3.A Appendix: Integrated-Circuit Doubly Balanced Mixer—Details

To understand the IC BM we begin with the bipolar-transistor differential pair in Fig. 3.14. For small excursions of the base voltage, the collector current I_1 can be expressed as

$$I_1 = \frac{I_0}{2} + \frac{v_1}{r_e} = \frac{I_0}{2}\left(1 + \frac{v_1}{V_T}\right), \tag{3.11}$$

where r_e is the differential emitter resistance, obtained be differentiating I with respect to v in Eq. (3.7), $r_e = V_T/I$. The objectives will be to make one signal proportional to v_1, which is easily done, and the other proportional to I_0, a little more difficult, and to eliminate all but the product term. To this end we generate I_0 for the differential pair in which the multiplication will occur by creating it in another differential pair. The current so generated, I_1 in Fig. 3.14, is proportional to v_1 and becomes the total emitter current for the second pair, as shown in Fig. 3.A.1. The collector current I_{21} can then be written

$$I_{21} = \frac{I_1}{2}\left(1 + \frac{v_2}{V_T}\right) = \frac{I_0}{4}\left(1 + \frac{v_1}{V_T}\right)\left(1 + \frac{v_2}{V_T}\right) \tag{3.A.1}$$

$$= \frac{I_0}{4}\left[1 + \frac{v_1}{V_T} + \frac{v_2}{V_T} + \left(\frac{v_1}{V_T}\right)\left(\frac{v_2}{V_T}\right)\right]. \tag{3.A.2}$$

Note that I_{22} could be represented by a similar equation except that the terms involving v_2 would be negative. To eliminate the undesired terms, we generate I_{22} and two more currents like I_{21} but with terms having different signs, as shown in Fig. 3.A.2. Table 3.A.1 shows the signs of the various terms as they appear in the various currents. From the table can be seen that the differential output voltage contains only the desired product.

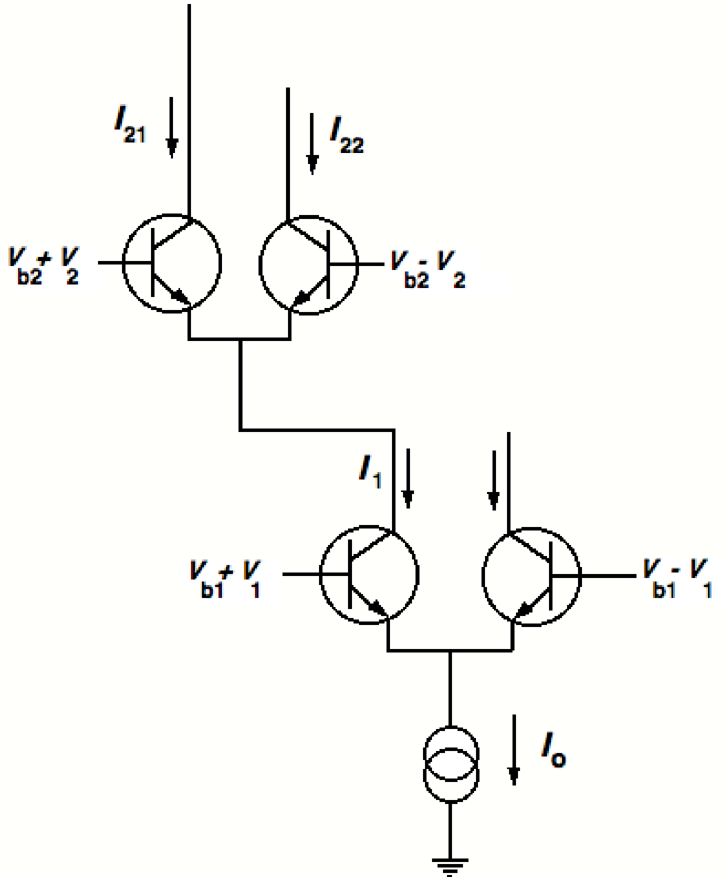

Fig. 3.A.1 Two Basic Circuits Interconnected for the IC Doubly Balanced Mixer.

