Optical Materials

W18.1   Optical Polarizers

A polarizer is basically an optically anisotropic material for which the transmission depends on the direction of polarization of the light relative to the crystal axes. The ability to control the polarization permits one to build such optical elements as modulators and isolators.

Suppose that a plane electromagnetic wave propagates along the z direction. The electric field vector lies in the xy plane and may be characterized by two complex amplitudes: $\mathbf{E}_0 = E_{0_x}\hat{i} + E_{0_y}\hat{j}$. The intensity of the light (i.e., its power per unit area), is written as

$$I = \sqrt{\frac{\epsilon}{\mu}}|E_0|^2 = \sqrt{\frac{\epsilon}{\mu}}(|E_{0_x}|^2 + |E_{0_y}|^2) = I_x + I_y, \tag{W18.1}$$

where I_x and I_y are the intensities of x and y polarized light. If I_x and I_y are the same, the light is said to be *unpolarized*. If they are different, the light may be *linearly polarized*. The degree of linear polarization, P_L, is given by

$$P_L = \frac{I_x - I_y}{I_x + I_y}, \tag{W18.2}$$

where it is assumed that $I_x \geq I_y$ so as to make $0 \leq P_L \leq 1$. If $I_y = 0$, then $P_L = 1$ and there is 100% linear polarization. If $P_L = 0$ the light is unpolarized. If $0 < P_L < 1$, the light is partially linearly polarized.

A more detailed description of the light involves information concerning the relative phases of the electric field components as well as the intensity and degree of polarization. It is convenient to construct the complex column vector

$$\chi_0 = \begin{bmatrix} E_{0_x} \\ E_{0_y} \end{bmatrix} \tag{W18.3}$$

and form the two-dimensional matrix, called the *density matrix*,

$$\chi_0\chi_0^+ = \begin{bmatrix} E_{0_x}E_{0_x}^* & E_{0_x}E_{0_y}^* \\ E_{0_y}E_{0_x}^* & E_{0_y}E_{0_y}^* \end{bmatrix}. \tag{W18.4}$$

(If the light is fluctuating in time, one generally performs a time average and replaces $\chi_0\chi_0^*$ by $\langle\chi_0\chi_0^*\rangle$.) Note that the matrix is Hermitian (i.e., its transpose is equal to

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its complex conjugate). A general complex two-dimensional matrix needs eight real numbers to specify its elements, but the Hermitian condition reduces this number to four. This matrix may be expanded in terms of four elementary Hermitian matrices. The Pauli spin matrices (used coincidentally to describe the electron spin operator in Appendix WC) and the identity matrix are chosen for this purpose. Thus multiplying the column vector χ_0 by the row vector χ_0^+ formed from the two complex conjugate elements gives

$$\rho_0 = \chi_0 \chi_0^+ = \tfrac{1}{2}(S_0^0 \mathbf{I} + \mathbf{S}^0 \cdot \boldsymbol{\sigma}), \tag{W18.5}$$

where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{W18.6}$$

The real numbers $S_i^0 (i = 0, 1, 2, 3)$ are called the *Stokes parameters* and fully characterize the state of polarization, including the relative phase relations. They are given by

$$S_0^0 = |E_{0_x}|^2 + |E_{0_y}|^2, \tag{W18.7a}$$

$$S_3^0 = |E_{0_x}|^2 - |E_{0_y}|^2, \tag{W18.7b}$$

$$S_1^0 = E_{0_x} E_{0_y}^* + E_{0_y} E_{0_x}^*, \tag{W18.7c}$$

$$S_2^0 = i(E_{0_x} E_{0_y}^* - E_{0_y} E_{0_x}^*). \tag{W18.7d}$$

From Eq. (W18.1) one sees that S_0^0 is proportional to the intensity, I. The quantity $P_L = S_3^0/S_0^0$ is the degree of linear polarization and $P_C = S_2^0/S_0^0$ is the degree of circular polarization. The degree of total polarization is given by $P_T = \sqrt{P_C^2 + P_L^2}$. The Stokes parameter S_1^0 contains information concerning the relative phase of the x- and y-polarized light, or equivalently, between the right- and left-circularly polarized light.

