Robust Oscillations and Bifurcations in Cellular Neural Networks

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Abstract

A software package for studying discrete time Neural Networks dynamics is presented as an efficient analysis tool having also capabilities for designing special purposes cellular neural networks (CNN's) such as period controlled oscillators , noise generators and chaotic based systems. Based on an information theory approach, an entropy associated with a given structure of the network was defined as a global descriptor of the system dynamics. Any kind of discrete time neural model with any size is allowed, including CNN's as a particular case. By performing two or one dimensional analysis trough the weights space, some new properties of the opposite-template CNN's were discovered, e.g. the existence of robustness domains which means that small controllable change in weights values implies no change in the network's entropy. Using this software package, robust chaotic networks capable of generating white-noise like signals were also discovered.

1 Introduction

Cellular Neural Networks (CNN's) [1] are analog parallel computers which have proved very suitable for VLSI implementations [2]. Most of the CNN's applications are based on relaxation dynamics, the network converging towards a stable state which is the solution of the problem presented as an initial state or as input signals.

Oscillatory dynamics, including chaotic evolutions were also reported in CNN's [3] and there are opinions [4] that biological perception and memory functions are based on such kind of dynamics rather than on relaxation one. Using mathematical tools related with the field of Neurodynamics [5], some aspects related with oscillatory CNN's were studied [3] [6] and some interesting behaviors were also emphasized. From an engineer point of view developing methods for designing CNN's with oscillatory and chaotic

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behavior dynamics represents a fruitful possibility to embed already cited applications like secure transmission systems [7] or noise-like generators into an easy to be implemented paradigm like the CNN one.

In order to avoid long time learning cycles in reccurent networks some "prior knowledge" about the relationship between network's structure and it's dynamics is needed. In this paper an experimental approach to the above mentioned problem is presented, based on searching trough one or two-dimensional sections in the weights space. Some interesting considerations about weight robustness, bifurcation regime and chaotic behaviors were derived using our experimental method.

2 Mathematical Model

A Discrete Time Cellular Neural Network (DTCNN) can be modelled as a particular case of a full interconnected reccurent network by taking the weight matrix W as a band matrix. The dynamics of the discrete-time neural network is modelled by one of the following equation:

$$X(t+1) = W * F(X(t)) + I, \qquad t = \overline{t_0, ..t_0 + T_a} \tag{1}$$

where the structure of the network is defined by the N dimensional weight matrix $W = \{w_{i,j}\}$, by the threshold vector I and by the vector of activation functions $F(X) = (f(x_0), f(x_1), ..f(x_{N-1}))$. X represents the network state vector and Y is the output (activation) vector. For each neuron $y_i = f(x_i)$, where i is the neuron index; $i = \overline{0, ..N-1}$.

3 Tools for the dynamic behavior description

Simulation program description

In order to obtain "prior knowledge" about the dynamics of the CNN's, one or two variable weights (including thresholds) can be selected as free parameters and also the domain where these parameters can vary. For each fixed set of parameters, a well defined structure $\mathcal{N}$ of the neural network is runned during the analysis period T_a and the trajectory described by the neuron's states in the N dimensional space is recorded. Based on this trajectory an *entropy* is then evaluated as a global descriptor of the network dynamics. Transitory regime is avoided by performing a "hidden" running for the first t_0 periods. The set of entropies computed for different parameters is then displayed in one or two-dimensional fashion, depending on the number of free parameters that was chosen.

Neural network's entropy

A Neural oscillator can be viewed as an information generator, each distinct status vector being associated with a symbol s_j, $\quad j = \overline{j = 0, ..Ns}$ from an alphabet of Ns possible symbols. For each symbol a probability $p_j = \lim_{T_a \to \infty} ns_j / T_a$ is assigned, where ns_j is the number of symbols s_j generated by the network during the analysis period T_a. Network's $\mathcal{N}$ entropy is then defined as: $H(\mathcal{N}) = -\sum_{j=0}^{Ns} p_j \log p_j$. Because the transitions in the state space are defined in a deterministic manner by equation (1), for periodic oscillations $Ns = T$ and for each symbol $p_j = 1/Ns$ so $H(\mathcal{N}) = \log Ns$.

For aperiodic or chaotic oscillations, $H(\mathcal{N}) \rightarrow \infty$, thus a measure-theoretic entropy inspired by [8] was introduced as follows: $h(\mathcal{N}) = \lim_{dx \rightarrow 0} lim_{T_a \rightarrow \infty} \frac{2^{H(\mathcal{N})}}{T_a}$, where dx is the smallest quantum of the quantified state space. For periodic oscillations $h(\mathcal{N}) = 0$ because the alphabet of symbols is finite while for aperiodic behaviors including chaotic ones this entropy has a sub-unitary positive value. For some chaotic behaviors, it is possible to generate at any moment a symbol that was never generated before and thus $H(\mathcal{N}) = T_a$, so $h(\mathcal{N})$ reaches it's maximum value which is 1. In our simulations we have used $Ns(T_a)$ as global entropic descriptor instead of $H(\mathcal{N})$.