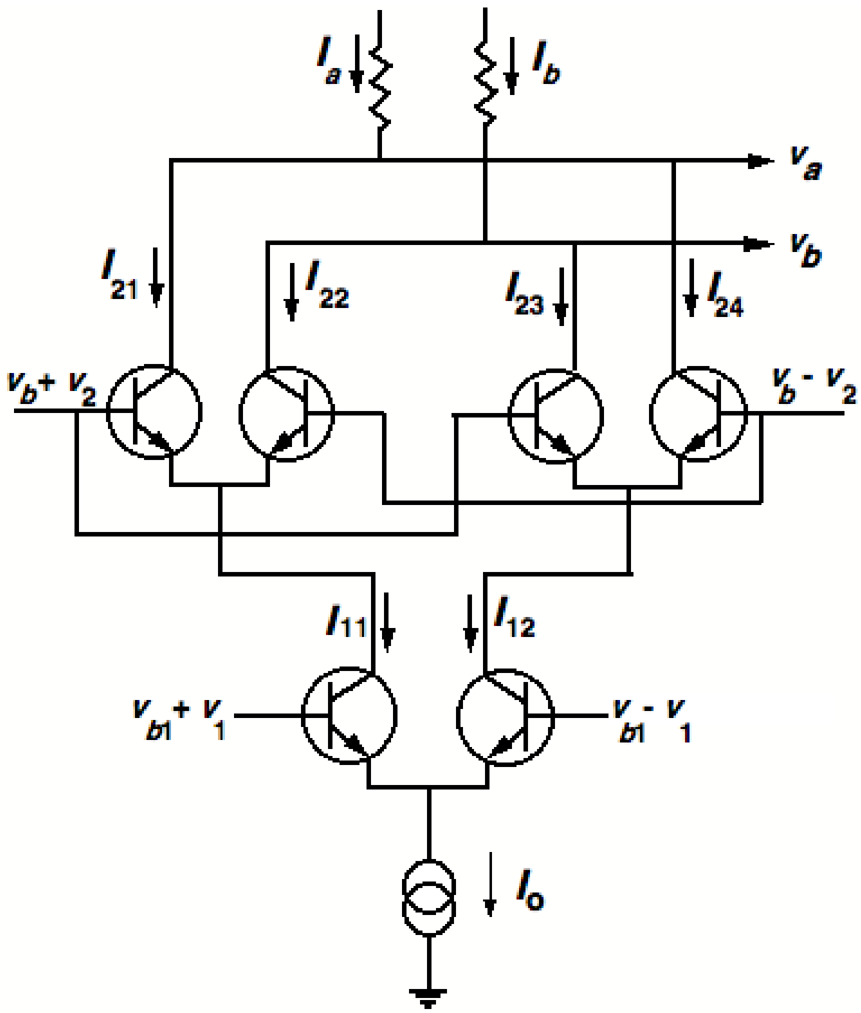

Fig. 3.A.2 The IC Doubly Balanced Mixer

$\frac{I_0}{4}\Big[$	1	$\frac{v_1}{V_T}$	$\frac{v_2}{V_T}$	$\left(\frac{v_1}{V_T}\right)\left(\frac{v_2}{V_T}\right)\Big]$
I_{21}	+	+	+	+
I_{22}	+	+	-	-
I_{23}	+	-	+	-
I_{24}	+	-	-	+
$I_a = I_{21} + I_{24}$	+2	0	0	+2
$I_b = I_{22} + I_{23}$	+2	0	0	-2
$v_b - v_a$	0	0	0	+4

Table 3.A.1 Signs of the Currents and Voltages in Fig. 3.A.2 showing cancellation of undesired components and reinforcement of desired components.

