Inverse numerical filters
for linearisation of loudspeaker's response

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Abstract

In this paper a non-linear loudspeaker model, which accurately reproduces the low frequency behavior, is presented. This description, derived from an extension of the well known Small-Thiele equations, requires far less computational time and memory space than generic non linear structures. Moreover a noticeable further reduction of the number of operations and of the memory cells required has been achieved by means of a multirate architecture. Inversion of the proposed model allows digital prefiltering of the electrical signal in order to compensate for the non-idealities of the electro-acoustic conversion. The above filter structure implemented on a digital signal processor, placed between the audio signal source and the power amplifier allows effective compensation of loudspeaker linear (both magnitude and phase) and non-linear distortion. Measurement results obtained with a commercial woofer are discussed

1 Introduction

In this paper a numerical model of the non-linear behavior of loudspeakers is presented; it accurately reproduces the low frequency response, as it was checked by comparison with experimental data.

From the loudspeaker's model, an inverse filter was derived, implemented in the form of a variable FIR filter on a commercial low-cost DSP. Inserting this filter along the signal path (before the power amplifier, which is considered inherently linear), the loudspeaker's moving coil is driven by a modified signal; the modification is designed to be opposite to the non-linear distortion, so that the actual displacement of the moving coil accurately matches the ideal motion prescribed by the original signal. This means that not only the harmonic distortion is reduced, but also the linear frequency response is equalised and even the phase response is smoothed. The implementation on the DSP module makes use of a multi-rate processing, ensuring optimal filtering in the most critical low-frequency region, and a less effective filtering at high frequencies, where a shorter filter length is required. The experimental results obtained using a commercial woofer for car audio applications show that the new algorithm is effective in reducing the harmonic distortion and in equalising the response. The paper fully describes the new mathematical theory for describing the not-linear response, the algorithm for producing proper inverse filtering, the measurement technique for characterising the loudspeaker (section 2) and the experimental results obtained with the actual implementation of the numerical inverse filter on a car-audio sound system (sections 3 and 4).

2 Loudspeaker non-linear modeling

Preliminary audio processor implementations aiming at a reduction of distortion are already described in literature following either a closed-loop or an open loop control approach [1], [2], [3], the latter being possible only thanks to the availability of accurate loudspeaker models. The field is extensively studied in the literature [4], [5], but to the best of our knowledge, the available models rely on measurement results rather than on first principles and construction parameters.

Until now loudspeakers have been modeled, following the well-known Small-Thiele approach [6], that describes their behavior at low frequencies and small input power, and produces a linear transfer function. Unfortunately it does not match our requirements; first of all because it is a liner model and moreover as it does not describe accurately real Sound Pressure Level (SPL) curves. In order to describe non-linear systems some general structures have been presented in literature [7], [8]. Among them, the Volterra series expansion [4] seems useful for describing weak non-linearities. Due to the great number of operations required, this structure cannot be adopted for real time applications.

Direct-radiation loudspeakers can be modeled at first using lumped-parameters equivalent circuits for the electric, mechanical and acoustic parts of the transducer, where the circuit elements can be computed directly from the Small-Thiele parameters measured by the manufacturer [6].

In order to define a non-linear woofer model suitable for a simple inversion and implementation on a low cost DSP, we rewrite the above Small-Thiele equations, pointing out the dependence of the force factor Bl=Bl(x), of the suspension stiffness K=K(x) and, eventually of the coil inductance L=L(x) from the instantaneous cone displacement x(t). The following relations are thus obtained:

$$Bl(x)i(t) = F(t) \qquad (1)$$

$$e(t) - R_e i(t) - L(x)\frac{di(t)}{dt} = Bl(x)\dot{x}(t) \qquad (2)$$

$$U(t) = S_r \dot{x}(t) \qquad (3)$$

$$F(t) - R_m \dot{x}(t) - M_m \ddot{x}(t) - K(x)x = p(t)S_r \qquad (4)$$

$$U(t) = G_a(t) * p(t) \qquad (5)$$

where the usual symbol are used for electrical, mechanical and acoustic quantities.

