# FUNDAMENTALS OF ADAPTIVE FILTERING 

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Readers are welcome to bring to the attention of the author at sayed@ee.ucla.edu any typos or suggestions for improvements. The author is thankful for any feedback.

## ERRATA

## Chapter 2

- Prob. 2.4: replace $\mathrm{E} \tilde{\boldsymbol{x}} W \tilde{\boldsymbol{x}}^{*}$ by $\mathrm{E} \tilde{\boldsymbol{x}}^{*} W \tilde{\boldsymbol{x}}$.
- Prob. 2.17: replace $\{\boldsymbol{v}, \boldsymbol{v}\}$ by $\{\boldsymbol{v}, \boldsymbol{w}\}$.


## Chapter 3

- Prob. 3.2, part (a): replace $c^{*} z=\alpha-c^{*} w^{o}$ by $c^{*} z=\alpha-c^{*} R_{u}^{-1} R_{d u}$.
- Prob. 3.3, part (c): replace "is now given by" by "is now related to".
- Prob. 3.7, part (d): replace $R_{\alpha}$ by $R_{z}$ and $\beta^{*}=b_{\mathrm{opt}}^{*}$ by $\left[\begin{array}{ll}1 & \beta^{*}\end{array}\right]=b_{\mathrm{opt}}^{*}$.
- Prob. 3.12, Fig. 3.5: the label $\boldsymbol{y}(t)$ should appear at the output of the channel $c(t)$; remove $\boldsymbol{y}(i)$.


## Chapter 4

- Prob. 4.9, part (a): replace " $<1$ " by " $\leq 1$ ".
- Prob. 4.21: replace the reference to Prob. 4.21 by a reference to Prob. 3.2.


## Chapter 5

- Prob. 5.25 , part (b): replace " $w_{i}$ that solves" by " $w_{i}$ with smallest perturbation to $w_{i-1}$ that solves".
- Prob. 5.26 , part (b): replace last $|h(i)|^{4}$ in $J(w)$ by $|h(i)|^{2}$. Also replace " $h\left(i_{o}\right)=1$ " by " $\left|h\left(i_{o}\right)\right|=1$ ".


## Chapter 7

- Prob. 7.3 , weight vector update: replace $\boldsymbol{U}_{i} w_{i-1}$ by $\boldsymbol{U}_{i} w_{i-1-\alpha(K-1)}$.


## Chapter 8

- Prob. 8.1, part (a): the expression for $\alpha$ should be scaled by $1 / 2$.


## Chapter 9

- Prob. 9.3, part (b): replace "negative-definite" by "indefinite".
- Prob. 9.4: replace $1-c$ by $1-\mu c$.
- Prob. 9.15, part (b): replace $\left\|\overline{\boldsymbol{u}}_{i}\right\|_{\bar{\Sigma}}^{2}$ by $\left\|\check{\boldsymbol{u}}_{i}\right\|_{\check{U}}^{2} \bar{\Sigma} \check{U}^{*}$. In the expression of part (d), remove the factor 2.
- Prob. 9.30, part (b): expression for $D$ should be $D=2 \mu \Lambda^{\alpha}-\mu^{2}\left(\Lambda^{\alpha}\right)^{2}$.
- Probs. 9.32 and 9.33: it is assumed in these problems that the nonstationary model is $\boldsymbol{w}_{i}^{o}=w^{o}+\boldsymbol{q}_{i}$ (i.e., it consists of random perturbations around a constant $w^{o}$ ) rather than as in item (2) of Prob. 9.31.
- Prob. 9.33: replace $\sigma_{v}^{2}$ by $\sigma_{\bar{v}}^{2}$.


## Chapter 10

- Prob. 10.5: the identity should read as follows: $u(i) *\left[e^{j \omega_{o} i} h(i)\right]=e^{j \omega_{o} i}\left(\left[e^{-j \omega_{o} i} u(i)\right] * h(i)\right)$.
- Prob. 10.9: first row of $C$ should be divided by $\sqrt{K}$ and not $K$.
- Prob. 10.11: replace $1 /(M-1)$ by $1 / M$.
- Last equation on page 607: rightmost term should be $e^{j \omega_{k} i}\left(\left[e^{-j \omega_{k} i} u(i)\right] * h(i)\right)$.
- Page 609: replace $R\left(z e^{j 2 \pi k / K}\right)$ and $R\left(e^{j\left(\omega+\omega_{k}\right)}\right)$ by $R\left(z e^{-j 2 \pi k / K}\right)$ and $R\left(e^{j\left(\omega-\omega_{k}\right)}\right)$. Also, $r_{k}(i)=$ $e^{j \omega_{k} i} r(i)$ and the last equation on the page should be $s_{k}^{\prime}(i) * r_{k}(i)=s_{k}^{\prime}(i) * e^{j \omega_{k} i} r(i)$.


## Chapter 11

- Prob. 11.13, part (a): replace (11.5.1) by (11.5.7). Also, replace " $\widehat{w}="$ by " $w_{s}=$ ".
- Prob. 11.16, part (a): replace " $\widehat{w}="$ by " $w_{s}=$ " and remove third line of [0 1] on the right.
- Eq. (11.9.11): replace $+b_{2}(i)$ by $-b_{2}(i)$.
- Prob. 11.28, part (b): ignore the correlations between $\left\{s_{1}, s_{2}\right\}$ and their shifted versions.