While the preceding has been a small-signal analysis, it is not difficult to see how a large signal into the lower stage could alternately turn off the left and right upper stages, thus inverting the polarity of the part of the output current that is proportional to v_2. This is similar to what happens in the diode DBM. Similarly, with both v_1 and v_2 large, the circuit could act like an ExOR gate, as was true for the diode circuit. Consider the relationship between the collector currents I_{11} and I_{12} and the input voltage v_1 under the large signal condition, v_1, $v_2 >> V_T$.

$$I_{11} = (I_0/2)(1 + \text{sign } v_1); \quad I_{12} = (I_0/2)(1 - \text{sign } v_1). \tag{3.A.3}$$

The currents are either equal to I_0 or to zero, depending on v_1. Let us call the condition where they equal I_0 the one state and the other condition the zero state. Then we can represent the state of the currents I_{11} and I_{12} by their "logic" states, $L(I_{11})$ and $L(I_{12})$. Let us also define the logic state of v_1, $L(v_1)$, to be one when the sign is positive and zero when it is negative. Then Eq.(3.A.3) can be written in shorthand notation as

$$L(I_{11}) = L(v_1)\,;\; L(I_{12}) = \overline{L(v_1)}\,. \tag{3.A.4}$$

This says that if v_1 is positive (large being understood), I_{11} is on and I_{12} is off. Continuing in the same manner, we can describe the state of the other current as a function of the large input voltages:

$$L(I_{21}) = L(I_{11}) \bullet L(v_2) = L(v_1) \bullet L(v_2)\,, \tag{3.A.5}$$

$$L(I_{22}) = L(I_{11}) \bullet \overline{L(v_2)} = L(v_1) \bullet \overline{L(v_2)}\,, \tag{3.A.6}$$

$$L(I_{23}) = \overline{L(I_{11})} \bullet L(v_2) = \overline{L(v_1)} \bullet L(v_2)\,, \tag{3.A.7}$$

$$L(I_{24}) = \overline{L(I_{11})} \bullet \overline{L(v_2)} = \overline{L(v_1)} \bullet \overline{L(v_2)}\,, \tag{3.A.8}$$

$$L(I_a) = L(I_{21}) + L(I_{24}) = \overline{L(v_1) \oplus L(v_2)}\,, \tag{3.A.9}$$

$$L(I_a) = L(I_{22}) + L(I_{23}) = L(v_1) \oplus L(v_2)\,, \tag{3.A.10}$$

where + and ⊕ represent logical OR and ExOr respectively. The algebraic sum of currents at I_a and I_b is equivalent to an OR function because, for the assumed large signals, only one of the constituent currents I_{2i} can be on at a time. Thus we see that both the IC- and diode-type DBMs act like ExOR circuits under large signal conditions, although the latter requires square-wave inputs to give a true triangular characteristic.

3.B Appendix: Op-Amps In Loop Filters

3.B.1 The Op Amp

The traditional op amp is a high-gain, high-input-impedance, amplifier intended for use in circuits in which part of the output signal is fed back to the input (as in Fig. 3.25).

At low frequencies the gain is very high, often more than 100 dB. If it were uncompensated, its transfer gain and phase would be as shown in Fig. 3.26, curves 1 and 2. Unfortunately, it would tend to oscillate when feedback was applied because the gain would be too high when the multiple, unavoidable, poles within its circuitry produced 180° of excess phase shift (at f_x). To control this, a single-pole roll-off is incorporated within the op-amp, beginning at perhaps a few Hz and producing the gain and phase shown in curves 3 and 4. Thus the gain can be reduced to a tolerable level while maintaining phase margin relative to -360°.

Besides the inverting input shown at v in Fig. 3.25, op amps have a non-inverting input. The output depends on the difference between the two inputs. For now we assume zero at the non-inverting input.

A current-feedback op-amp differs in that it is a transimpedance amplifier, producing an output voltage proportional to the current into the inverting (-) input [Franco, 1989; Little, 1990]. (The non-inverting input is not shown in Fig. 3.25.) The input impedance into the inverting input is low but that point is virtual ground in the configurations that we will study anyway. We will begin with the traditional op-amp.

