Fast Training Analog Approximator on the Basis of Legendre Polynomials

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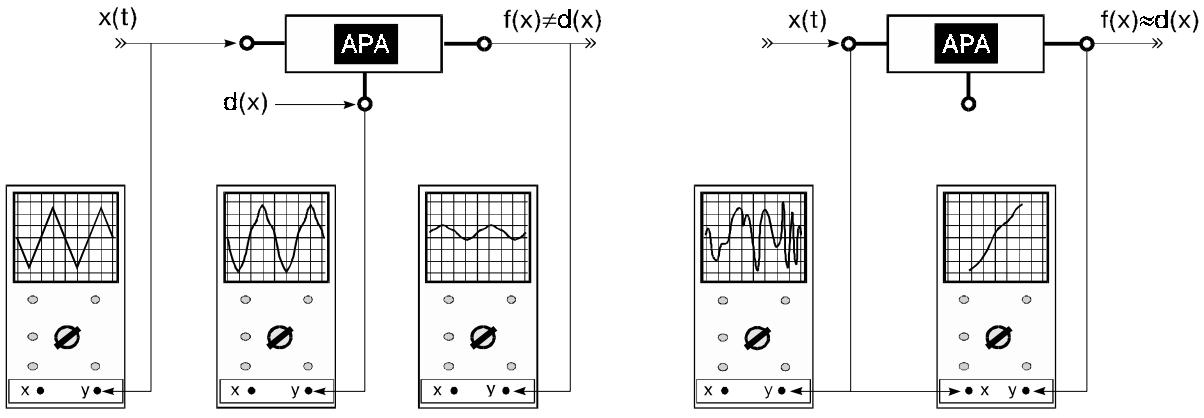
Abstract

In this paper the neural network based devices performing the operation of approximation or interpolation are described. The schemes and working characteristics of a breadboard based on analog radio components are presented.
Legendre polynomials were offered as basis functions for significant increasing of the speed of the approximator training. The scheme of analog synthesizer of Legendre polynomials was also suggested.

1: Introduction

The systems based on artificial neural networks (ANN) can find wide applications in various systems of monitoring, management and automatic control already in the nearest future. Although many techniques, such as digital and analog electronics, optical computers, chemical implementation, and bio-chips, are investigated for implementing neuro-computers, only digital and analog electronics, and in a lesser degree optical implementations, are at present feasible techniques .
It can be easily shown that the analog implementation of ANN is characterized by some potential advantages in comparison with the digital one: greater speed, smaller sizes and lesser power consumption (see e.g., [1]), and also large stability to damages of various units of systems. In this work the pure analog implementation of systems performing operations of approximation/ interpolation is described. The breadboard model was made on the basis of analog chips: operational amplifiers and analog multipliers and may be a prototype of specialized analog VLSI chip.
The aim of the work was to reveal the working characteristics of real (imperfect) analog adaptive nonlinear systems, and - opportunities of one-chip design of systems.



a) b)

Fig. 1. The analog polynomial approximator device: a)- in the regime of training and b)- after training.

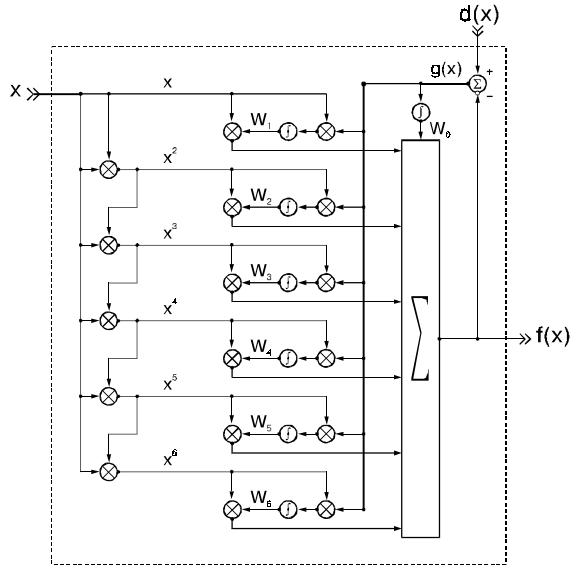


Fig. 2. The schematic diagram of APA

2: Analog Polynomial Approximator

Analog Polynomial Approximator (APA) is the system, performing operations of approximation or interpolation of given dependence of two signals - x and $d(x)$ (see Fig. 1). The signal $x(t)$ can accept various values in range from x_{min} to x_{max} , and the signal $d(x)$ is connected with x by a simple interrelation. The aim of the device is to minimize the mean-square deviation of the output signal $f(x)$ from the reference one $d(x)$ under the given dependence $x(t)$. The result is achieved by using the procedure of approximation or interpolation of function $d(x)$ by polynomial

$$f(x) = \sum_n W_n \cdot x^n .$$

APA is based on neural-like architecture with the "Widrow-Hoff delta rule" algorithm of training [2]. The described device was made on the basis of analog radio elements and functioning on a principle of continuous (not discrete in time) training.