From equations (3) and (5), by Laplace transformation, we obtain:

$$\tilde{U}(s) = S_r s\tilde{x}(s) = \tilde{G}_a(s)\tilde{p}(s) \qquad (6)$$

$$\tilde{p}(s) = \tilde{Z}_a(s)S_r s\tilde{x}(s) \qquad (7)$$

where $\tilde{Z}_a(s)$ is the acoustic impedance, representing the load. Let ρ_0 be the air density, c_s the sound velocity in the air, supposed constant, and r_a the radius of the effective radiating surface of the loudspeaker, $S_r=\pi r_a^2$. At low frequencies we can assume the expression of reported by [9]:

$$\tilde{Z}_a(s) = \frac{1}{\tilde{G}_a(s)} = -\frac{\rho_0}{2\pi c_s}s^2 + \frac{8\rho_0}{3\pi^2 r_a}s \qquad (8)$$

The relation (7) is non-linear, and in order to obtain a linear model of the loudspeaker, suitable to be inverted and mapped with digital filters, the range of the displacement x is divided into a set of fixed displacements $\hat{x}_n$. So doing we can approximate Bl(x), K(x) and eventually L(x), respectively with $Bl(\hat{x}_n), K(\hat{x}_n), L(\hat{x}_n)$, in order to linearize the above equations near $\hat{x}_n$. Hence from the Laplace transformation of the equations (1)-(8) the system transfer function for small variations near $x = \hat{x}_n$ can be obtained:

$$\varphi_{\hat{x}_n}(s) = \left.\frac{\tilde{p}(s)}{\tilde{e}(s)}\right|_{x=\hat{x}_n} \qquad (9)$$

This leads to the definition of a set of expressions describing the loudspeaker behavior with respect to its instantaneous displacement.

$$\varphi_{\hat{x}_n}(s) = \frac{\tilde{z}_a(s)\tilde{S}_r(s)sBl(\hat{x}_n)}{R_e + sL(\hat{x}_n)}\left[K(\hat{x}_n) + \right.$$

$$\left. + \frac{Bl^2(\hat{x}_n)s}{R + sL(\hat{x}_n)} + \tilde{J}(s)\right]^{-1} \qquad (10)$$

Therefore from the knowledge of the cone instantaneous displacement, or an approximation of it, as a function of the input signal, referred to as $\tilde{H}(s) = \tilde{p}(s)/\tilde{e}(s)$, we can fully describe the loudspeaker behavior. This could be obtained in principles by a linear approximation, i.e. by a transfer function producing the actual displacement as a function of the audio electric input e(t).

Unfortunately, this model is obtained directly from the five equations of the small-signal Small-Thiele model which do not describe faithfully SPL curves, that, on the other hand, can be accurately measured. SPL curves, which are obtained by measuring in an anechoic room the response of the loudspeaker when it is excited with pure sinusoidal tones, provide the true system transfer function measured under linear working conditions.

Therefore we propose to derive from the set of relations $\tilde{\varphi}_{\hat{x}_n}(s)$, reported above, only the informations concerning the non linear behavior, that means, the variations with respect to $\tilde{\varphi}_{\hat{0}}(s)$, and to use in place of $\tilde{\varphi}_{\hat{0}}(s)$ the transfer function derived from SPL measurements.

Let us define the function

$$\tilde{\psi}_{\hat{x}_n}(s) = \frac{\tilde{\varphi}_{\hat{x}_n}(s)}{\tilde{\varphi}_{\hat{0}}(s)} \qquad (11)$$

that describes the non linear distortions caused by the variation of parameters at different displacements. Then, a linear filter in cascade with the previous system can take into account the small signal behavior. Hereafter we will refer to $\tilde{\varphi}_{\hat{0}}(s)$ with $\tilde{\varphi}_{SPL}(s)$, since the former expression is obtained by SPL measurements. Thus we obtain the model sketched in *Figure 1*.