3.B.2 General Equations, Voltage Feedback

The active filter is shown in Fig. 3.25 in generic form with the feedback impedance Z_{FB} as yet unspecified. We will begin by assuming only that the amplifier has ideal infinite input impedance and develop the equations for that rather general case. We will then proceed to simplify the equations by making various other assumptions that apply in many practical situations. While the simpler equations will commonly be most useful, we will thus be aware of the modifications that will be necessary under conditions where they do not apply.

The output voltage can be expressed in terms of the op-amp input voltage as

$$u_2 = -G_a v \ . \qquad (3.B.1)$$

Since the same current flows through all the passive components, the voltage drops are proportional to their impedances:

$$\frac{u_2 - v}{v - u_1} = \frac{Z_{\mathrm{FB}}}{R_1} \ . \qquad (3.B.2)$$

From these last two equations we can eliminate v to obtain

$$\frac{u_2(1+1/G_a)}{-u_2/G_a-u_1} \equiv -\frac{u_2(1+G_a)}{u_2+G_a u_1} = \frac{Z_{\mathrm{FB}}}{R_1}, \tag{3.B.3}$$

$$\frac{u_2}{u_2+G_a u_1} = -\frac{Z_{\mathrm{FB}}}{(1+G_a)R_1}. \tag{3.B.4}$$

From this we now obtain the ratio u_2/u_1 :

$$u_2 = -\frac{Z_{\mathrm{FB}}}{(1+G_a)R_1}u_2 - \frac{G_a}{1+G_a}\frac{Z_{\mathrm{FB}}}{R_1}u_1 \ ; \tag{3.B.5}$$

$$\left(1+G_a+\frac{Z_{\mathrm{FB}}}{R_1}\right)u_2 = -G_a\frac{Z_{\mathrm{FB}}}{R_1}u_1 \ ; \tag{3.B.6}$$

$$-K_{\mathrm{LF}}F(s) \equiv \frac{u_2(s)}{u_1(s)} = \frac{G_a F_d}{1+G_a-F_d}\ , \tag{3.B.7}$$

where F_d is the desired response,

$$F_d \triangleq -\frac{F_{\mathrm{FB}}(s)}{R_1}. \tag{3.B.8}$$

The minus sign in (3.B.7) implies that part of the filter transfer function $K_{LF}F(s)$ is an inversion somewhere else in the loop. This detail allows $K_{LF}F(s)$ to represent both active and passive filters.

Equation (3.B.7) can be rearranged as

$$\frac{G_a}{-K_{\mathrm{LF}}F(s)} = \frac{(1+G_a)}{F_d} - 1 \tag{3.B.9}$$

to show that, if the amplifier's gain is much greater than the magnitude of the desired transfer function, then

$$\frac{G_a}{-K_{\mathrm{LF}}F(s)} \approx \frac{1+G_a}{F_d} \tag{3.B.10}$$

so that

$$-K_{\mathrm{LF}}\,F(s)\Big|_{|1+G_a|>>|F_d|} \approx \frac{G_a}{1+G_a}F_d\ . \tag{3.B.11}$$

If, also, $|G_a|>>1$, then

$$-K_{\mathrm{LF}}\,F(s)\Big|_{|1+G_a|>>|F_d|,\ |G_a|>>1} \approx F_d \ . \qquad (3.B.12)$$

When $(1+G_a)$ drops well below F_d, Eq. (3.B.9) shows that

$$-K_{\mathrm{LF}}\,F(s)\Big|_{|1+G_a|<<|F_d|} \approx -G_a \, . \qquad (3.B.13)$$

That is, the filter transfer function becomes the op amp's open-loop gain. We can see the transition in Fig. 3.26.

