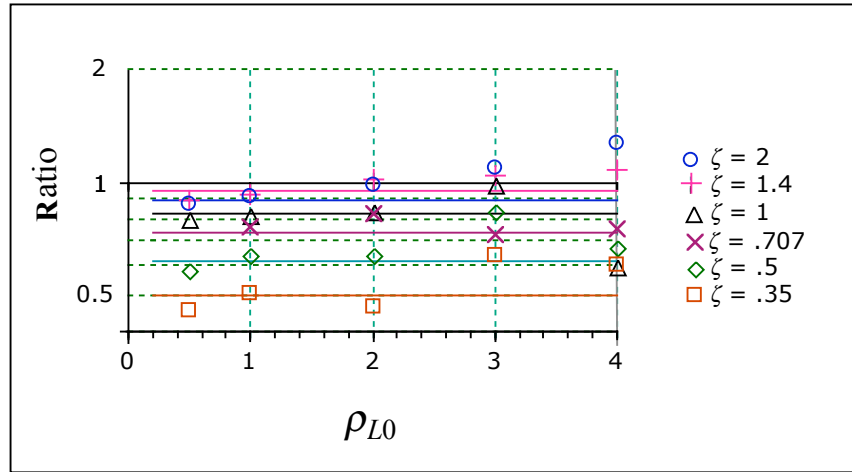
## 17.D.1 Variance from Simulation Compared to Data



**Fig. 17.D.1** Experimental and simulated true variance versus variance in  $B_{n0}$  for a high-gain second-order loop. Measured data is extracted from results by Charles and Lindsey

(1966). Refer to Section 17.1.3.2. {Script NStat2, Appendix 17.M.2}

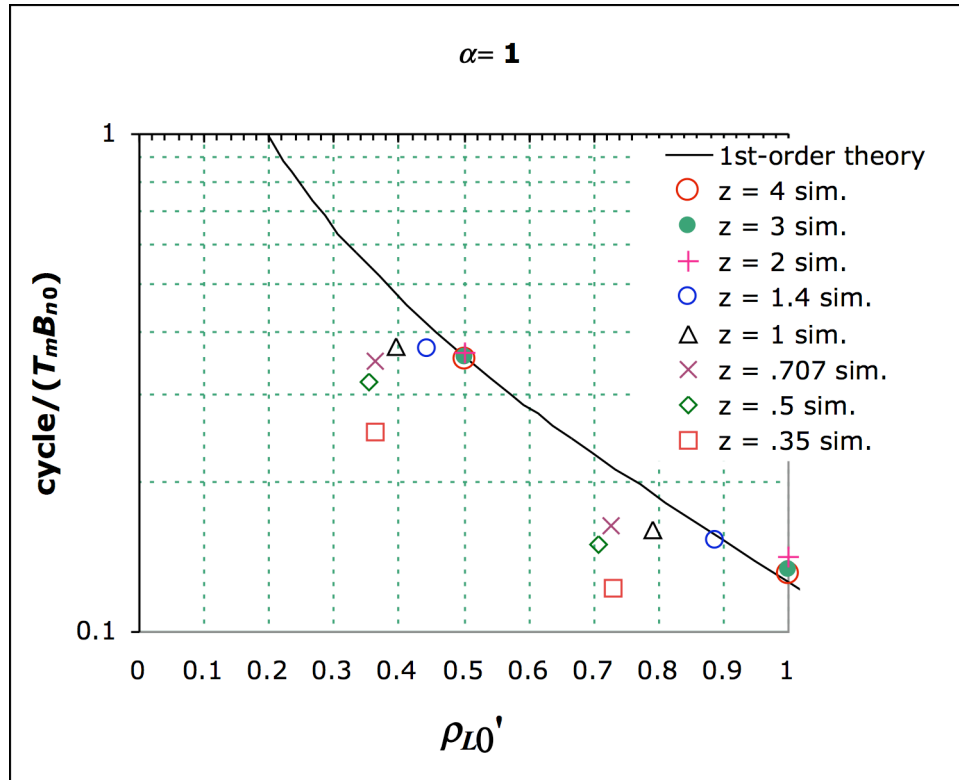
## 17.D.2 Cycle-Skipping Data From Simulations



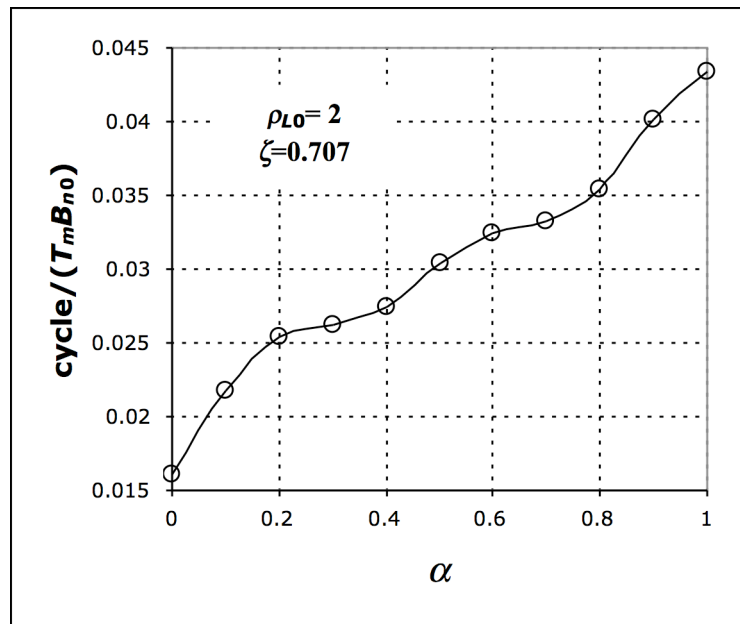
**Fig. 17.D.2** Ratio of  $1/(T_m B_{n0})$  for  $\alpha = 0$  to First-Order Solution from Eq. (17.18), theory

