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### **I. SOLUTIONS TO SELECTED EXERCISES**

(IN PROGRESS)

### **II. ADDITIONAL EXERCISES**

The exercises in II.1 refer to specific error control or treatment designs, whereas the exercises in II.2 give a general description of the experimental situation and the reader is asked to identify the specific error control and/or treatment design (some of these exercises were used in MS qualifying examinations (closed book) at Virginia Tech).

#### **II.1:**

1. An experiment has been performed to compare 4 treatments, A, B, C, D, say. A CRD has been used with 20 experimental units, and a total of 60 observations have been obtained. The experimenter has identified the following comparisons as important:

A vs. average of B and C  
A vs. D  
B vs. C.

He has performed what he thinks is an appropriate analysis of the data, but he comes to you with the following questions:

- a.) “I have used an  $F$ -test with 3 and 56 d.f. to test the hypothesis that there are no differences among the treatments. Is that correct? I am surprised that with 15 replications for each treatment I do not get a significant result.”
- b.) “I have obtained the contrast sums of squares for the 3 contrasts identified above (using SAS), but they do not add up to the sum of squares for treatments. What is wrong?”
  - (i) Answer question a.) and explain to him why or why not what he did is correct. In case you think that he made an error, explain the correct

procedure by writing out the ANOVA table, giving sources of variation, d.f., E(MS), and appropriate SAS statements to obtain this ANOVA table.

- (ii) Answer question b.) and explain why in this case the contrast SS do not add up. For each contrast give an expression for its estimator, its standard error, and its sum of squares.
2. An experiment was conducted to compare 5 different management techniques for apple trees with respect to yield. The trial was laid out as a CRD and each management technique was applied to 10 trees. Prior to the trial the trees had been grown under one and the same management technique. For each tree the yield (in kilograms) for the 4-year period preceding the trial is available. At the end of the experimental period the yield (in kilograms) is obtained for each tree.
- (i) What are the experimental and observational units?
- (ii) In what way might the pre-trial yield be used in the evaluation of the experiment?

Suppose you are presented with the following partial ANOVA table:

Source	d.f.	Type I SS	Type III SS
Treatments		100	120
Pre-trial yield		93	
Error		132	
Total		325	

- (iii) Fill in the rest of the ANOVA table.
- (iv) Give the  $F$ -ratio for testing the hypothesis that there are no differences among the management techniques.
- (v) Has the use of the pre-trial yield been successful in reducing the variance for the treatment comparisons?
- (vi) How would you find out whether there exists a linear relationship between pre-trial and post-trial yield?
3. The following results ( $y$  = observation,  $x$  = covariate) were obtained from a  $2^3$  factorial in a CRD with 3 replicates for each treatment:

Trt. Comb.	(y, x)		
$a_0b_0c_0$	(7.1, 5.9)	(7.5, 7.0)	(6.7, 5.8)
$a_1b_0c_0$	(8.2, 5.6)	(7.9, 5.8)	(8.5, 6.3)
$a_0b_1c_0$	(9.0, 7.1)	(9.5, 6.4)	(10.1, 7.0)
$a_1b_1c_0$	(9.2, 6.1)	(9.0, 7.0)	(9.6, 7.3)

$a_0b_0c_1$	(12.1, 7.1)	(13.0, 6.9)	(13.2, 8.0)
$a_1b_0c_1$	(11.9, 6.2)	(11.5, 7.2)	(11.0, 6.8)
$a_0b_1c_1$	(14.3, 6.4)	(15.2, 7.3)	(14.8, 5.7)
$a_1b_1c_1$	(15.7, 7.5)	(16.0, 6.7)	(16.5, 8.1)