On Fig. 2 the electrical scheme of the system is presented. The device consists of:

1. Synthesizer of power-type functions from the source signal x in accordance with the formula:

$$y_n = x^n .$$

2. A neuron with the linear activation function:

$$f(x) = \sum_n W_n \cdot y_n = \sum_n W_n \cdot x^n .$$

3. The "Widrow-Hoff delta rule" training system which performs continuously in time the updating of the weights W_n with the purpose of minimization of mean-square deviation e, where

$$e = \sum_k \left[d(x_k) - f(x_k)\right]^2 = \int_T \left[d\left(x(t)\right) - f\left(x(t)\right)\right]^2 dt .$$

In the approximation of a sufficiently slow training the weights updating dW_n can be written in a form:

$$\delta W_n \sim -\frac{\partial e}{\partial W_n}\delta t = 2(d(x) - f(x)) \cdot x^n \delta t .$$

The breadboard model basic element is a chip of analog multiplying (KR525PS2A - Soviet), (see Fig. 3.). The investigation showed that for normal operating of the system it is not necessary to adjust all the mismatches: x_o , y_o , z_o on each microchip, but it is enough to adjust only z_o in the learning circuit of each cascade of the system. Another mismatches will be compensated by

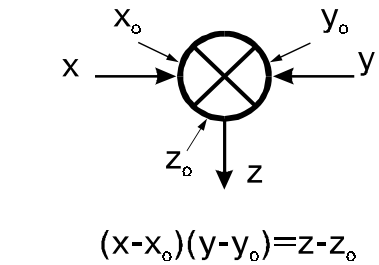


Fig. 3. The analog multiplier

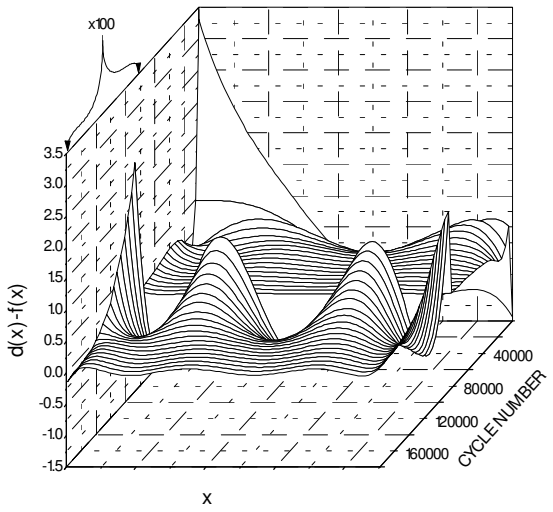


Fig. 4. The dependence of the deviation from the cycle number for APA on the basis of power-type functions.

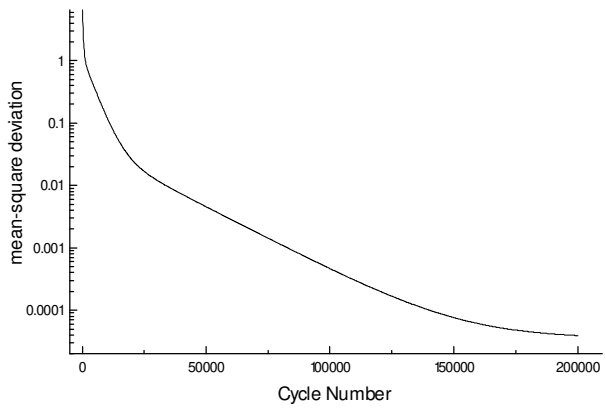


Fig. 5. The dependence of the mean-square deviation signal from the cycle number for APA on the basis of power-type functions.

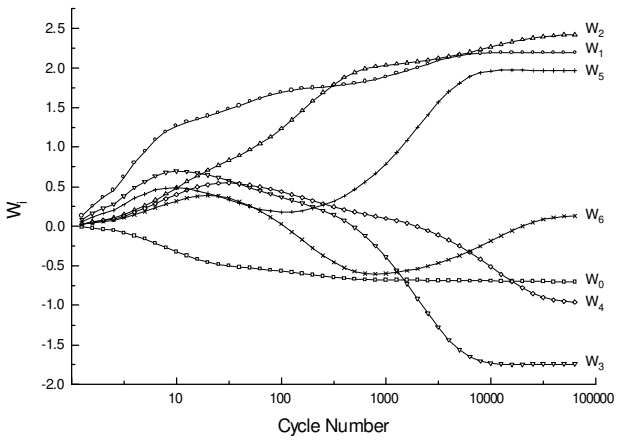


Fig. 6. The dependence of weights W_i from the Cycle Number for APA on the basis of power-type functions.

previous cascades of the system.