One-dimensional analysis

By changing only one weight parameter within a defined domain two kind of one-dimensional analysis representation can be performed:

- In *Feigenbaum plots* (FP or bifurcation diagrams), $\{\| X(t) \|\}$ is plotted versus free parameter (weight), where $\{\| X(t) \|\}$ is the set of Euclidian norms of the network states computed during the analysis period.
- In *one-dimensional entropic plots* (1DEP's), the global entropic descriptor Ns is plotted versus free parameter.

Two-dimensional analysis

In *two-dimensional entropic maps* (2DEM's), each pixel $p(x, y)$ of the map has a brightness proportional with the global entropic descriptor computed for the network $\mathcal{N}$ defined by two fixed parameters (weights) associated with the (x, y) coordinates of it. The upper left corner pixel is associated with minimum values of the free parameters while the lower right corner pixel with the maximum ones. Black pixels represents stationary dynamics (fixed points attractors) while white pixels are associated with rich dynamic behavior including chaos.

Robustness zones and bifurcations

The connex zones in the 1DEP's and 2DEM's are called *robustness zones* . They are segments in 1DEP's and constant brightness patterns in 2DEM's. The greater the area of such a zone is, the more robust is the network for changes of the weight parameters. The maximum stability for weight perturbations is obtained by choosing the weight parameters corresponding with the "gravity" center of the robustness zone. Bifurcations are changes in the dynamical behavior and they can be easily recognized in 1DEP's as discontinuities and in 2DEM's as borders between patterns.

4 Simulation Results

The simulation results presented bellow are based on choosing an opposite template CNN as basic architecture. Small sizes ($N = 3$) networks were chosen in order to make comparisons with continuous time CNN's simulations described in [6].

Neural network's structure

Three different activation functions were used for the results presented bellow:

- Linear saturated with programable gain m : $f1(x) = \frac{|m*x+1|-|m*x-1|}{2}$
- Bipolar sigmoidal function, with programable gain ms : $f2(x) = tanh(\frac{ms*x}{2})$
- Saturated modulus function, is a non-monotonic activation function we have introduced and it is very similar with a class of fuzzy-membership function.

Others activation functions are also available, including the fixed-point overflow function used by L.Chua in [9] and the cubical function similar with the one used in the van-der-Pol oscillators. It is also possible to simulate finite-precision arithmetic with a given number of bits.

The weight matrix was chosen as follows: $$W = \begin{bmatrix} 1 & -1 & 0 \\ 1 & p0 & -1 \\ 0 & 1 & p1 \end{bmatrix} \quad (2)$$

where $p0$ and $p1$ are the free parameters ranging in the domain $(-3, 3)$ for the simulations presented bellow. Free parameters $p1$ and $p0$ corresponds with horizontal and vertical axes of the 2DEM's.

Two-dimensional analysis results

In Fig. 1 four 2DEM's obtained for different activation functions are presented. For the first three 2DEM's , monotonic activation functions were used while for the fourth the non-monotonic saturated modulus activation function was used.The non-monotonic activation function implies rich dynamic behavior also emphasized in [9] for another kind of non-monotonic nonlinearity (the fixed-point overflow).By examining the four 2DEM's the following conclusions were derived:

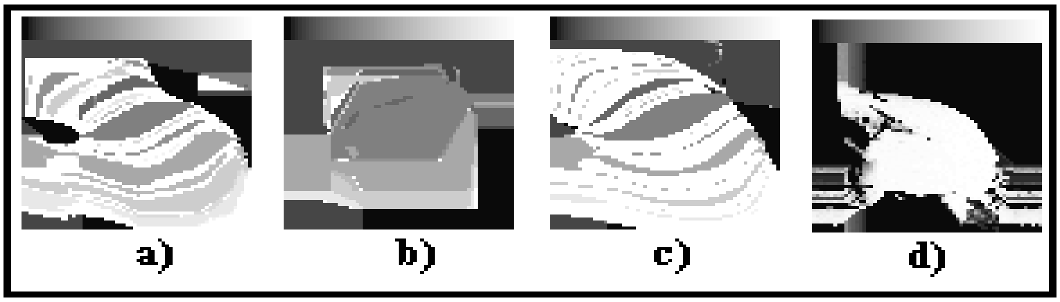

Figure 1: Two dimensional analysis for different activation functions: a) Linear saturated ($f1(x)$) with $m = 1$; b) Linear saturated with $m = 3$; c) Sigmoidal with $ms = 1$; d) Saturated modulus

- By using high gain activation functions (Fig.1b) the probability of chaotic or aperiodic behavior (small white areas) occurrence is reduced, and it seems to be a relationship between the array of a robustness zone and the associated entropy. Indeed, the more bright a zone is, the more smaller is it's area.
- By using small gain activation functions (as in Fig.1a and Fig.1c) high period and chaotic oscillations occurrence (large white zones) is encouraged but the exact shape of the nonlinearity seems to have very little influence on the global dynamic behavior (small differences between figures 1a) and 1c)), the gain parameter being the same.