- (i) Estimate all main effects and interactions and obtain their standard errors.
  - (ii) Obtain the ANOVA table.
  - (iii) Compare the results from (i) and (ii) with those that would have been obtained by ignoring the covariates.
4. Suppose somebody comes to you for advice for designing an experiment. The client wants to compare 4 drugs using mice as experimental units. She has 24 mice available for the experiment. How should she perform the experiment?
- (i) In order to decide whether a CRD or RCBD should be used, what questions would you ask first?
  - (ii) Suppose the mice are not too different from each other, what error control design would you use?
  - (iii) What would be an appropriate model to analyze the data?
  - (iv) Suppose the mice come from 6 different litters, each litter containing 4 mice. What error control design would you use?
  - (v) What would be an appropriate model for the data from this experiment?
  - (vi) How would you test whether there are differences among the litters? **(Warning!)**.
  - (vii) Suppose for the design given in (iv) the client obtains 2 measurements of the same characteristic for each mouse. What would you call this design and what would be an appropriate model to analyze the data? Sketch the ANOVA for this model, giving sources of variation, d.f., and E(MS).

(MORE TO COME)

## II.2:

- 1\*. A horticulturalist wants to conduct a fertilizer experiment for growing lettuce. He has available 2 fields (on separate farms), each field consisting of 16 equal-sized plots, and the plots on each field are quite homogeneous. He wants to use combinations of 2 rates of nitrate (N), 2 rates of phosphate (P), and 2 varieties of lettuce (V). he is mainly interested in obtaining information about the main effects N and P, and the interactions  $N \times P$ ,  $N \times V$ ,  $P \times V$ . He believes that the three-factor interaction  $N \times P \times V$  is negligible. On each plot he wants to grow the same number

of plants and apply the fertilizer during the growing season. The amount of lettuce (in kg) produced is used as the observation for each plot.

The horticulturalist comes to you to discuss possible designs for his experiment and get your opinion on statistical matters. Specifically, he proposes three options:

Plan 1: Randomly assign each variety to one field and randomly assign each (N,P)-combination to 4 plots in each field.

Plan 2: Divide each field into half and randomly assign (in each field) each variety to one half of the field. In each half, i.e, 8 plots, then randomly assign each (N,P)-combination to 2 plots.

Plan 3: In each field randomly assign each (N,P,V)-combination to 2 plots.

- (i) For each plan give the names of the treatment and error-control design (identify, where appropriate, intrinsic and nonspecific blocking factors), write out an appropriate linear model, sketch the ANOVA table giving sources of variation and d.f., and indicate whether and how you can test hypotheses about the desired main effects and interactions.
- (ii) From a practical point of view, the horticulturalist prefers Plan 1. Plan 2 comes next, and Plan 3 is the least desirable for him. Explain which plan you would recommend based on statistical and practical considerations.
- (iii) For the plan chosen by you give the variances of the estimators for N, P, NP, NV, and PV.
- (iv) Devise and describe an alternative to the 3 plans given above that achieves the stated objectives of the study.

- 2\*. A food scientist wants to conduct an experiment to assess the effects of cold storage, light conditions and container material on food quality in supermarkets. She has contracted with 3 supermarkets that have each made available to her 4 refrigerated display cases (RDC) which can be controlled individually for temperature and light.

For a particular food product the scientist chooses 2 temperatures (3°C, 6°C), 2 light intensities (high, low), and 4 container types (sealed plastic, sealed cardboard covered with wax, sealed cardboard, open cardboard). In each supermarket she assigns randomly to each RDC a particular temperature-light combination such that each combination occurs once in each supermarket. Then she places one container from each type within each RDC. After a certain period of time she takes a quality measurement for the product in each container.

- (i) Describe what experimental design is being used. Give details (for example, if there are any blocking factors, explain whether they are intrinsic or nonspecific factors).

- (ii) Give an appropriate model for the observations from this experiment.
- (iii) Sketch the ANOVA table, giving sources of variation and d.f.
- (iv) In terms of the linear model given in (ii) specify the parameter contrast for the difference between the effects of  $3^{\circ}\text{C}$  and  $6^{\circ}\text{C}$ . Give the estimator for this contrast and an expression for its standard error.
- (v) Explain how you would test the hypothesis that there is no interaction between the factors temperature and container type.

(MORE TO COME)