Consider the transmission of unpolarized light through a polarizer. Assume for the moment that the principal axes of the polarizer are aligned with the x and y axes. After transmission, the field is changed to $\mathbf{E} = E_x \hat{i} + E_y \hat{j}$, where the new amplitudes are related to the old amplitudes by

$$E_x = E_{0_x} e^{i\phi_x} p_x, \qquad E_y = E_{0_y} e^{i\phi_y} p_y. \tag{W18.8}$$

The parameters p_x and p_y are dimensionless attenuation constants, depending on the absorption coefficients when the electric field is directed along the principal optical axes. Thus $p_x = \exp(-\alpha_x L)$ for a polarizer of thickness L, and similarly for p_y. These coefficients may be frequency dependent, a phenomenon called *dichroism*. Henceforth, as a simplification, it will assumed that the phase factors ϕ_x and ϕ_y are zero.

The Stokes parameters may be arranged as a four-element vector and the effect of the polarizer will then be described by a four-dimensional matrix called the 4×4

Mueller matrix, M,

$$
\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} p_x^2 + p_y^2 & 0 & 0 & p_x^2 - p_y^2 \\ 0 & p_x p_y & 0 & 0 \\ 0 & 0 & p_x p_y & 0 \\ p_x^2 - p_y^2 & 0 & 0 & p_x^2 + p_y^2 \end{bmatrix} \begin{bmatrix} S_0^0 \\ S_1^0 \\ S_2^0 \\ S_3^0 \end{bmatrix} \equiv \begin{bmatrix} A & 0 & 0 & B \\ 0 & C & 0 & 0 \\ 0 & 0 & C & 0 \\ B & 0 & 0 & A \end{bmatrix} \begin{bmatrix} S_0^0 \\ S_1^0 \\ S_2^0 \\ S_3^0 \end{bmatrix}.
$$

$$(W18.9)$$

If the principal axes were rotated with respect to the x and y axes by angle θ, this could be described by rotating the M matrix by the rotation matrix T:

$$
T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin 2\theta & 0 & \cos 2\theta \\ 0 & 0 & 1 & 0 \\ 0 & -\sin 2\theta & 0 & \cos 2\theta \end{bmatrix},
$$

$$(W18.10)$$

and the Mueller matrix becomes

$$
M(\theta) = TMT^{-1} = \begin{bmatrix} A & B \sin 2\theta & 0 & B \cos 2\theta \\ B \sin 2\theta & A \sin^2 2\theta + C \cos^2 2\theta & 0 & (A - C) \sin 2\theta \cos 2\theta \\ 0 & 0 & C & 0 \\ B \cos 2\theta & (A - C) \sin 2\theta \cos 2\theta & 0 & A \cos^2 2\theta + C \sin^2 2\theta \end{bmatrix}.
$$

$$(W18.11)$$

Various types of polarizing sheets have been devised. They are generally based on the use of dichromophore molecules (i.e., molecules that produce dichroism). The *H-sheet*, invented by E. H. Lamb, consists of molecules of polyvinyl alcohol (PVA) stretched along a particular direction, to which an iodine-based dye is added. When light has its electric field parallel to the long axis of the molecules, they become polarized and develop large fluctuating electric-dipole moments. This sets up large local fields near the molecules and their excitation is readily transferred to the iodine-based dye molecules, where the energy is absorbed and thermalized. Light oriented perpendicular to the molecules does not cause as large a polarization and is therefore not transferred to the dye efficiently. Consequently, the perpendicularly polarized light is transmitted with higher efficiency than light oriented parallel to the PVA molecules. The PVA molecules are in laminated sheets consisting of cellulose acetate butyrate for mechanical support and chemical isolation.

Later the *J-sheet* was introduced, consisting of needlelike dichroic crystals of herapathite oriented parallel to each other in a matrix of cellulose acetate. A variation of this is the *K-sheet*, in which rather than achieving dichroism by adding a stain (an additive that absorbs at a particular color or colors), hydrogen and oxygen are removed by a dehydration catalyst. The material is stretched to produce aligned polyvinylene polymers. Another variation, the *L-sheet*, relies on organic dye molecules to achieve the dichroism. Typical dye molecules are aminil black, Erie green, Congo red, and Niagara blue. It is also possible to embed thin parallel metal wires in a substrate to create a polarizer. Typically, fine Al wires are placed in substrates of glass, quartz, or polyethylene.

For a dichromophore molecule or crystallite to be successful, it must exhibit a large anisotropy. In combination with the dye molecule it must be strongly absorbing for one state of polarization and weakly absorbing for the other state.

