20.O APPENDIX Offset Interference Data Correlation

See Section 20.2. Experiments have been performed to ascertain the effect of discrete interferers that are offset from the desired signal and we might expect similar effects, on the average, if the discrete interferer were to be replaced by a very narrow band of noise of the same power and if the interference were kept small enough for lock to be maintained.

Blanchard (1974) has shown that a discrete interferer causes the loop to develop an average phase error φ_e. The same occurs in simulations with eccentric noise. The data in Table 20.M.1 supports the validity of the simulations. Rows 1-3 give loop parameters and the offset ω_{off} of the interferer or the center of the noise from the signal. Rows 4 and 5 show φ_e according to Blanchard for discrete interferers. Row 4 is based on his theory in which a modulation index is computed and, from that, φ_e. In row 5, φ_e is measured. Rows 6 and 7 show phase errors from simulations where the interferer is a narrow band of noise with an equivalent video bandwidth of r times the noise bandwidth of the loop. All data is for 10 dB signal-to-noise ratio at RF, which is high enough to prevent cycle skipping.

Table 20.O.1. Comparison of loop phase errors φ_e due to eccentric interference.

1	$\alpha \Rightarrow$	1	1	0.95
2	$\zeta \Rightarrow$	2	2	4
3	$\omega_{\text{off}} / (2\zeta\omega_n) \Rightarrow$	4	2	2
		Phase Offset, φ_e in milliradians		
4	Theory, discrete	11.8	20.1	20.1
5	Measured, discrete	12.2	23.0	23.0
6	Simulated, $r = 0.04$	11.3	21.1	20.8
7	Simulated, $r = 0.02$	11.9		

20.S APPENDIX Band-Limited Simulation Data

Figures 20.S.1 and 20.S.2 show values of output phase variance and normalized skip rate for several loops and two *S*/*N* ratios. They are shown for filter (video) bandwidths that are twice the loop noise bandwidth, ten times that bandwidth, and unlimited. Table 20.S.1 lists the bandwidth of importance for these and other simulations. Below the table are methods used to obtain the net noise bandwidth of the combined filter and loop. The skip rate is normalized in this bandwidth and the noise density is translated into *S*/*N* using this bandwidth.

In the cases where second-order loops are used with fourth-order filters, the wider of the loop or filter bandwidth has been used for the net bandwidth. This is done because of the complexity of computing the true net bandwidth. Filter-to-loop bandwidth ratios of 0.1, 2, and 10 have been used here. The approximation is probably not very good with a ratio of only 2, but we can more easily see the effects of assuming broadband noise when it is, in fact, band limited under this condition.

We note that the skip rate is more affected by the input bandwidth than is the variance and, also, that the performance of the second-order loop with $\alpha = 0$ is less affected by the filtering than are loops whose gain falls off more slowly beyond the loop bandwidth.

Table 20.S.1 Relative Values of the Various Bandwidths Used in the Simulations

		$r = \dfrac{[B_n]_{\text{Filter}}}{[B_{n0}]_{\text{Loop}}}$	$\dfrac{[B_n]_{\text{Filter}}}{[B_{n0}]_{\text{Net}}}$		$\dfrac{[B_{n0}]_{\text{Loop}}}{[B_{n0}]_{\text{Net}}}$	
		Filter Order ⇒	first	fourth	first	fourth
First-Order Loop		.01	1.01	1.00	101.00	100.01
		.04	1.04	1.00	26.00	25.03
		.1	1.1	1.01	11	10.08
		.4		1.12		2.8
		.5	1.5		3	
		1	2	1.56	2	1.56
		2	3	2.49	1.5	1.24
		4	5	4.45	1.25	1.11
		10	11	10.42	1.1	1.04
		40	41	40.41	1.025	1.01
		100	101	100.41	1.01	1
		1000		1000.4		1
		∞	∞	∞	1	1
Second-Order Loop, $\zeta =$ 0.707	$\alpha = 0$	.1	0.940	1	9.395	10
		.5	1.131		2.262	
		1	1.560		1.560	
		2	2.512	2	1.256	1
		10	10.17	10	1.017	1
		∞	∞	∞	1.000	1
	$\alpha = 1$	0.1	0.928	1	9.276	10
		0.5	1.115		2.231	
		1	1.545		1.545	
		2	2.500	2	1.250	1
		10	10.47	10	1.047	1
		∞	∞	∞	1.000	1

