**Answers to exercises of Chapter 3 (Global Network Properties)**

1. Name several typical characteristics of vertices and describe how they can be estimated. In particular, give examples for vertex properties that can be estimated using only information about the vertex and its neighbors and properties that relate to the topology of the network as a whole.

Properties of individual vertices are described in Section 3.2. We distinguish between local and global properties. While the former can be estimated solely based on information about a given vertex and its neighborhood, the latter requires knowledge of the entire network topology. Examples of local properties are the vertex degree or the clustering coefficient (Section 3.2.5). An example of a global property is the betweenness centrality of a vertex (Section 3.2.7).

2. Give examples of typical degree distributions. How is the degree exponent defined and how can it be estimated from a given empirical network.

Many biological networks exhibit a strongly inhomogeneous degree distribution, often consistent with a power-law of the form $p(k) \sim k^{-\gamma}$. In the simplest case, the degree exponent $\gamma$ can be estimated from a (binned) histogram of the vertex degrees. Possible pitfalls and alternative approaches are described in Section 3.2.3.

3. Why does a negative correlation between vertex degree and clustering coefficient indicate a hierarchical structure?

The clustering coefficient of an individual vertex quantifies the density of connections between its neighbors. A community, or module, is typically defined as a subset of vertices with a significant internal density of links. In a modular and hierarchical network the vertices with low degree are usually connected to other vertices inside their community, therefore, the number of connections between the neighbors is typically high. However, vertices with large degree are linked to vertices in different communities so that, on average, their neighbours are poorly connected to each other.

4. Are functionally related vertices always connected? If not, what are alternative approaches to define "relatedness" of vertices within a complex network?

As functionally related vertices are not necessarily connected, alternative approaches to define "relatedness" are discussed in Section 3.2.6. For example, two vertices may be considered related if they are connected to the same set of neighbors (corresponding to the Matching index). Other approached are discussed in Section 3.4.2 on community structures and modularity in networks.

5. What are 'small-world' networks? Is the Erdös–Rényi model a 'small-world' network?

The concept of small-world networks is discussed in Section 3.2.2. The Erdös–Rényi model is considered to exhibit the small-world property, as its average pathlength scales approximately as the logarithm of the size of the network.

6. What is the average clustering coefficient of Erdös–Rényi random networks?

For the ER-network model, the fact that two vertices share a common neighbor does (by construction) not affect their probability to be connected. The average clustering coefficient of Erdös–Renyi random networks thus equals the probability that any two vertices are connected. See also the Sections 3.2.5 and 3.3.1.

7. Outline how the Barabási–Albert network model is constructed. What are the main differences to Erdös–Rényi and Watts–Strogatz network models?

The Barabási–Albert network model is, unlike the other models, based upon a growth process: At each timestep, new vertices are added to the network. One of the most important features of the growth process is preferential attachment, i.e. the probability of a vertex to acquire new links is proportional to the number of links it has already acquired. A detailed description is given in Section 3.3.3.

8. Outline how an ensemble of random networks can be constructed that preserves the degree distribution of a given empirical network.

A commonly used strategy to randomize a given empirical network is to swap links between two vertices: A randomly chosen pair of links (a-b) and (c-d) are replaced by the pair (a-d) and (c-b). By construction, the degree distribution remains invariant, as neither vertex acquires or looses links. See also Section 3.5.1.

9. Outline a scheme to randomize a network, such that the three-vertex motif distribution is preserved.

Such a strategy was employed by Milo *et al.* (Ref. [44]). In the simplest case, only such randomizations are allowed that preserve the local properties (here: the motif) of the vertices.

10. What does the term “significant” mean in the context of statistical validation of network properties?

In any kind of statistical test, the term “significant”, or “statistically significant”, usually implies that a feature is unlikely to have occurred by chance, given a specified null-hypothesis is true. The term is only defined with respect to a null-hypothesis. Unfortunately, to construct non-trivial null-hypothesis for network analysis is often not a straightforward task. See Section 3.5.