Neural Mechanics

Robert L. Fry

The Johns Hopkins University/Applied Physics Laboratory
Laurel, MD USA 20723-6099
robert_fry@jhuapl.edu

Abstract — **Elements of classical, statistical, and even quantum mechanics can be found in the described neural model through analogous constructs of position, momentum, Gibbs distributions, partition functions, and perhaps most importantly, observability. Such analogies suggest that the subject model represents a type of *neural mechanics* that is distinguished from other mechanical formulations of physics in two important regards. First, physical constants are not constant, but rather represent Lagrange factors that vary over time in response to learning. Secondly, neural systems attempt to optimize the very information-theoretic objective functions upon which their structure is founded. This paper provides an overview of an approach to neural modeling and understanding and highlights correspondences between this model, mechanical formulations of physics, and computational neurophysiology.**

1. Overview

It is posited that information theory, as it stands, is an incomplete theory and that a dual theory exists that characterizes the transduction of information within physical devices as opposed to the transmission of information between physical devices which is the main emphasis of traditional information theory. Traditional information theory, together with its dual theory, may provide a succinct logical explanation of the physical theories of quantum, statistical, and even classical mechanics. However, the scope of this paper is more limited in that it highlights some basic properties of such a dual theory of information as they relate to the analysis of neural computation. A surprising result is that many known computational properties of cortical neurons can be deduced from the developed theory independent of biological assumptions. These properties include action potentials, Hebbian learning, nonlinear synaptic conductances, and other known computational properties of neurons which are described. The proposed model predicts other biologically-plausible properties as well which may validate the proposed model through focused physiological studies.

One can conceive that physics, as we know it, arises as a natural and logical consequence of a set of information laws that have been only partially formalized. In particular, consider the information and physics hierarchies shown in Figure 1. The left side of Figure 1 shows an information hierarchy which has the concept of distinguishability at the root and complexity at the highest level. The right side represents a hierarchy of mechanical formulations in physics; again with distinguishability at its root. Each intervening layer in both hierarchies arise through logical necessity from underlying and more fundamental layers. For example, Spencer-Brown [1] developed a mathematics of distinguishability with Boolean logic arising as one of its many consequences. Cox [2],[3] uses functional analysis to show that probability is the only logically consistent measure which generalizes to degrees of truthfulness or falseness on Boolean propositions or distinctions. Both Cox [3] and Jaynes [4] use a similar functional argument to show that entropy is the only logically consistent measure of degree of uncertainty on the part of an observer regarding which proposition of a set is the "true" state of nature.

$$\begin{Bmatrix}\text{Distinguishability}\\ \text{Structure of Logic}\\ \text{Probability}\\ \text{Entropy}\\ \text{Complexity}\end{Bmatrix} \Rightarrow \begin{Bmatrix}\text{Distinguishability}\\ \text{Quantum Mechanics}\\ \text{Statistical Mechanics}\\ \text{Classical Mechanics}\\ \text{Neural Mechanics}\end{Bmatrix}$$

$$\xleftarrow{\text{Info. Th.}}\text{Information Mechanics}\xrightarrow{\text{Physics}}$$

Figure 1

Finally, Rissanen [5] and Feder [6] show that the minimum descriptor length criterion generalizes formulations where maximized entropy would otherwise be used in the formation of a model by an observer of the observed information, e.g., an observer should form the model that has minimal complexity, yet is consistent with all observed information.

Distinguishability may also provide the basis for physical laws as suggested on the right side of Figure 1. More fundamentally, though, one can posit that basic information laws, perhaps realized through some type of *information mechanics*, dictates the essential details of both hierarchies shown in Figure 1 where both ultimately arise from a common notion of distinguishability. The idea that distinguishability and information furnish the basis of physics is not new and has been quantified to a large extent by Kantor [7], suggested by Wheeler [8], and discussed by many other physicists. Distinguishability is also a profound and persistent theme in Chinese philosophy being recognized as far back as 600 B.C. as found in the teachings of Lao Tsu [9] and also embodies the notion of the Yin and the Yang, i.e., that ultimately, concepts in its various forms arise in mutual relation to one another.

Distinguishability may then necessarily provide an information-theoretic basis for understanding neural computation. Neurons manifest distinguishability in two important regards that define their intrinsic nature as an observer and an assertor. First, neurons observe spatiotemporal patterns of input synaptic activity. These activity patterns induce the neuron to learn where learning is tantamount to symmetry breaking on the part of the neuron. That is, the neuron is driven to distinguish between patterns that generate action potentials and patterns that do not. These two classes of patterns of course only exist in mutual relation to one another. An action potential may be deemed an outward manifestation of a neural decision-making process. Regarding the second type of neural distinguishability, there is mounting evidence that neurons distinguish input potentials in a similar manner. That is, if a pre-synaptic potential is of sufficient magnitude, then a post-synaptic dendritic potential is generated which can then influence the somatic potential and which in turn affects the decision-making process of the same neuron. Even these two manifestations of distinguishability are complementary to one another in that one manifestation dictates what the neuron will assert (answer) while the other dictates what the neuron will inquire (asked). This duality is ubiquitous to many quantitative information-theoretic aspects of neural function which are summarized later in this paper.