[Eq. (17.21)] lines and simulation points. Poorer correspondence at high  $\rho_{L0}$  may be due

to less abundant data there. Refer to Section 17.2.2.



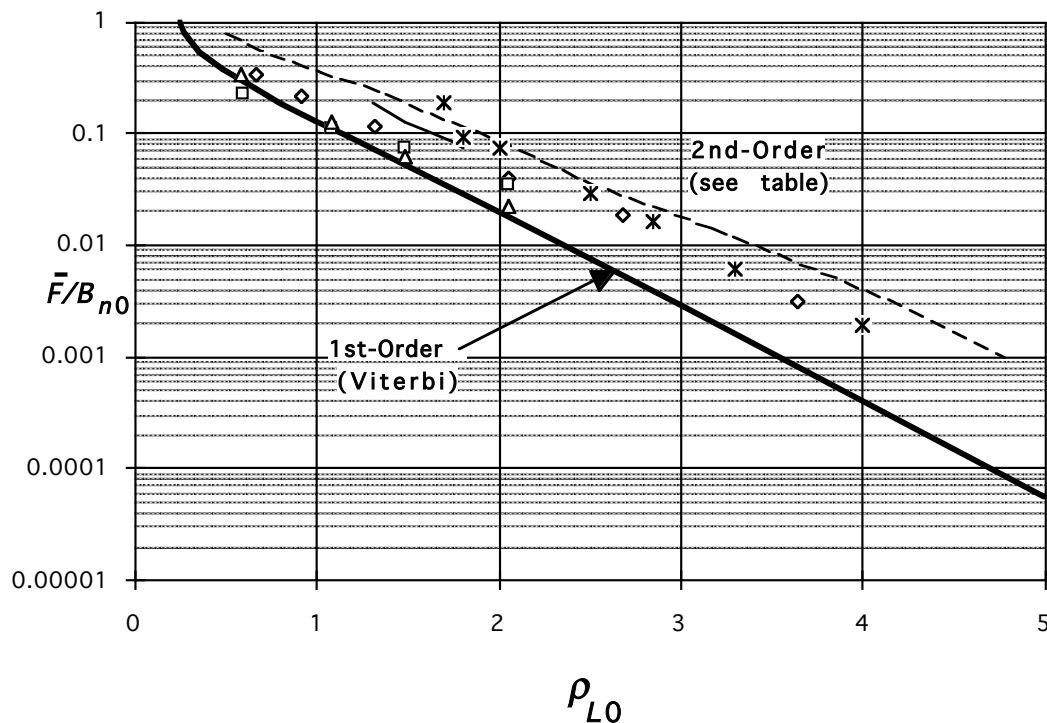
**Fig. 17.D.3** Expansion of Fig. 17.15 for low signal-to-noise.



**Fig. 17.D.4** Mean time to first skip for intermediate values of  $\alpha$  from simulations at  $\rho_{L0} = 2$  and  $\zeta = 0.707$ . Refer to Section 17.2.4.

### 17.D.3 Data for High-Gain Second-Order Loops

Refer to Section 17.3. Values of  $\bar{F}/B_{n0}$  from various sources are shown in Fig. 17.D.5. The dashed line is the approximate locus of experimental data points given by Ascheid and Meyr (1982). Their study also shows the effect of mistuning on the rate of cycle skipping. Rowbotham and Sanneman's (1967) data are from an approximate simulation of second-order, type-2 (i.e., with integrator-and-lead filter), loops. These points do not exactly give  $\bar{F}$ , but something related to it, the reciprocal of the mean time required, starting at 0, 0 in the phase plane, to cross  $\pm\pi$  and not return for  $t = 4/\omega_{n0}$ . This does not count multiple skips.



Symbol	$\alpha$	$\zeta$	Source
$\Delta$	1	1.4	Rowbotham & Sanneman
$\circ$		0.35	
$\diamond$	0.9788	0.707	Ascheid & Meyr
— — — — —	1		Smith
—————	0.9756		

**Fig. 17.D.5 Rate of Cycle Skipping for a First-Order Loop and Data Points for High-Gain Second-Order Loops**

Smith's (1966) data is also from a simulation. He counted a cycle as being skipped whenever  $\pm(2n+1)\pi$  rad was crossed except when the crossing was a reversal of the

previous crossing. Smith found that, for low  $\rho_L$ , many cycles tended to be skipped at one time but that the number skipped together was reduced when the loop filter pole occurred at a non-zero frequency. Most of his points are for a type-2 loop. However, the short solid curve represents five data points with a non-zero pole frequency and there is a noticeable reduction in  $\bar{F}$  even though the pole is at only one twentieth of the zero frequency.

