1 Network Clustering

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Solutions to Exercises

Solution 1. Denote by $w(C)$ the sum of the weights of vertices in C for any $C \subseteq V = \{1, 2, \ldots, n\}$. Pick $v \in V$ such that $w(N[v])$ is a maximum. Add v to C (initialized empty), delete v and non-neighbors of v from G. Repeat this process until V is empty. Resulting C is a maximal clique and hence there does not exist another clique $C' \supset C$ with $\sum_{i \in C'} w_i \geq \sum_{i \in C} w_i$. Note that if we have vertices with negative weights, these cannot belong to a maximum weighted clique and hence can be deleted before-hand. Although trivial, it is important to note that a clique of maximum weight need not be a maximum size clique in G, but it will be a maximal clique when the weights are all positive. Finally, when the weights are unity, this is simply the greedy algorithm for maximal clique based on degrees.

Solution 2. Using the "cliquishness" formula we obtain the vertex weights as,

$$< w_1, w_2, \ldots, w_{12} > = < 1, \frac{2}{3}, \frac{1}{3}, 0, \frac{1}{3}, 0, \frac{2}{5}, \frac{2}{3}, \frac{1}{2}, \frac{1}{2}, 0, 0 > .$$

Table 1.1 Progress of weight-greedy maximal clique algorithm.

iter.	C	V	v such that $w(N[v])$ is max
0	$\emptyset$	$\emptyset$	5 (tie between 5,7)
1	{5}	{2,3,7,8}	7 (tie between 7,8)
2	{5,7}	{8}	8
3	{5,7,8}	$\emptyset$	-

Note that as V changes with each iteration, so do $N(v)$ and $w(N[v])$. In this example, we use static weights, that is w_i remain unchanged during the course of the algorithm. The "cliquishness" definition of vertex weights actually allows us to compute w_i for $i \in V$, dynamically in each iteration, and use those to compute $w(N[v])$. Furthermore, note that our solution is neither the maximum size clique $\{7, 8, 9, 10\}$, nor the maximum weighted clique $\{1, 2, 3\}$.

Solution 3. Given positive integer k, construct the power graph G^k, and apply the weight-greedy maximal clique heuristic to G^k assuming unit weights.

Solution 4. Apply the greedy minimal independent dominating set algorithm discussed in the chapter, on the power graph G^k.

Solution 5. Easy. Review the answer to the next question to understand what the clusters represent.

Solution 6. Given the connected graph $G = (V, E)$, construct a weighted complete graph $G_C = (V, E_C)$ with edges weighted by the shortest path length in G between the end points. Apply the heuristic referred to in the previous question to G_C. Suppose the maximum edge weight in a cluster V_i in G_C is k_i, then V_i is a k_i-clique in G. Hence, we have a 2-approximation algorithm for the *min-max k-clique clustering problem.*