- Using saturated modulus function an interesting chaotic behavior was discovered. The chaotic attractor is presented in Fig.2a .The autocorrelation function (Fig. 2c) for 512 samples (Fig. 2b) shows that the signal has the statistical properties of a white-noise signal. The measure-theoretic entropy reaches it's maximum value; $h(\mathcal{N}) = 1$. More interesting, there is a large robustness zone (see the white shape in Fig.1d) for which the above cited properties are the same.

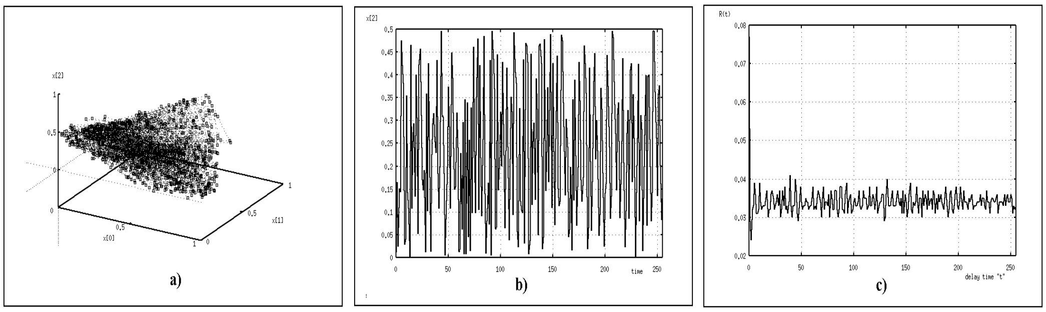

Figure 2: Robust chaotic network: a) The chaotic attractor; b) Projection on the x_2 axis (time sequence); c) Autocorrelation function

One dimensional analysis results

One dimensional analysis can be viewed as making vertical or horizontal sections trough the 2DEM's. In Fig.3a) and b) the results of the two kinds of one-dimensional analysis are presented by making vertical sections trough Fig.1a for $p1 = 0$. Three different behaviors are emphasized in FP and 1DEP as follows bellow:

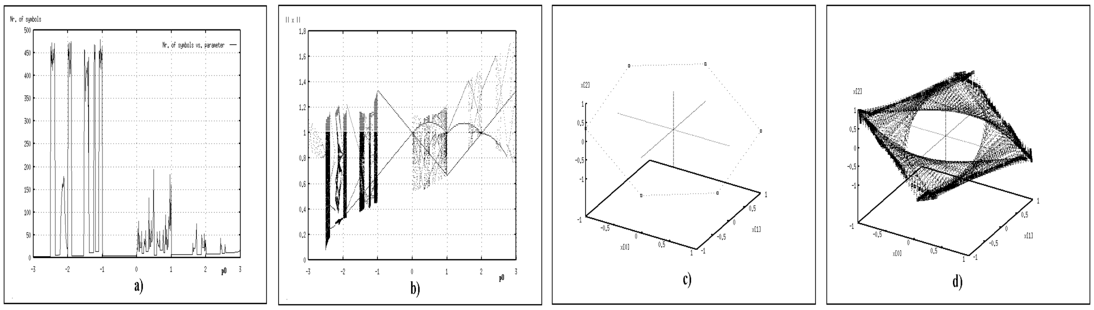

Figure 3: One dimensional analysis results: a) 1DE Plot; b) Feigenbaum plot; c) A periodic attractor ; d) A chaotic attractor

- Zone 1 (The six black vertical bands in FP) is characterized by high entropies $H(\mathcal{N})$.By doubling T_a the measured-theoretic entropies were computed and they have positive values, situation that is specific for chaotic oscillations [8]. A chaotic attractor corresponding with the sixth band ($p0 = -1.05$) is displayed in Fig. 3d .

- Zone 2 (dotted bands in FP) is also characterized by larger $H(\mathcal{N})$ entropies, but measured-theoretic entropies are 0, emphasizing periodic oscillations with large periods ($Tmax \approx 200$) .

- Zone 3 (horizontal segments in the 1DEP) is characterized by small entropies (e.g. periods). These segments corresponds with relatively larger robustness zones and they are alternating with bands from the zones above mentioned. For example, taking $p0 = 1.35$, period 6 oscillations are achieved (Fig.3c).

5 Conclusions

Using an information theory approach, a set of experimental tools was developed. Experiments carried out with these tools allowed us to emphasize some global relationships between the structure and the dynamics of the cellular neural networks. Previously discovered phenomena in continuous time CNN's [3], [6] and in digital filters [9] were rediscovered also in discrete-time systems. By introducing a new kind of nonlinearity function ("saturated modulus") a interesting chaotic behavior was discovered in DT-CNN's. The *robustness zone* concept we have introduced and the experimental results related with it, joined with the already proved VLSI implementability of the DT-CNN's are results which can enrich the range of oscillatory CNN's applications.

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