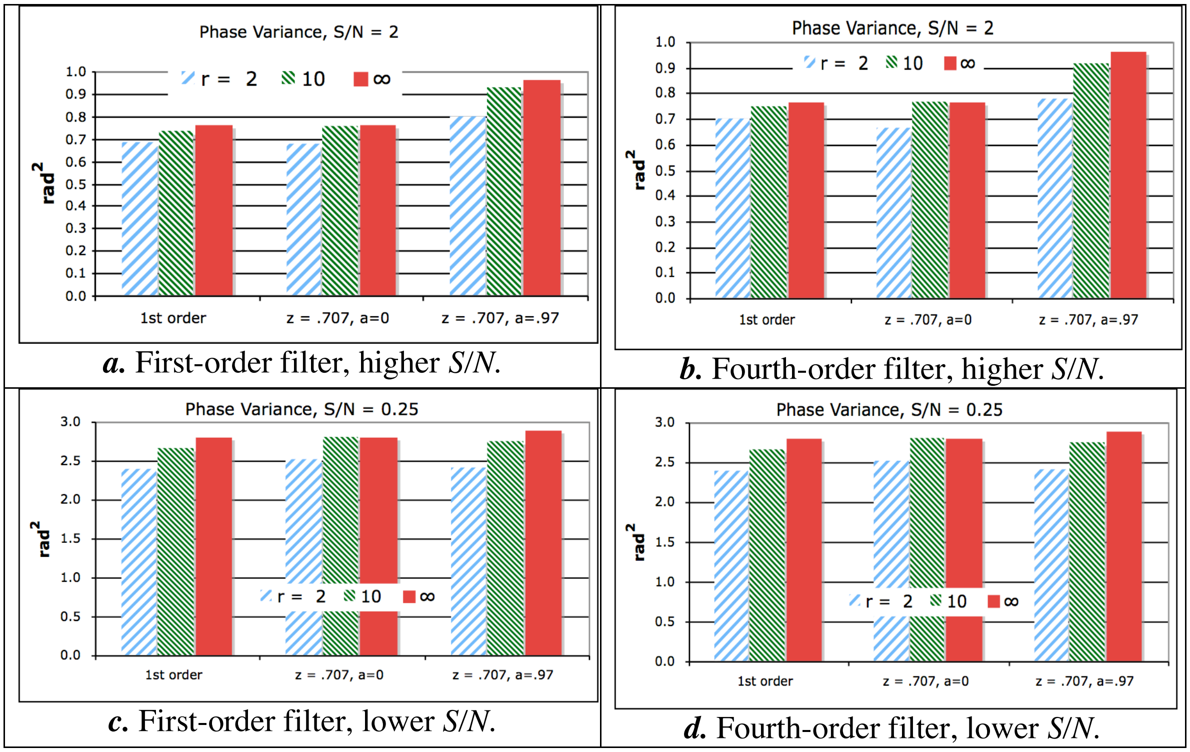
Notes on Noise Bandwidth (bw) Calculations

First-order Loop

- First-order Filter: Noise bws and net bw computed from formulas.
- Fourth-order Filter: Loop noise bw computed.
  Filter noise bw experimentally adjusted.
  Net noise bw from integration of loop response to filter noise bw.