An example of the spectral dependence of the polarization parameters on wavelength is given in Fig. W18.1, where p_x^2 and p_y^2 are presented for the polarizer KN-36 (a
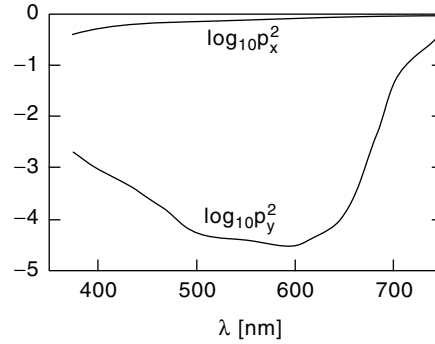
Figure W18.1. Spectral parameters p_x^2 and p_y^2 plotted as a function of the wavelength λ for the polarizer KN-36. (Adapted from E. Collett, *Polarized Light*, Marcel Dekker, New York, 1993.)

commercial polarizer of the K-sheet variety). The filter is called a *neutral polarizer* because these parameters are approximately flat across the visible spectrum.

It should be noted that the concept of a polarizer may be extended to any device that modifies the Stokes parameters of the transmitted light. A large number of physical parameters is associated with the Mueller matrix of the device. Full characterization of a general polarizer is rarely given.

W18.2 Faraday Rotation

In Section W18.1 polarization of light was obtained by means of dichroism. In this section attention is given to how the direction of polarization may be changed with little attenuation. The polarization of an electromagnetic wave is rotated when it propagates through a medium along the direction of a magnetic field, a phenomenon called *Faraday rotation*. The angle of rotation, θ_F, is determined by the magnetic induction or flux density, $\mathbf{B} = B\hat{k} = \mu_0 H\hat{k}$, the length of propagation, z, and the Verdet constant of the material, V:

$$\theta_F = VHz. \tag{W18.12}$$

The process is illustrated in Fig. W18.2.
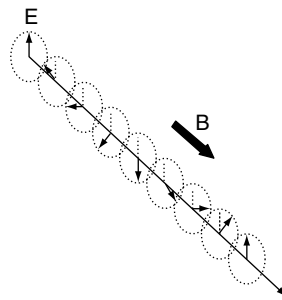
Figure W18.2. Rotation of the electric polarization vector of light propagating along a magnetic field.

To obtain an expression for V, one may model the electrons as a collection of Lorentz oscillators interacting with the light and the magnetic field imposed. The model is general enough to include both bound and free electrons. The classical equation of motion for an oscillator is

$$\left[\frac{d^2}{dt^2} + \gamma\frac{d}{dt} + \omega_0^2\right]\mathbf{r}(t) = -\frac{e}{m_c}\left(\mathbf{E}(t) + \frac{d\mathbf{r}}{dt} \times \mathbf{B}\right), \tag{W18.13}$$

with $\mathbf{B}$ along the positive z direction. For free electrons m_c is the cyclotron effective mass of the electrons (see Problem W18.1), whereas for bound electrons m_c is replaced by the free-electron mass, m. If the electrons are bound, then ω_0 represents an electronic resonance frequency of the medium, while for free electrons it may be taken to be zero. Assuming harmonic variations for $\mathbf{E}(t)$ and $\mathbf{r}(t)$ of the form $\exp(-i\omega t)$, one obtains the following equations for the amplitudes x and y:

$$(\omega_0^2 - \omega^2 - i\omega\gamma)x = -\frac{e}{m_c}(E_x - i\omega By) \tag{W18.14a}$$

$$(\omega_0^2 - \omega^2 - i\omega\gamma)y = -\frac{e}{m_c}(E_y + i\omega By). \tag{W18.14b}$$

Letting $x_{\pm} = x \pm iy$, $E_{\pm} = E_x \pm iE_y$, and $\omega_c = eB/m_c$ (the cyclotron frequency) gives

$$x_{\pm}(\omega) = -\frac{e}{m_c}\frac{E_{\pm}}{\omega_0^2 - \omega^2 - i\omega\gamma \mp \omega\omega_c}. \tag{W18.15}$$

The polarization vector of the medium is expressed similarly as

$$P_{\pm} = -nex_{\pm} = \chi_{\pm}\epsilon_0 E_{\pm}, \tag{W18.16}$$

where n is the concentration of oscillators. The relative permittivity or dielectric constant is $\epsilon_{r_{\pm}} = 1 + \chi_{\pm}$.