Justification of the observed computational properties of biological neurons from an information-theoretic perspective has largely remained an elusive goal. One can speculate that perhaps information theory, as a mathematical tool, is somehow incomplete. That is, essential aspects of a more complete theory of information may exist that are important to understanding neural computation and which have not been formalized. In his original development of information theory, Shannon [10] provides modest clues regarding the nature of this incompleteness. In fact, Shannon explicitly obviates in both [10] and [11] an important aspect of information theory that is not considered in his seminal work. This aspect consists of that of the *meaning* of information and in fact in [11], Weaver develops the idea that the concept of meaning can be quantified using the same information measures developed by Shannon for the transmission of information.

Shannon dealt primarily with the transmission of information between a source and a receiver. However, the objective consideration of meaning requires the analysis of the transduction of information by a physical device in that meaning represents a continuous-valued assignment to observed information on the part of a measurement device (as noted by Weaver [11]). Both transmission and transduction are of intrinsic importance to the analytic treatment of processes of information flow *between and within* physical systems. This paper highlights the fact that neurological systems epitomize this perspective. In particular, interneural information exchange is accomplished through the use of action potentials which represent discrete events in both time and space. Alternatively, intraneural information transduction occurs through a sequential process of: (*i*) *Observation*; (*ii*) *Assimilation*; (*iii*) *Conclusion*; and ultimately, (*iv*) *Decision*. This paper summarizes how interneural transmission can be treated in a more or less classical manner, while intraneural transduction requires the acknowledgment of a complementary aspect of information theory that specifically addresses semantic aspects of information theory in an objective manner. This complementary aspect of information theory is the primary focus of this paper

and is thought to give rise to the many relationships between the described neural model and traditional mechanical formulations in physics.

2. Highlights of the Described Neural Model

Although there is insufficient time to exhaustively cover every detail and ramification of the subject model, the essential approach and the primary results are delineated. Neural processing in space and time are both briefly covered under the aegis of distinguishability. Spatial domain processing is seen to have a statistical mechanics formulation including Gibbs' distributions, partition functions, and temperature-like construct. Alternatively, time domain processing requires the consideration of position and momentum constructs by way of analogy to classical mechanics. Neural observability is seen to be a key issue in both time and space processing and hence lends itself to a quantum-mechanical interpretation.

It is posited that individual neurons are rational in that they use probability (or a monotonic function thereof in the form of somatic energy), in their rational decision making. Action potential can be viewed as a logical decision arising as a consequence of a rational and structured decision-making process on the part of the neuron, or equivalently, an assertion in response to observed spatiotemporal activity regarding a posed question that is intrinsic to the neuron. From this perspective, a neuron functions as a passive observer or measurement device that awaits observable information upon which it will subsequently (*i′*) *Measure*, (*ii′*) *Process*, (*iii′*) *Infer*, and then outwardly (*iv′*) *Decide*. Decisions take the form of an action potential or the lack thereof. The methodology by which the neuron derives its intrinsic question and decision rule must formally correspond to learning. For the sake of conciseness in the following discussions, denote the subject model neuron by q.

The following questions can be asked of the conceptual neural model described so far.

(1) How does the propositional question intrinsic to q become defined? This question is synonymous with the question of how a neuron, which functions as a measurement device, is driven to determine what it will measure.

(2) Assuming that the interrogative proposition has been formed, how then does q draw conclusions and generate rational decisions to this intrinsic question given that particular patterns of spatiotemporal activity are observed?

(3) Finally, how should neural behavior vary over time in response to statistical changes to its external or environmental input? Moreover, in acknowledgment of the eventual loss or denial of information on which decisions are made, how should q maintain function which is somehow useful to the organism in the long-term?

If one precisely delineates the information observable to q, then it has been shown [12]-[15] that a maximum entropy (ME) formulation dictates the architectural structure of q which formally corresponds to a Hopfield [16] neuron paradigm for specified neural measurement functions. The set of measurements which the neuron can perform are specified by the vector $f = (x_1 y, x_2 y, x_3 y ,..., x_n y, y)^T$, where $x_j y$ denotes the Boolean conjunction of input x_j and the output decision y. The presence of the output y alone in the vector f signifies neural observability of its own output. Note that each component of f must be interpreted as a neural measurement in both space and time since the conjunctive operator will sometimes evaluate *true* and sometimes *false*.