3.B.3 General Equations, Current Feedback

The current feedback op amp is a more recent version of the traditional op amp. Refer again to Fig. 3.25. In this case $v = 0$ (it has the same DC value as the non-inverting input — the input appears to be shorted). The output voltage equals the current out of the inverting port multiplied by the op-amp's transimpedance, Z_{21}. Thus we can write the current out of the inverting port as

$$\frac{u_2}{Z_{21}} = I_- = -\frac{u_1}{R_1} - \frac{u_2}{Z_{\mathrm{FB}}} \ , \qquad (3.B.14)$$

which we solve as

$$K_{\mathrm{LF}}F(s) \triangleq \frac{u_2}{u_1} = -\frac{1}{R_1\left(\dfrac{1}{Z_{21}} + \dfrac{1}{Z_{\mathrm{FB}}}\right)} = F_d \frac{1}{1+\dfrac{1}{G_a}} \ . \qquad (3.B.15)$$

where $G_a = Z_{21}/Z_{FB}$. This has the same form as Eq. (3.B.11) but an advantage claimed for these amplifiers is that the gain can be changed by varying R_1 without affecting the bandwidth. In the previous type of op amp, the input resistor R_1 forms part of a voltage divider that is the feedback circuit (Eq. (3.B.2). The closed-loop bandwidth (Fig. 3.27) depends on the gain, which depends on R_1. In this type, the effective short at the input isolates the feedback from R_1. If $|G_a|>>1$ then $K_{\mathrm{LF}}F(s) \approx F_d$ and if $|G_a|<<1$ then $K_F F(s) \approx -Z_{21}/R_1$ so, much as with voltage feedback, the desired transfer function F_d is obtained until the frequency increases to the point where the gain drops too much and then the closed-loop transfer function becomes equal to the open-loop transfer function.

3.B.4 High-Frequency Poles

The filter will normally be designed using Eq. (3.B.12) but the transition from (3.B.12) to (3.B.13) at f_y in Fig. 3.27 represents an additional pole in the transfer function. Likewise, additional poles in various parts of the op-amp circuitry cause phase shift to accumulate, often quite rapidly once a critical frequency is reached, as shown at f_x. These frequencies must be high enough compared to the bandwidth of the PLL that they do not have a significant detrimental effect.

3.B.5 Filter Stability

While the filter characteristics are important to the performance and stability of the PLL, the stability of the active filter itself is also important. Stability considerations for the filter loop are similar to those for the PLL so they may be more easily understood after loop stability has been studied in Chapter 5. Nevertheless, the material is presented here because it is an essential part of loop-filter design.

The open-loop G of the active-filter loop (through the op amp and back through Z_{FB}) with a voltage-feedback op-amp can be obtained from Eq. (3.B.1) and (3.B.2). If a voltage $v = v'$ is applied at the input, by (3.B.1) it will cause an output

$$u_2 = -G_a v' . \tag{3.B.16}$$

By (3.B.2), with $u_1 = 0$, the value of $v = v''$ resulting from this value of u_2 is given by

$$\frac{-G_a v' - v''}{v'} = \frac{Z_{\text{FB}}}{R_1}, \tag{3.B.17}$$

from which can be obtained the open-loop transfer function,

$$\frac{v''}{v'} = -\frac{G_a}{\frac{Z_{\text{FB}}}{R_1} + 1} = \frac{G_a}{F_d - 1} . \tag{3.B.18}$$

Unity open-loop gain occurs when the numerator and denominator have the same magnitude. For a general lag-lead filter and $|F_d| >> 1$, this corresponds to point x in Fig. (3.B.1). The phase of F_d will be approximately 0° in this flat-gain region so the open-loop phase will be the phase of G_a and this should be more than -360° for stability (much more if the filter is to be well behaved). Otherwise the open-loop gain will be greater than one when the phase shift reaches -360°, which will very likely cause instability.