The wave vector is different for right- and left-circularly polarized light: $k_{\pm} = \omega\sqrt{\epsilon_{r_{\pm}}}/c$. Introducing the dielectric function for zero magnetic field,

$$\epsilon_{r_0} = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + i\omega\gamma}, \tag{W18.17}$$

where ω_p is the plasma frequency, one finds that

$$\epsilon_{r_{\pm}} = 1 - \frac{1 - \epsilon_{r_0}}{1 \pm (\omega\omega_c/\omega_p^2)(1 - \epsilon_{r_0})}. \tag{W18.18}$$

To first order in B, the difference in the wave vectors is

$$k_+ - k_- = \frac{\omega_c}{c}\left(\frac{\omega}{\omega_p}\right)^2\frac{(1 - \epsilon_{r_0})^2}{\sqrt{\epsilon_{r_0}}}. \tag{W18.19}$$

After propagating a distance z through the medium, this leads to a phase-angle difference,

$$\theta_F = (k_+ - k_-)z = \frac{e}{m_c c}\left(\frac{\omega}{\omega_p}\right)^2\frac{(1 - \varepsilon_{r_0})^2}{\sqrt{\epsilon_{r_0}}}Bz. \tag{W18.20}$$

The Verdet constant is therefore

$$V = \frac{e}{m_c c} \left(\frac{\omega}{\omega_p} \right)^2 \frac{(1 - \epsilon_{r_0})^2}{\sqrt{\epsilon_{r_0}}} \approx \frac{ne^3}{m_c^2 c \epsilon_0} \frac{\omega^2}{(\omega^2 - \omega_0^2)^{3/2} \sqrt{\omega^2 - \omega_0^2 - \omega_p^2}}, \quad \text{(W18.21)}$$

where the damping constant is neglected in the last expression.

This formula displays the factors influencing the size of the Verdet constant: the concentration of oscillators, the cyclotron effective mass of the carriers, and the resonance frequency relative to that of the light. In semiconductors, the effective mass could be small and the value of V could be large. In the neighborhood of an electronic resonance, the value of V could likewise become large.

Typical values for the Verdet constant for several nonmagnetic materials are presented in Table W18.1. It is customary to express V in arc-minutes/Oe·m, where 1 Oe = $1,000/4\pi$ A/m. A magnetic induction of $B = 4\pi \times 10^{-7}$ T corresponds to a field intensity H of 1 A/m. The Faraday and Kerr effects in magnetic materials are discussed in Chapter 17 of the textbook.[†] Magneto-optical applications are also given there.

An optical isolator may be constructed from a polarizer and Faraday rotator that rotates the polarization vector by 45°. If light is partially reflected from some interface

TABLE W18.1 Verdet Constants for Several Non-magnetic Materials

Material	λ (nm)	V (arc-min/Oe·m)	
Diamond	589.3	2.3	
NaCl	589.3	3.5	
KCl	589.3	2.8	
SiO$_2$	589.3	1.7	
B$_2$O$_3$	633	1.0	
Al$_2$O$_3$	546.1	2.4	
SrTiO$_3$	620	14	
ZnSe	476	150	
	496	104	
	514	84	
	587	53	
	633	41	
Tb$_2$Al$_5$O$_{12}$	520	−103.9	(300 K)
	520	−343	(77 K)
	520	−6480	(4.2 K)
KH$_2$PO$_4$ (KDP)	632.8	1.24	

Source: Data from M. J. Weber, *Handbook of Laser Science and Technology*, Vol. 4, CRC Press, Boca Raton, Fla., 1986; and D. R. Lide, ed., *CRC Handbook of Chemistry and Physics*, 75th ed., CRC Press, Boca Raton, Fla., 1994.

after it passes through the isolator, the direction of its electric field vector will be reversed by the reflection. As it propagates backward through the Faraday rotator, the electric field vector will experience a further 45° rotation. Since the field will then be perpendicular to the polarizer, it will be blocked by it. This prevents the reflected light from propagating backward and possibly causing damage to optical components.

W18.3   Theory of Optical Band Structure

Band-structure engineering may be applied to more complex structures than were considered in Section 18.6. In this section an analysis is given of one such structure, consisting of a one-dimensional periodic array. Each unit cell of the array contains two layers of transparent material with different indices of refraction. The propagation of electron waves in one-dimensional periodic structures is studied in Chapter 7, and it forms the basis for understanding the band theory of solids. Here the concept is extended to the optical case.