A simple Gibbs distribution and corresponding partition function arise from this formulation which are described by

$$P(\boldsymbol{x}, y) = \frac{\exp[-\lambda^{\mathrm{T}} \boldsymbol{x} y + \mu y]}{Z} \tag{1}$$

and

$$Z = \sum_{x'\in X} \sum_{y'\in Y} \exp[-\lambda^{\mathrm{T}} \boldsymbol{x}' y' + \mu y'] \quad , \tag{2}$$

respectively. However, there are several problems with this approach that include: (*i*) the under constrained nature of the problem due to the lack of the supervised specification of moments required for an ME formulation; (*ii*) its biological implausibility due to mathematical intractability in solving the nonlinear ME problem; and (iii) the lack of a clear and meaningful objective function for obtaining values for the Lagrange factors which formally correspond to the synaptic efficacies and decision threshold, respectively. References [12] and [13] show that a maximized mutual information (MMI) criterion eliminates all these problems and yields a simple computational modality that is in close agreement with physiological models and measurements. Some areas of agreement are listed in Table I and will be highlighted during the paper presentation. The resulting adaptation equations and neural structure describe neural function in space.

Neural processing in time can be addressed in a similar fashion using the same MMI objective function used for ascertaining the neural architecture just described. The most important consequence of this analysis is the prediction of a set of adaptable parameters τ which are conjugate to the synaptic efficacies λ, where both sets of parameters are modified through a common Hebbian adaptation mechanism. Whereas λ serves to effect neural MMI in space, the adaptation of τ serves to effect MMI in time through the temporal equalization of *meaningful* synaptic inputs relative to the soma, where the term *meaningful* here denotes those inputs which have been successfully encoded by q and play a direct role in its decision-making process. Table II highlights some important results regarding neural temporal adaptation which requires the consideration of a construct analogous to momentum where mass corresponds to the synaptic efficacy, velocity corresponds to the time-derivative of the input potential, and the neural output $y(t)$ enables measurement of the momentum by q. Another important result regarding neural processing in time, is the derived requirement for a stochastic mechanism that is necessary to effect the discussed spatial and temporal adaptation mechanisms and which is consistent with integrate-and-fire dynamical neural models. This stochastic mechanism may functionally arise through inputs that are *non-meaningful* to the neuron where the term non-meaningful denotes the uncoded synaptic inputs that are complementary to the meaningful inputs and which ultimately give rise to a Wiener process at the soma.

Spatial and temporal MMI considerations describe the transduction of information *within* the model neuron. In addition, a further consideration of this transduction process indicates how transmission of information occurs *between* neurons. The entirety of information flow between neurons can be understood through the following four exhaustive information exchange cases.

(1) An action potential is sent and then measured by another neuron waiting to see it.
(2) An action potential is sent, but the other neuron is not waiting to measure it.
(3) No action potential is sent, but the other neuron waits to measure it anyway.
(4) No action potential is sent and the other neuron is not waiting to measure it anyway.

Those familiar with coding theory can recognize the above as being realizable through a pulse-code modulation scheme or PCM. There are many advantages of such a coding scheme [17] in a neural setting including efficient use of the average neural transmit power, simplicity of signal modulation and demodulation, optimality over virtually all other modulation schemes, precise signal regeneration, almost optimal performance for low average signal-to-noise ratios, and minimal intersymbol interference from other proximal transmission lines such as in a dense neural environment. Perhaps most importantly, the information capacity C in units of bits/s of each interneural channel must abide by the Shannon [10] limit

$$C = B \log_2 \left(1 + \frac{P_{avg}}{N} \right) \quad ,$$

where the channel has bandwidth B and white Gaussian noise with in-band noise power N is added to the transmitted action potential subject to an average neural transmit power constraint P_{avg}. Equation (3) holds regardless of the type of modulation scheme used to transmit information through the channel. Table III suggests biological correspondence of various aspects of neural function with computational functions derived within the framework of the subject neural model, including the implementation of PCM through nonlinear postsynaptic activations in response to input action potentials.

3. Summary

This paper describes how many neural properties can be derived and described in terms of a type of neural mechanics by way of analogy to other mechanical formulations in physics. The subject model is easily extended to networks of connected neurons which is then described by a composite Gibbs distribution and a corresponding energy function which is the sum of the energies of the component neurons. The neural significance and interpretation of *meaning* in the present model is clear. Meaning corresponds to the continuous-valued somatic evidence $\zeta(x) = \lambda^T x - \mu$ computed by the neuron in response to observed information and used by the neuron in rational decision making.