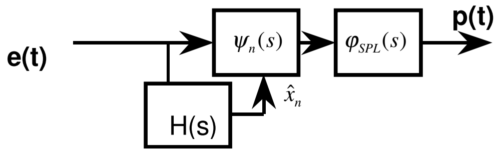

Figure 1. Loudspeaker non-linear model

The proposed model works fine within the whole audio spectrum, if properly adjusted to a particular loudspeaker. To this purpose the Small-Thiele parameters and the measured SPL curves are needed. Moreover the profiles of Bl(x), K(x), L(x) and H must be known. It must be pointed out that while the latter proves to be critical (its derivation will be discussed in the following subsection) a rough approximation of Bl(x) and of K(x) is sufficient, and L(x) is not relevant for woofers. An acceptable approximation is

$$Bl(x) = Bl(0)\left[\alpha\left(\frac{x}{x_{max}}\right)^2 + \beta\left(\frac{x}{x_{max}}\right)^2 + 1\right] \qquad (12)$$

$$K(x) = K(0)\left[\gamma\left(\frac{x}{x_{max}}\right)^2 + \delta\left(\frac{x}{x_{max}}\right)^2 + 1\right] \qquad (13)$$

where $\alpha<0$, $\gamma>0$ express the fractional reduction of the magnetic field and the increase of the suspension stiffness respectively, and x_{max} stands for the maximum displacement at nominal input power. The coefficients β and δ account for non symmetrical profiles.

2.1 H(x) filter design

As mentioned in the previous section a linear filter is required in order to estimate as accurately as possible, the instantaneous displacement of the driver x(t), given the electric audio signal e(t). Unfortunately, all the non-linearities caused by the variations of Bl(x), K(x), L(x), affect the position of the cone. Anyway, we shall suppose to be in the case of small non-linearities, so that an appropriate linear filter can produce an output signal resembling the driver displacement. It must be noted that any error in the phase response of the synthesized filter produces a delay (positive or negative) between the real and the calculated position of the cone causing a remarkable deterioration of the proposed model.

A few solutions to compute H(s) were examined, but none of them was completely satisfactory.

The chosen solution was derived by dividing both terms of equation (7) by e(s), so as to obtain:

$$\frac{\tilde{p}(s)}{\tilde{e}(s)} = \tilde{Z}_a(s) S_r s \frac{\tilde{x}(s)}{\tilde{e}(s)} = \tilde{Z}_a(s) S_r s \tilde{H}_x(s) \qquad (14)$$

Thus it is possible to derive an expression of from the transfer function directly measured from the SPL.

3 Audio digital processor for loudspeaker systems

The inversion of the model described in the previous section allows to design and implement, using a low cost DSP, a digital audio processor capable to reduce both linear and non-linear loudspeaker distortion. The processor must be inserted within the audio chain, before the power amplifier but necessarily after the volume gain control of the pre-amplifier. As previously discussed, the loudspeaker can be modeled by means of a linear law whenever it works within small displacements near $\hat{x}_n$. This relation can be inverted to obtain an inverse filter, where all filters are considered as a function the index n. In fact, digital filters, are used, rather than continuous-time filter, since the system is implemented on a DSP.

An estimate of the cone position $\hat{x}_n$, derived from the signal directly applied to the loudspeaker, is required for processing each input signal sample with the proper filter. A possible approach is to neglect the last sample of the audio signal, which cannot be calculated before choosing the corresponding filter. It is easy to understand that the error thus introduced is, in the worst case, as large as the one introduced by deriving the driver displacement through a linear filter $\tilde{H}(s)$, driven by the input signal e(t). In fact, in this latter case all loudspeaker non-linearities would be neglected, while only those corresponding to the correction system are neglected in the proposed filter.
The direct implementation of the structure proposed would require a computational cost far too high to be implemented on a low cost DSP. Therefore a multirate structure was designed, where low frequency components are decimated and processed at a lower bit rate, while filters operate the other components that do not require the compelling constraints, necessary at very low frequencies. Thus the whole system of *Figure 2* has been implemented on a 320C542 Digital Signal Processor with only 10

Kword of on chip Dual Access RAM and no other external memory (this being the major limitation).