#### 17.D.4 Cycle-Skipping with Mistuning, Comparison To Other Results

Refer to Section 17.4.2.

##### 17.D.4.1 Effect on $\bar{F}$

Figure 17.D.6 shows simulated and experimental values of  $\bar{F}$  in the presence of mistuning, which produces a steady-state phase offset. Conditions for the lower curve correspond to those for the middle curve of Fig. 17.17, although the data is not repeated from Fig. 17.17. The other data are for the same conditions excepting a mistuning. This experimental data from Ascheid and Meyr [1982, fig. 19] is given for a relative mistunings,  $\Delta\Omega/\Omega_{PI}$ , of 0, 0.5 and 0.9. Using Eq. (8.25), which they reference, the corresponding value of  $\Delta\Omega/\omega_n$  are 5.78 and 10.4. The simulation results are more pessimistic than the measured data, i.e. the skip rates are higher. However, there is a much better match between simulation and measured data when  $\Delta\Omega$  is reduced by about 14.6%, as shown by the dashed curves. If the  $\Omega_{PI}$  was determined experimentally, rather than being derived from loop parameters, then 1.5 % of the difference can be explained by the approximation in Eq. (8.25) rather than Eq. (8.23).

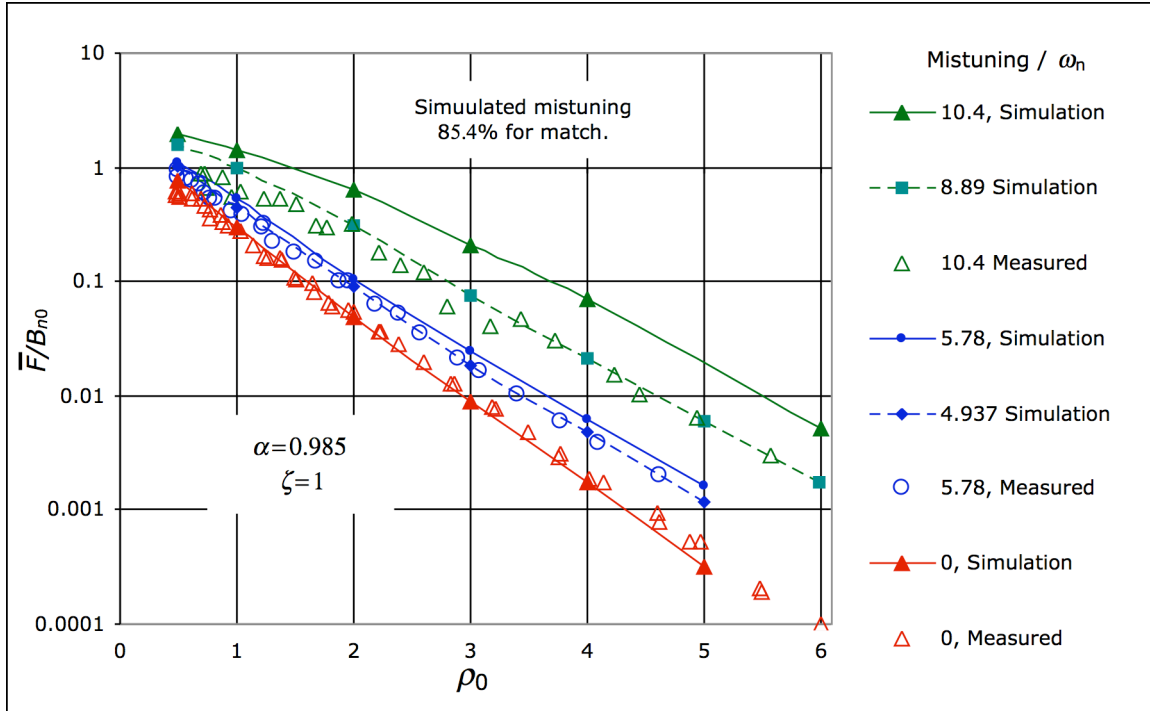


Fig. 17.D.6 Simulated and Measured Data with Mistuning.

17.D.4.2 Effect on  $T_m$ 

Holmes (1971) obtained  $T_m$  by a method reminiscent of that in Section 6.11. He plots  $T_m \omega_L$ , which appears to be  $2T_m B_{n0}/\text{cycle}$  in our terms, against  $\rho_0$  for several values of  $\zeta$ . We can obtain a data point from his results that appears to correspond to the conditions of Fig. 17.20 at an abscissa of 0.25. His result is slightly more pessimistic (shorter  $T_m$ ) than ours, in contrast to the experimental results described above for  $\bar{F}$ , by about 14%. (He does not specify the value that he used for his variable  $F = \omega_p/\omega_z$  but the formula he uses for  $\zeta$  implies that  $\alpha$  is large, as it is in Fig. 17.20.)