Consider the passage of light through two materials in the case where the photon energy is less than the bandgap. Barring any other absorption processes, both materials would, separately, be transparent. Next construct a stratified structure in which alternate layers of the two materials are stacked in a periodic fashion. It will be shown that for some wavelengths, propagation cannot occur and the structure acts as a mirror. Other colors, however, will pass through and the structure therefore acts as a color-selective filter. These effects come about due to the destructive and constructive interference of reflected light waves, in much the same way as electronic band structure results from the interference of scattered electron waves in solids.

Let the indices of refraction for the two materials be n_1 and n_2, and let the thickness of layer n_1 be b and the thickness of layer n_2 be $a - b$. The structure has a periodicity of size a (Fig. W18.3). For transverse waves propagating along the x direction, the problem of wave propagation reduces to solving the Helmholtz equation $[\nabla^2 + k^2(x)]E = 0$, where $k_1 = \omega n_1/c$, $k_2 = \omega n_2/c$, and E is the electric field of the light. The solution in medium 1 is

$$E(x) = A_j e^{ik_1(x-ja)} + B_j e^{-ik_1(x-ja)} \qquad \text{if } ja < x < ja + b, \qquad \text{(W18.22a)}$$
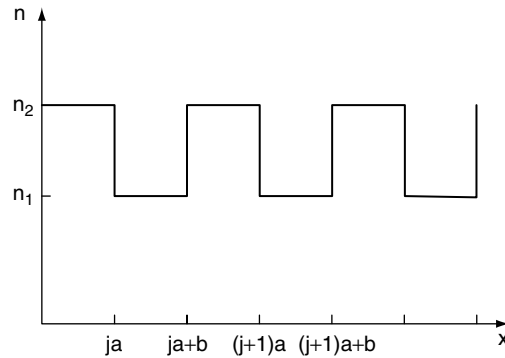

Figure W18.3. Stratified layers of optically transparent materials.

and in medium 2 is

$$E(x) = C_j e^{ik_2(x-ja)} + D_j e^{-ik_2(x-ja)} \qquad \text{if } ja + b < x < ja + a. \qquad \text{(W18.22b)}$$

Matching E and dE/dx at $x = ja + b$ yields

$$A_j e^{ik_1 b} + B_j e^{-ik_1 b} = C_j e^{ik_2 b} + D_j e^{-ik_2 b}, \qquad \text{(W18.23a)}$$

$$k_1 A_j e^{ik_1 b} - k_1 B_j e^{-ik_1 b} = k_2 C_j e^{ik_2 b} - k_2 D_j e^{-ik_2 b}. \qquad \text{(W18.23b)}$$

Repeating the match at $x = (j + 1)a$ yields

$$A_{j+1} + B_{j+1} = C_j e^{ik_2 a} + D_j e^{-ik_2 a}, \qquad \text{(W18.24a)}$$

$$k_1 A_{j+1} - k_1 B_{j+1} = k_2 C_j e^{ik_2 a} - k_2 D_j e^{-ik_2 a}. \qquad \text{(W18.24b)}$$

Let

$$\xi_1 = e^{ik_1 a}, \qquad \xi_2 = e^{ik_2 a}, \qquad \eta_1 = e^{ik_1 b}, \qquad \eta_2 = e^{ik_2 b}. \qquad \text{(W18.25)}$$

After eliminating C_j and D_j from the equations above, one arrives at the recurrence formula

$$\begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix} = M \begin{pmatrix} A_j \\ B_j \end{pmatrix}, \qquad \text{(W18.26)}$$

where the 2×2 transfer matrix $\mathbf{M}$ is

$$M = \frac{1}{4k_1 k_2} \begin{pmatrix} (k_1 + k_2)^2 \eta_2^* \eta_1 \xi_2 & (k_2^2 - k_1^2) \eta_1^* \eta_2^* \xi_2 \\ \quad -(k_1 - k_2)^2 \xi_2^* \eta_1 \eta_2 & \quad -(k_2^2 - k_1^2) \xi_2^* \eta_1^* \eta_2 \\ (k_1^2 - k_2^2) \eta_2^* \eta_1 \xi_2 & -(k_2 - k_1)^2 \eta_1^* \eta_2^* \xi_2 \\ \quad -(k_1^2 - k_2^2) \xi_2^* \eta_1 \eta_2 & \quad +(k_1 + k_2)^2 \xi_2^* \eta_1^* \eta_2 \end{pmatrix}. \qquad \text{(W18.27)}$$