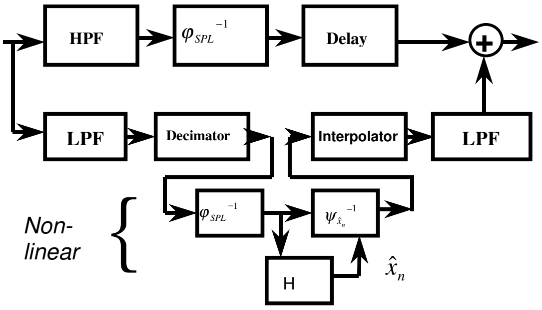

Figure 2. Multirate non linear audio processor.

4 Results

For the experiments a 110-mm commercial double coil sub-woofer was used. It has an operating frequency range of [20Hz, 2kHz]. A high-precision audio analyzer was used to evaluate the Total Harmonic Distortion (THD) and the Sound Pressure Level (SPL) produced by the loudspeaker. Measurements were performed in an anechoic room with a highly linear microphone, while stimulating the system with pure sinusoidal tones. A remarkable flattening of the amplitude (*Figure 3*), and a linearisation of the phase (*Figure 4*) of the reproduced SPL has been achieved. It must be noted that the anti-aliasing filters of the low-quality ADC used produce the reduction of SPL at very low frequencies.

Moreover harmonic distortion is reduced as shown in *Figure 5* over the frequency range of [20Hz, 90Hz]. At higher frequencies the processor is substantially linear.

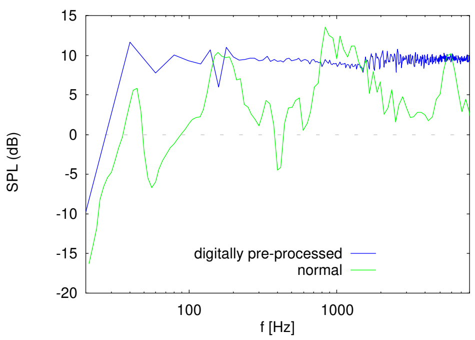

Figure 3. Comparison between the SPL (amplitude) of the considered loudspeaker with and without the audio processor (3 Volt input signal amplitude).

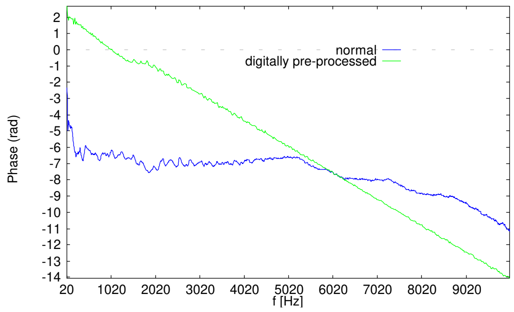

Figure 4. Comparison between the phase responses. (3 Volt input signal amplitude).

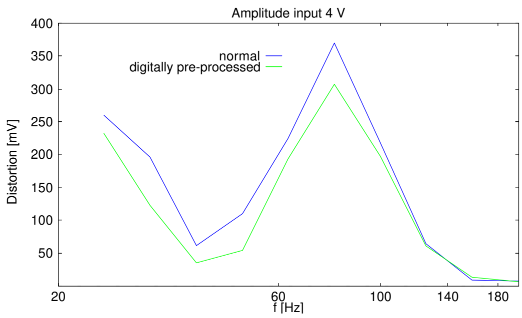

Figure 5. Comparison between the THD (3 Volt input signal amplitude).

5 Conclusions

A non-linear loudspeaker model, which reproduces the low frequency behavior with good approximation is presented. It was inverted and approximated in order to allow the realization of an anti-distortion audio processor based on a 320C542 Digital Signal Processor.

Measurements performed on the audio signal with and without the insertion of the designed audio processor demonstrate that the audio processor features better performances in the reproduced sound together with higher reliability.

6 Acknowledgments

The authors would like to thank Ing. Gino Gobbi and the Technical Staff of ASK Industries, for their encouragements and suggestions.

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