The dependence of signal deviation from the cycle number is shown on Fig. 4. At the first moment (back edge of a surface) the system is not trained ($f(x)$=0, for any x). As a result of the system training the maximum deviation of $d(x)$ from $f(x)$ is smaller than 0.1%, that took approximately 2×10^5 cycles of training (several minutes in the experiment) (see Fig. 5).

3: Fast training analog approximator on the basis of Legendre polynomials

As it was shown above, rather large number of cycles of training (and respectively much time) for APA system learning is required. The learning time can be considerably reduced by choosing not power-type functions, but orthogonal at given range basis for function approximation. In the case of sawtooth waveform signal x(t) (in the regime of training) it will be the orthogonal at the range (x_{min} , x_{max}) Legendre polynomials' basis (see Fig. 7).

This choosing leads to the fact that the training of each degree of freedom of the system does not result in increasing the errors of other degrees of freedom.

The offered system consists of:

a) the synthesizer of Legendre polynomials,
b) the neuron with the "Widrow-Hoff delta rule" algorithm of training.

The electrical scheme of the device is shown on Fig. 8.

The principle of the Legendre polynomials' synthesizer operation is based on a recurrence relation:

$$P_n = 2x \cdot P_{n-1} + a_n \cdot P_{n-2},$$

where a_n are determined from the conditions:

$$\int_{-1}^{1} P_n \cdot P_{n-2}\, dx = 0.$$

The computer's modeling shows that analog Legendre polynomials based approximator is much faster trained (about 3 orders of magnitude) in comparison with the approximator on the basis of power-type functions

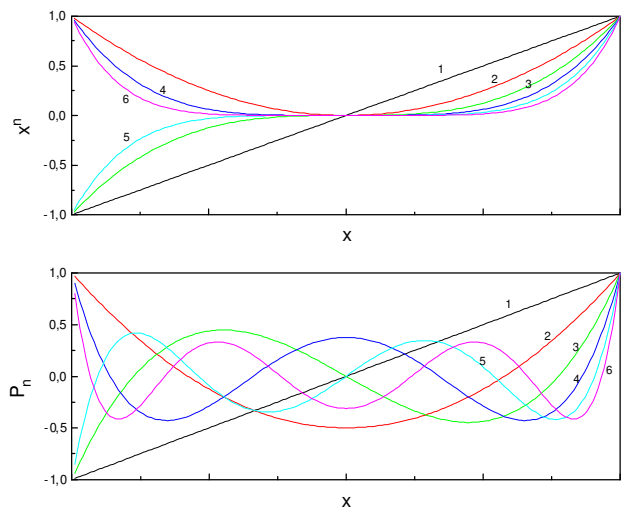


Fig. 7. The power-type functions x^n and Legendre polynomials P_n .