Note that $\mathbf{M}$ is independent of the index j. The sum of the diagonal elements is called the *trace*:

$$\text{Tr}(\mathbf{M}) = \frac{1}{4k_1 k_2}[(k_1 + k_2)^2 (\eta_2^* \eta_1 \xi_2 + \eta_2 \eta_1^* \xi_2^*) - (k_1 - k_2)^2 (\xi_2^* \eta_1 \eta_2 + \xi_2 \eta_1^* \eta_2^*)]. \qquad \text{(W18.28)}$$

The determinant of the $\mathbf{M}$ matrix is 1.

The eigenvalues of the $\mathbf{M}$ matrix are defined as the roots of the characteristic equation

$$\begin{vmatrix} M_{11} - \mu & M_{12} \\ M_{21} & M_{22} - \mu \end{vmatrix} = 0 = \mu^2 - \mu \,\text{Tr}(M) + 1, \qquad \text{(W18.29)}$$

and are

$$\mu_{\pm} = \frac{1}{2}\text{Tr}(M) \pm \sqrt{\left(\frac{1}{2}\text{Tr}(M)\right)^2 - 1}. \qquad \text{(W18.30)}$$

The product of the two eigenvalues is equal to 1, the determinant. If both eigenvalues are real, one of them is larger than 1 and the other is smaller than 1. On the other

hand, if one of the eigenvalues is complex, the other is its complex conjugate and each eigenvalue has magnitude 1. If the eigenvalue is real, repeated application of the transfer matrix will cause the amplitudes A_j and B_j to grow exponentially with increasing j, leading to an unphysical situation. Under such circumstances, propagation is not possible. The condition for propagation is therefore that $\mu_{\pm}$ be complex [i.e., that $(Tr\mathbf{M})^2 < 4$]. This will define what is called a *propagation band*. The condition may be recast as the condition

$$\{(k_1 + k_2)^2 \cos[(k_2 - k_1)b - k_2 a] - (k_1 - k_2)^2 \cos[(k_2 + k_1)b - k_2 a]\}^2 < (4k_1 k_2)^2.$$
(W18.31)

In Fig. W18.4 the allowed propagation band for the special case $b = a/2$ is illustrated. Let

$$k = \frac{k_1 + k_2}{2}, \qquad q = \frac{k_2 - k_1}{2}, \qquad x = \frac{ka}{2}, \qquad y = \frac{qa}{2}. \tag{W18.32}$$

Then the propagation-band conditions are

$$y^2 \cos^2 y < x^2 \cos^2 x, \qquad y^2 \sin^2 y < x^2 \sin^2 x. \tag{W18.33}$$

Some wavelengths are able to propagate through the structure and others are blocked.

Typical materials for use in these devices, which may serve as either mirrors or filters, are TiO_2 ($n = 2.4$) and SiO_2 ($n = 1.46$). Other combinations are MgF_2 ($n = 1.38$) and ZnS ($n = 2.35$) or MgF_2 with TiO_2. A one-dimensional array of air holes in a Si strip on top of an SiO_2 substrate has been fabricated[†] which displays a 400-nm gap centered around $\lambda = 1.54$ μm.

To withstand bursts of light that may arise in pulsed lasers, one generally wants matched coefficients of thermal expansion and high thermal conductivity. The reason is that mismatched thermal expansion between successive layers will generate strains upon heating that could produce dislocations at the interface. Repeated thermal expansion may enlarge these dislocations and could eventually crack the material. The high
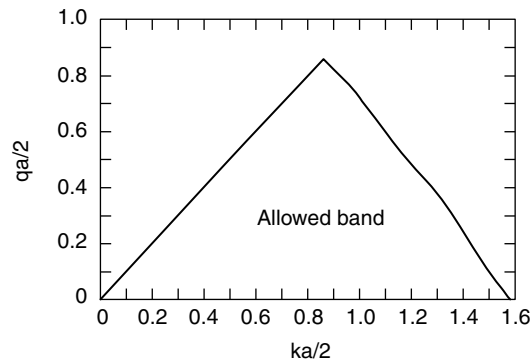
Figure W18.4. Region of parameter space for the propagation band.

[†] J. S. Foresi et al, *Nature*, **390**, 143(1997).

thermal conductivity permits the material to cool rapidly. Optical damage is considered further in Section W18.4.