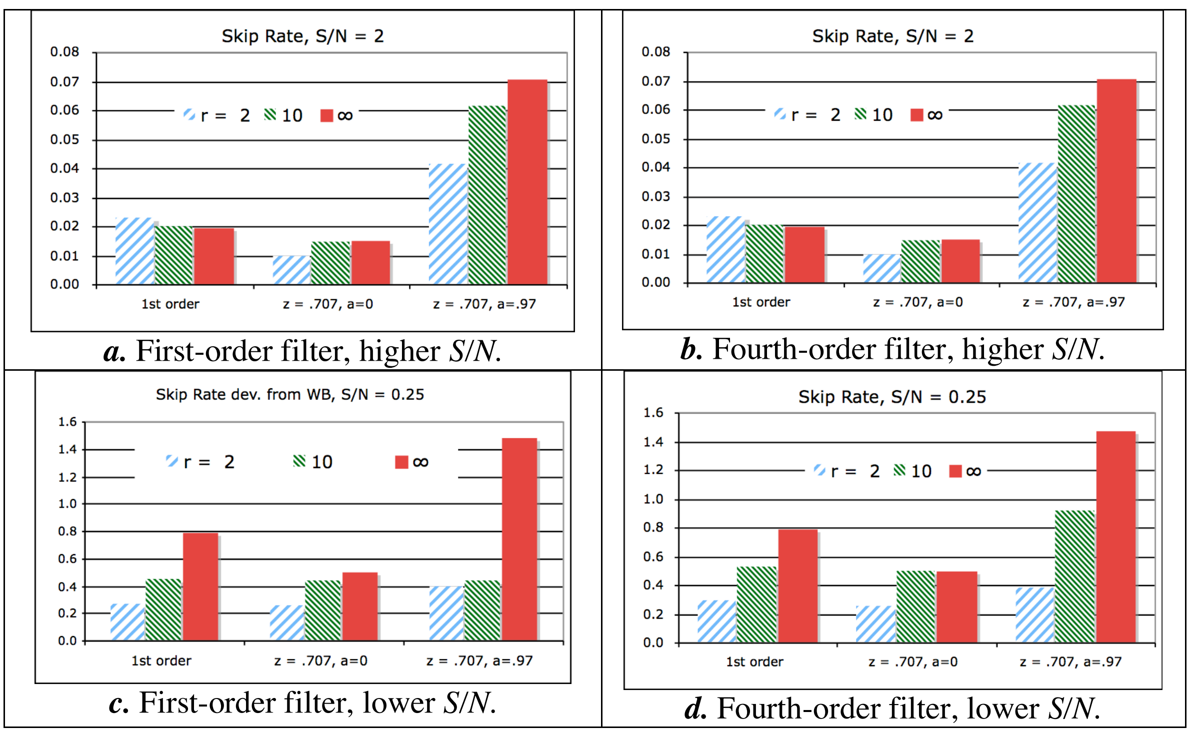
Second-order Loop

- First-order Filter: Noise bws from formulas, net bw from contour integration.
- Fourth-order Filter: Loop noise bw computed.
  Filter noise bw experimentally adjusted.
  Net noise bw set to either loop or filter noise bandwidth.



a. First-order filter, higher *S/N*.
b. Fourth-order filter, higher *S/N*.
c. First-order filter, lower *S/N*.
d. Fourth-order filter, lower *S/N*.

Fig. 20.S.1 Phase variance for three filter ratios and two $[\rho_0]_{net}$ values, noise band limited by first-order filter at *a* and *c* and by a fourth-order filter at *b* and *d*. {*}



a. First-order filter, higher *S/N*.
b. Fourth-order filter, higher *S/N*.
c. First-order filter, lower *S/N*.
d. Fourth-order filter, lower *S/N*.

Fig. 20.S.2 Normalized skip rate for three filter ratios and two $[\rho_0]_{net}$ values, noise band limited by first-order filter at *a* and *c* and by a fourth-order filter at *b* and *d*. {*}

* Scripts `NB1` in Appendix 20.M.2, `NB1bw` in Appendix 20.M.3, `NB2` in Appendix 20.M.4, `NB2bw` and `NB2bwi` in Appendix 20.M.5, `NStat1` and `NStat2` in Appendix 17.M.2.