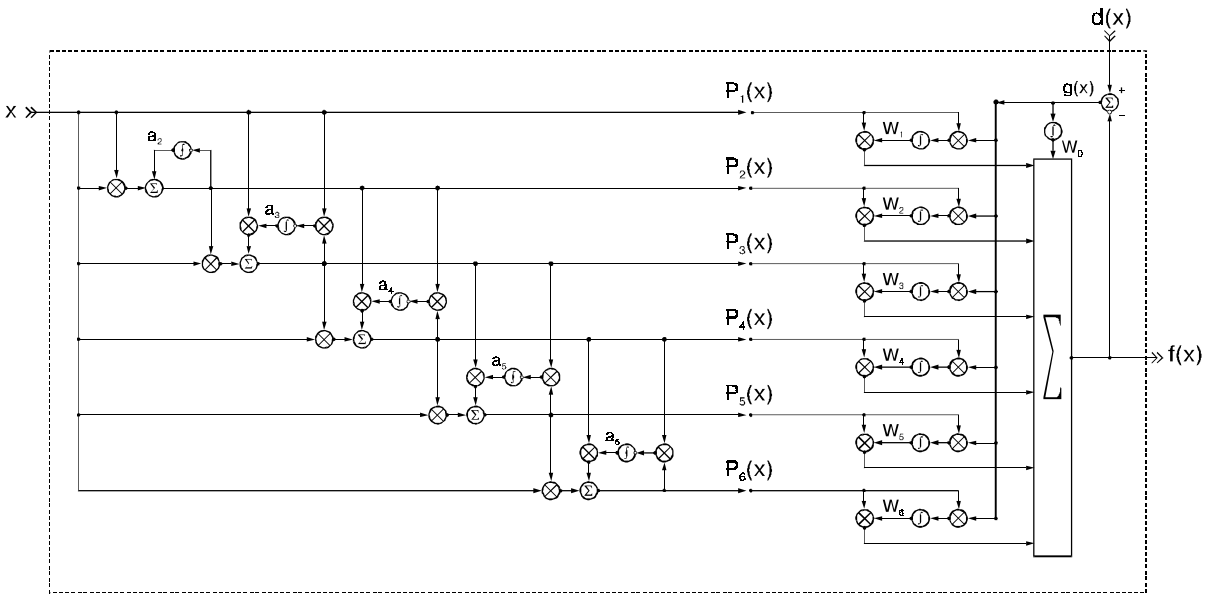


Fig. 8. The electrical scheme of the analog approximator on the basis of Legendre polynomials.

described in the first part of the article (see Fig. 4-6, 9-11).

It should be noted, however, that additional circuits of analog multipliers "zero" correction are necessary for reliable and precise operation of the system represented on Fig. 8 (these circuits are not shown on the scheme).

4: Conclusions

The suggested systems carrying out the function of approximation or interpolation were investigated experimentally and by the methods of computer's modeling. Surprisingly good working characteristics of these devices was manifested in result.

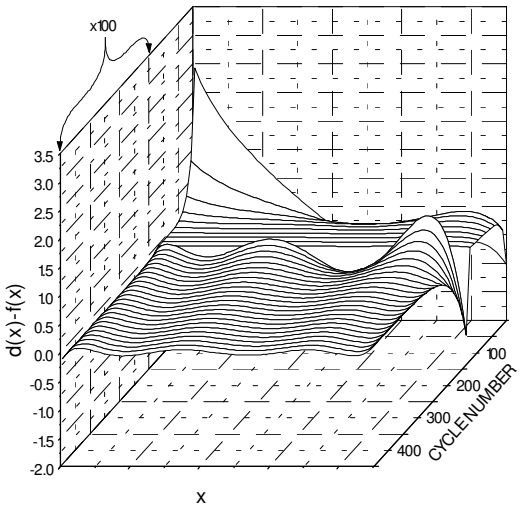


Fig. 9. The dependence of the deviation signal from the cycle number for analog Legendre polynomials based approximator.

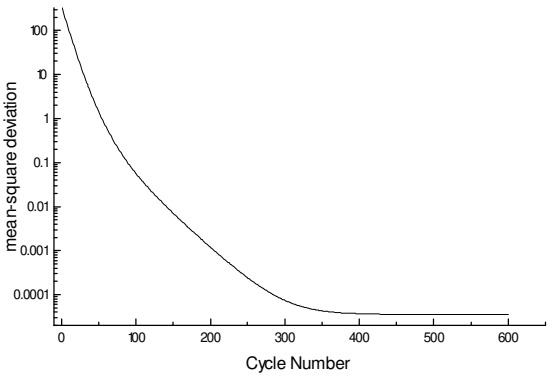


Fig. 10. The dependence of mean- square deviation from the cycle number for analog Legendre polynomials based approximator.

In author's opinion they can find application in high precision analog engineering as an adaptive nonlinear element compensating nonlinearity of radioengineering units or correcting frequency characteristics of radioengineering systems.

References

[1] C. Mead, Neuromorphic Electronic Systems, *Proceedings of the IEEE*, Vol. 78, No. 10, 1990.

[2] B. Widrow and M. E. Hoff, *Jr.,"Adaptive switching circuits",* 1960 IRE Western Electric Show and Convention Record, Part 4, pp. 94-104, 1960.

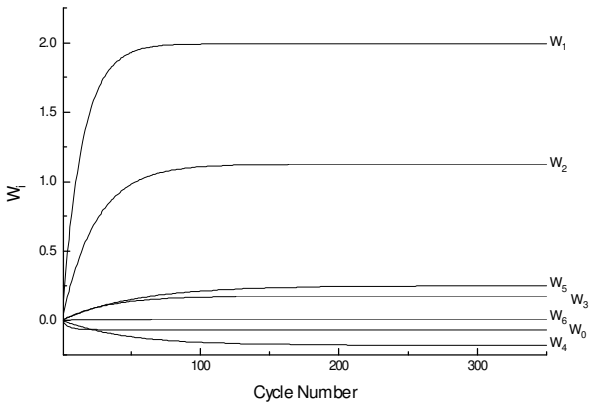


Fig. 11. The dependence of weights W_i from the Cycle Number for analog Legendre polynomials based approximator.