The extension of the periodic structure to two or three dimensions has led to the construction of what are called *photonic crystals*. By creating an array of holes in a dielectric slab a photonic crystal operating in the microwaves has been built.[†] By stacking Si rods in a face-centered tetragonal array with air filling the interstices, it has been possible to fabricate[‡] a photonic crystal with a bandgap in the infrared (10 to 14.5 μm). Similarly, a periodic array of air-filled spheres in a titania crystal has been fashioned to serve as a photonic crystal in the visible region of the spectrum.[§]

Just as electrons may be localized in a medium with random scatterers, the same is true of electromagnetic radiation. Localization in the microwave region has been demonstrated by using a three-dimensional metal-wire network with random scatterers.[¶] It is clear that band-structure engineering is still at its early stage of development and that new and exciting developments are rapidly emerging in the field.

W18.4 Damage

Laser damage to optical components, such as laser crystals, mirrors, polarizers, fibers, electro-optic crystals, and prisms, is of concern in applications involving high power, in both pulsed and continuous wave (CW) operation. Due to the optical absorption, the materials heat up. Materials with a low heat capacity and low thermal conductivity are more likely to reach high temperatures. In layered structures the mismatch in thermal expansion coefficients can lead to crack formation and propagation. Typically, bulk damage results for 10-ns pulses when the power density is in the range 200 to 4000 TW/m^2.

One of the prime concerns is the phenomenon of self- focusing. This can occur in a medium with a positive value of the nonlinear index of refraction, $n_2 I$. A laser beam generally has a cross-sectional intensity profile with a higher intensity, $I(R)$, near the axis than away from it. A typical form for the profile is Gaussian; that is,

$$I(R) = \frac{2P_0}{\pi f^2} e^{-2(R/f)^2}, \tag{W18.34}$$

where R is the radial distance, P_0 the power in the beam, and f a measure of the beam radius. The nonlinearity causes a larger value for the index of refraction, $n(R) = n_1 + n_2 I(R)$, near the axis, when $n_2 > 0$. The medium behaves as a lens, and this tends to focus the radiation [i.e., make $f(z)$ decrease with increasing propagation distance, z]. However, there is a competing effect due to diffraction, which tends to defocus the radiation. This defocusing effect becomes stronger the smaller the value of f. There exists a critical value of P_0 for which the focusing effect of the nonlinearity dominates over the defocusing effect of diffraction and the beam focuses. When it does so, the focal spot can become as small as a wavelength of light and the intensity can become

[†] E. Yablonovitch et al, *Phys. Rev. Lett.*, **67**, 2295 (1991).

[‡] S. Y. Lin et al, *Nature*, **394**, 251 (1998).

[§] J. Wijnhoven and W. Vos, *Science*, **281**, 803 (1998).

[¶] M. Stoychev and A. Z. Genack, *Phys. Rev. B*, **55**, R8617 (1997).

very large. A crude estimate of the critical power may be obtained by setting $f = 1/k$, where k is the wave vector, and setting $n_1 \approx n_2 I$. This gives $P_{cr} \sim n_1/n_2 k^2$.

Often, the electric field of the light can exceed the strength of the typical electric fields in the solid and electrons can be accelerated to high energies, causing radiation damage such as atomic displacements. The highly concentrated beam could cause local melting, vaporization or ionization.

The situation is exacerbated when there are preexisting cracks or dislocations in the material. When subjected to the (uniform) electric field of the laser, the local electric field in the vicinity of the defect could be nonuniform, with particularly strong fields being generated near sharp features. The same effects occur near a lightning rod, where the strongest field occurs near the sharpest point. Local breakdown is likely to occur near the defect, often inflicting additional damage there.

Defects are usually introduced into optical components during their fabrication stage. For example, YAG is seen to have edge dislocations, helical dislocations, and zigzag dislocations. Laser crystals are often plagued by secondary phases of crystals mixed in with the primary phase. Bubbles are often present. These larger features can also serve as scattering centers which deplete the laser beam of power and couple their signals to other optical components. For this reason it is important that the optical components be largely free of defects before being used in high-power applications.

REFERENCES

Polarized Light

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PROBLEM

W18.1 The effective-mass tensor for an electron is diagonal in the *xyz*-coordinate system and has elements m_1^*, m_2^*, and m_3^*. A magnetic induction **B** is directed in an arbitrary direction. If the cyclotron resonance frequency is eB/m_c, find an expression for m_c.