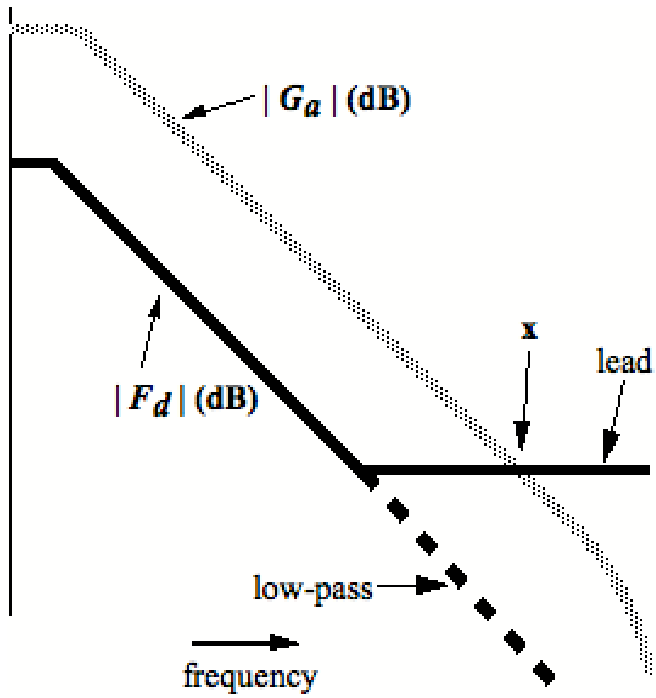

Fig. 3.B.1 Op-amp open-loop and desired closed-loop gains.

If the closed-loop filter characteristic is low-pass, as shown by the dashed line, then the denominator of Eq. (3.B.18) will approach -1 as F_d becomes small compared to one. Then the open-loop gain becomes equal to $-G_a$. Many op-amps are designed to be stable with unity feedback, so circuits using such op-amps will be stable.

The open-loop G of the current-feedback op-amp can be obtained beginning with an input current I_- that produces an output voltage u_2'',

$$u_2'' = Z_{21} I_- = Z_{21} u_2' / Z_{FB} \,, \qquad (3.B.19)$$

where I_- is derived from the output voltage u_2' through the feedback impedance Z_{FB}. The open-loop transfer function is thus

$$\frac{u_2''}{u_2'} = -\frac{Z_{21}}{Z_{FB}} = -G_a \,. \qquad (3.B.20)$$

This differs from Eq. (3.B.18) in that the denominator lacks a minimum value of one. Therefore, if a lag filter (Fig. 3.23*e*) is used with a current-feedback op-amp there is a danger of instability because $|G_a| = 1$ can occur in the region where G_a has developed considerable excess phase shift. Physically, the impedance of the feedback capacitor continues to decrease as frequency increases so the output voltage is converted to ever increasing current, which maintains the open-loop gain as $|Z_{21}|$ falls. If a lead configuration is used (Fig. 3.23*c*), if ω_z is much less than the frequency where G_a acquires a phase of -360°, and if R_2 is in the recommended range of feedback resistances for the op-amp, the circuit should be stable.

Example 3.1, Active Filter

Requirement: Integrator-and-lead filter with f_p <10 Hz, f_z = 5000 Hz, $K_{LF}F(f >> f_z) = 20$.

A solution: Use Fig. 3.23*c*. Choose a convenient value of R_2 = 10 kΩ. From Eq. (3.39),

$$C = \frac{1}{R_2\omega_z} = \frac{1}{\left(10^4 \text{ V/A}\right)\left(2\pi \times 5 \times 10^3/\text{sec}\right)} = 3.18 \times 10^{-9} \text{ C/V} \approx 3300\text{pF} .$$

High-frequency gain is R_2/R_1 so R_1 = 500 Ω to give 20. The op-amp must have more than the desired gain at 10 Hz if the pole is to be lower than 10 Hz. Using Eq. (3.20), the value of $k = 1/(R_1C)$ can be obtained from the high-frequency gain and the gain at 10 Hz can then be obtained using k.

$$\left|K_{LF}F(f >> 5000\,\text{Hz})\right| = 20 \approx \frac{k}{\omega_z} = \frac{k}{2\pi 5000\,\text{rad/sec}} ;$$

$$k = 2\pi \times 10^5 \text{ rad/sec} ;$$

$$\left|K_{LF}F(f = 10\,\text{Hz})\right| = \left|\frac{k}{s}\right| = \frac{2\pi \times 10^5 \text{ rad/sec}}{2\pi \times 10 \text{ rad/sec}} = 10^4 \Rightarrow 80\,\text{dB} .$$

The DC gain of the op-amp must exceed 80 dB, therefore. For stability of the op amp, its open-loop phase-shift must be >> -360° when its open-loop gain is 20 (Fig. 3.B.1). In addition, the frequency at which that gain is 20 must be high enough that the additional pole at that frequency will not be of importance to the loop, certainly much higher than 5000 Hz.