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λ	Lagrange multiplier vector corresponding to synaptic efficacies and describing the question intrinsic to the neuron in generating its answers.
μ	Lagrange multiplier corresponding to the decision threshold. It describes how answers are generated by the subject neuron in response to measured information and in turn serves to modify the intrinsic neural question.
$\lambda^T x(t)$	Time-varying somatic potential induced by dendritic inputs which can be coded by the subject neuron.
$I(X;Y)$	Maximized mutual information of the neuron realized through the simultaneous maximization of $H(Y)$ and minimization of $H(Y\|X)$.
$H(Y)$	Maximized through the adaptation of the decision threshold μ, and serves to optimize neural transmission of information.
$H(Y\|X)$	Minimized through the adaptation of λ and conditionalized principal components analysis; important to neural reception of information.
$\zeta(x)$	(*i*) Formally corresponds to *statistical evidence* as defined by the log of the odds function and mathematically described by log $\Pr\{Firing\|x\}/\Pr\{No\ Firing\|x\}$. (*ii*) A *sufficient statistic* for firing; i.e., it contains all the information necessary for the neuron to make a decision. (*iii*) Provides a maximum-likelihood decision rule in the limit as $T \to 0$. (*iv*) Provides the optimum nonlinear minimum mean-squared-error estimator for y. (*v*) $E_q = -\zeta(x)y$ is the neural energy function. (*vi*) The potential $\zeta(x)$ must be computed at some point within the soma.
$P(y=1\|x)$	The probability of action potential as conditionalized on the current input x. This is a sigmoidally shaped function of the evidence $\zeta(x)$ and the auxiliary time-varying temperature parameter T.

Table I: Key results of the analysis of neural function in space along with some noteworthy properties of the evidence function $\zeta(x) = \lambda^T x - \mu$.

Condition:	Output: $y(t)$	Efficacy: λ_i	Derivative: $\partial x(t-\tau_i^d)/\partial t$	Explanation
1	○	○	○	No adaptation due to lack of output
2	○	○	●	No adaptation due to lack of output
3	○	●	○	No adaptation due to lack of output
4	○	●	●	No adaptation due to lack of output
5	●	○	○	No or little adaptation due to minimal or nonexistent synaptic efficacy
6	●	○	●	
7	●	●	○	No input or equilibrated adaptation
8	●	●	+/–	Temporal adaptation occurs

Table II: Summary of the eight conditions under which the input momentum as defined by $p_i = \lambda_i x_i(t-\tau_i)y(t)$ is zero. The symbol ○ denotes zero; ● denotes non-zero. The notation +/– denotes the positive or negative derivative, respectively, of the input action potential. Temporal adaptation only occurs in case 8.

Neural Function and Association	**Speculated Computational Importance**
Modified Oja's equation	**A means of implementing the optimal conditionalized learning rule:** $\lambda(t+\Delta t)=\lambda(t)+\pi\zeta(\boldsymbol{x})[\boldsymbol{x}(t)-\gamma^2\zeta(\boldsymbol{x})\lambda(t)]$
Hebbian paradigm	**Provides for feedback for maximized mutual information through spatiotemporal adaptation. This allows the minimization of conditional entropy *H*(*Y*|*X*) and temporal equalization of coded inputs.**
Decision threshold	**Provides for optimal decision making to maximize the output entropy denoted by *H*(*Y*). Adaptation is driven by the rule:** $\mu(t+\Delta t)=\alpha\mu(t)+(1-\alpha)\zeta(\boldsymbol{x})$
Synaptic efficacies	**Correspond to the Lagrange multipliers for microcanonical neural distributions.**
Temporal delay parameters	**Are the temporal equivalent to the synaptic efficacies. They serve to equalize arrival times of potentials at the soma. Temporal adaptation is driven by** $\frac{d\tau_i^d}{dt}=y(t)\lambda_i\frac{dx_i(t-\tau_i^d)}{dt}$
Nonlinear postsynaptic conductance	**Implements quantization of induced somatic charge as measured by the subject synaptic efficacy and provides for PCM reception.**
Sigmoidal activation function	**Implements optimal inference and learning through stochastic adaptation and neural decision making.**
Action potential	**Represents the simplest decision/indicational form for transmission of neural "decision" and also represents a very energy-efficient way of transmitting information between neurons through PCM.**
Distal coincidence detection	**Provides for temporal adaptation at sites providing the best temporal resolution and sensitivity.**
Dendritic spines	**Provide substrate for channel group delay and advance modification and are the temporal analog to efficacy modification. In fact, spines may implement both functions.**