## Chapter 12

- Prob. 12.3: replace $P_{-1}=\Pi$ by $P_{-1}=\Pi^{-1}$.
- Prob. 12.5, part (a): $w$ should multiply $\operatorname{col}\left\{R_{N-1}, u_{N}\right\}$ on the right-hand side.
- Prob. 12.8, part (a): remove the ${ }^{*}$ from the second $U_{N}$ in recursion for $P_{N}$. In part (c), $w_{-1}=\bar{w}$.


## Chapter 13

- Prob. 13.10, part (a): replace $f^{2}$ by $a^{2}$.


## Chapter 14

- Alg. 14.6.1 and Prob. 14.5: replace $\xi$ by $\zeta$. Also, in Alg. 14.6.1, $\gamma_{M}^{-1}(i)=\zeta_{M}^{f}(i) \zeta_{M}^{-b}(i) / \lambda^{M}$.
- Prob. 14.10: it is assumed that $u_{i} \Psi=u_{i-1}$.
- Prob. 14.11, downdating step: replace $g_{i-1}^{L} \gamma_{L}^{-1 / 2}(i-1)$ by $-g_{i-1}^{L} \gamma_{L}^{-1 / 2}(i-1)$.
- Prob. 14.14, part (a): 2nd column of pre-array should read as follows:

$$
\left[\begin{array}{cccc}
{\left[\begin{array}{ccc}
u^{(1)}(i) & u_{i-1}^{(1)} & \ldots \\
\bar{L}_{i-1} & u^{(N)}(i) & u_{i-1}^{(N)}
\end{array}\right] \bar{L}_{i-1}}
\end{array}\right], \quad \text { where } \quad \bar{L}_{i-1}=\operatorname{diag}\left\{\bar{L}_{i-1}^{(1)}, \ldots, \bar{L}_{i-1}^{(N)}\right\}
$$

- Prob. 14.15: definitions should read as follows:

$$
\begin{aligned}
& u_{i}^{(3)}=\left[\begin{array}{lll}
u(i) u(i-1) & \ldots & u(i-M+2) u(i-M+1)
\end{array}\right] \\
& u_{i}^{(4)}=\left[\begin{array}{lll}
u(i) u(i-2) & \ldots & u(i-M+3) u(i-M+1)
\end{array}\right], \quad u_{i}^{(M+2)}=[u(i) u(i-M+1)] \\
& w^{o(3)}=\left[\begin{array}{lll}
w^{o}(0,1) & \ldots & w^{o}(M-2, M-1)
\end{array}\right] \\
& w^{o(4)}=\left[\begin{array}{lll}
w^{o}(0,2) & \ldots & w^{o}(M-3, M-1)
\end{array}\right], \quad w^{o(M+2)}=\left[w^{o}(0, M-1)\right]
\end{aligned}
$$

## Chapter 15

- Prob. 15.4: replace $\zeta_{m}^{b / 2}(i-1)$ by $\zeta_{m}^{b}(i-1)$ in expression for $\rho$.
- Page 914 , second equation: replace $\bar{f}_{M}(i)$ by $\bar{b}_{M}(i)$.
- Prob. 15.9: in the state estimator equations of parts (a), (b), and (c), a factor of $\lambda^{1 / 2}$ should multiply $p^{\bar{b}}(i+1 \mid i), p^{f}(i+1 \mid i)$, and $p(i+1 \mid i)$.


## Chapter 16

- Eq. (16.4.24): replace $u(i, M)$ by $u(i-1, M-1)$.
- Prob. 16.6: a matrix $\Psi$ should multiply the first two terms on the right-hand side from the left.
- Prob. 16.7: the reference is to Prob. 14.2. Also, replace $\beta_{M}(i)$ by $\breve{\beta}_{M}(i)$.
- Prob. 16.12: replace $\xi_{M}^{l}$ by $\xi_{M}^{l}(i)$. Also, in part (b), replace $a$ by $|a|$.
- Prob. 16.16: replace $\mathcal{V}_{k}(z)$ inside the sum by $\mathcal{V}_{m}(z)$.


## Chapter 17

- Eq. (17.2.18): replace $P_{N}$ by $P_{N-1}$.
- Prob. 17.3, recursion for $w_{i}$ : replace $\alpha$ by $\alpha^{-1}$.
- Prob. 17.5, part (a): remove $\epsilon$ from the denominator.
- Prob. 17.13, part (d): replace $\left\|u_{i}\right\|^{2}>\epsilon>0$ by $\left\|u_{i}\right\|^{-2}>\epsilon>0$. Also, replace $w_{i} \rightarrow w$ by $w_{i} \rightarrow w^{o}$.
- Prob. 17.14, part (c), rephrase as follows: "Follow arguments similar to part (d) of Prob. 17.13 to conclude that $w_{i} \rightarrow w^{o}$.
- Prob. 17.15 , part (b): replace $\mu$ by $\mu(i)$ inside the boxed expression.
- Prob. 17.21, part (b): min should be max.
- Prob. 17.22: in the definition of $J(w)$, remove the right-most equality that involves $v(i)$.
- Prob. 17.24, part (c), $m$ should be $M$.