3.B.6 Non-Inverting Input

We can extend our development of active filter performance to the non-inverting case. The op amp responds to the difference between the inverting (-) and non-inverting (+) inputs. We now allow a voltage v_+ to be present on the + input to the op amp. We can treat this as a change in voltage reference from ground to v_+ and write the input and output voltages relative to this new reference. Equation (3.23) then becomes

$$\frac{u_2 - v_+}{u_1 - v_+} = \frac{G_a F_d}{1 + G_a - F_d} \triangleq -H_a \ , \tag{3.B.21}$$

which gives

$$u_2 = v_+(1+H_a) - u_1 H_a \,. \tag{3.B.22}$$

Thus the response to a signal on the op amp's + input equals unity minus the response from u_1.

How might we obtain a differential response, the same response from two inputs except that one is the negative of the other? From (3.B.22) we can see that such a response can be obtained from a voltage u_1' if it is related to v_+ by

$$v_+ = \frac{H_a}{1+H_a} u_1' \,. \tag{3.B.23}$$

Thus we want to introduce u_1' to the op amp by means of a voltage divider that gives Eq. (3.B.23). However, H_a is a function of frequency. Nevertheless, for large G_a, Eq. (3.B.21) becomes

$$H_a \approx -F_d = \frac{Z_{\mathrm{FB}}(s)}{R_1} \,, \tag{3.B.24}$$

which describes the voltage divider that we actually use and with which Eq. (3.B.23) becomes

$$v_+ = -u_1' \frac{F_d}{1-F_d} \,. \tag{3.B.25}$$

The divider is illustrated in Fig. 3.B.2. Equation (3.B.22) now becomes

$$u_2 = -u_1' F_d \frac{1+H_a}{1-F_d} - u_1 H_a \,, \tag{3.B.26}$$

which gives the desired results as long as G_a is large enough so $H_a \approx -F_d$.

Under the assumed conditions, the common mode response is zero. That is, if $u_1 = u_1$', $u_2 = 0$. To the degree that the impedances (R_1 and Z_{FB}) on the two sides of the op amp are not equal, however, the circuit will have a non-zero common-mode response [Endres and Kirkpatrick, 1992].

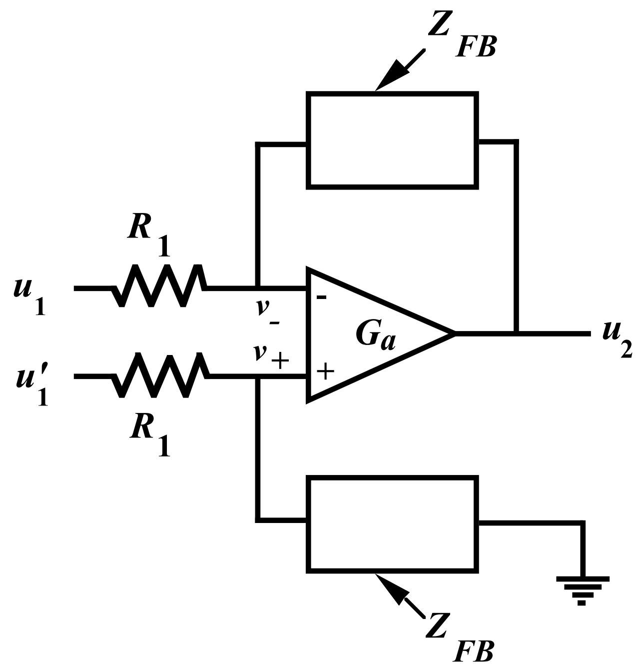

Fig. 3.B.2 Op-amp using both inputs.