# Computation with chaos: A paradigm for cortical activity *

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**Abstract**

A device comprising two interconnected networks of oscillators exhibiting spatiotemporal chaos is considered. An external cue stabilizes input specific unstable periodic orbits of the first network thus creating an "attentive" state. Only in this state is the device able to perform pattern discrimination and motion detection. We discuss the relevance of the procedure to the information processing of the brain.

## 1 Introduction

Electrical activity of cerebral cortex can be measured non invasively from the scalp. The electroencephalograms (EEGs) thus obtained are local averaged activity of millions of cells. According to the behavioural states of the brain, the EEG shows well defined ethiology. Some pathologies such as for example the "petit-mal" epilepsy or Creutzfeld-Jackob coma generate very characteristic waves. Thus before the advent of more sophisticated medical imagery techniques the EEG was a diagnostic tool for assessing neurological diseases. It still is a common tool for assessing sleep disorders.

In the last decade the techniques of nonlinear time series analysis were applied by Babloyantz and her colleagues to the human EEG [1, 2, 3, 4]. They could show that in several behavioural states as well as in "petit-mal" epilepsy and Creutzfeld-Jackob coma the EEG may be described by deterministic chaotic dynamics. Several groups have confirmed the existence of chaotic dynamics in the human cortex (see refs. [5, 6] and references therein).

Drawing on these findings a model of cerebral cortex was recently constructed [7]. In this model the various behavioural states of the cortex are seen as spatiotemporal chaotic cortical activity of increasing coherence generated by the nature of the input from the thalamus. The space average of network activity over an area of the cortex provides EEG-like signals which are akin to

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behavioural states of human brain activity. Such models led to the concept that the dynamics of cerebral cortex is of spatiotemporal chaotic nature. Thus it seems that information processing in the brain is performed at a global network level. The information processing capacity of neuronal networks was demonstrated recently by measuring the active neuronal population in the *Aplysia* abdominal ganglion network during spontaneous and evoked behaviors using multineuronal optical measurements [8]. It was found that a distributed organization involving a large number of neurons of the network may generate the two behaviors. These functional behaviors stem from altered activities of a large ensemble of neurons and thus they are a network property. This assumption raises the following question. If brain dynamics is of chaotic nature then how does such a system process information ?

The importance of providing an answer to this question is twofold. First, it may give a clue for the understanding of brain processes. Second, the relevant concepts could be integrated into artificial neural networks, in order to produce a more efficient computational device by mimicking more closely brain dynamics and its awesome capacity for storage and rapid processing of a great amount of complex information.

Indeed the classical neural networks generally rely on fixed point attractor dynamics, thus limiting the capacity of small or moderate size networks. On the other hand, chaotic dynamics, even with few degrees of freedom, can in principle provide an infinite means of coding, as it is a "reservoir" of an infinite number of unstable periodic orbits [9, 10, 11]. In our aproach the periodic orbits are used as coding devices.

The aim of this paper is to propose a neural network model based on chaotic dynamics which not only encompasses the parallel computation seen in brain dynamics but also introduces layered chaotic structures which exhibit somewhat more brainlike architecture, and features such brain attributes as a state of attentiveness. Indeed cerebral cortex is a multilayer structure with specific interconnections between different layers [12]. Each layer is a network of densely connected neurons. Moreover different layers are thought to have functional specificities in the processing of information and it is widely assumed that information is embedded in the links or synapses joining the neurons. On the other hand, in all sensory processing the first task is to become receptive to incoming information. Once such an "attentive" state is reached the information to be treated could flow through the specific pathways. For instance in a busy airport it is perfectly possible to become oblivion to the environmental noise. A conversation extracted from the background noise will convey information only if one pays attention deliberatly. This is also the case for the visual system. It is possible to look intensively at an object without seeing it. Seeing requires a deliberate will to become attentive. Once this attentive state is reached the brain is ready to process the incoming information. Usually the attentive state is reached very rapidly and has a finite time span.

The model we propose is an interconnected two layered device. Each layer is a chaotic network of oscillatory units. A small external or internal input

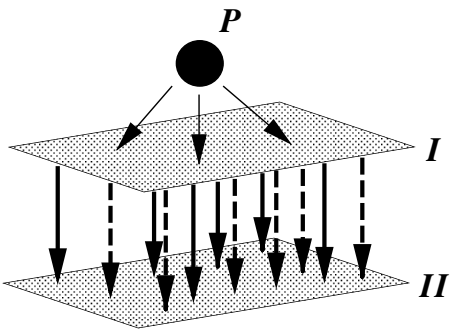


Figure 1: Chaotic categoriser. The pacemaker $P$ sends micro-pulses to layer $I$ thus stabilizing one of the unstable periodic orbits. The oscillators of layers $I$ and $II$ are connected with a one-to-one correspondence. The distribution of active links (solid arrows) and inactive ones (dashed arrows) is determined by a pattern to be processed. A response is measured from output layer $II$.

brings the device into an attentive state. We show that once the device is in an attentive state, it can perform such tasks as pattern categorisation, motion detection and discrimination between clockwise and anticlockwise rotation. The dynamics of the device makes use of unstable periodic orbits contained in a chaotic attractor which are stabilized by small fluctuations with the help of an algorithm proposed by Ott, Grebogi and Yorke (OGY) [13, 14]. The number of such orbits is infinite, thus even small chaotic networks may in principle process an infinite amount of information.

We begin by introducing the model, and proceed to discussing the pattern discrimination and motion detection. In a final section we discuss the relevance of the model as a paradigm of brain function.

## 2 Chaotic Categorizer

Let us consider a device, hereafter called a chaotic categorizer, made of two interconnected layers as shown in Fig. 1 [15].

Each layer comprises $N \times N$ oscillating elements. The elements of the two layers are connected in a one to one correspondence with links which are active and represent a given pattern only if an external stimulus activates the first layer. The details of this procedure will become apparent in the sequel.

The pacemaker $P$ sends micropulses only to the layer $I$ as shown in Fig. 1. In the absence of external stimuli the activities of the two layers are independent and both show spatiotemporal chaotic behaviour.

The device is described by the following differential equations.

$$
\begin{aligned}
\frac{dZ_{jk}}{dt} &= Z_{jk} - (1+i\beta)|Z_{jk}|^2 Z_{jk} + (1+i\alpha)\, D \sum_{l,m} C_{jklm} Z_{lm} + P_{jk}(t) \\
\frac{dW_{jk}}{dt} &= W_{jk} - (1+i\beta)|W_{jk}|^2 W_{jk} + (1+i\alpha)\, D \sum_{l,m} C_{jklm} W_{lm} \qquad (1) \\
&\quad + \gamma I_{jk}(Z_{jk} - W_{jk}) \qquad\qquad (j,k,l,m = 1,\ldots,N).
\end{aligned}
$$

The variables $Z_{jk}$ are the complex amplitudes describing the oscillators in the layer $I$, whereas the $W_{jk}$ refer to the corresponding variables in the layer $II$. The first two terms in each equation are related to the complex Ginzburg-Landau description of amplitude equations for each oscillator. The diffusive connectivity $C_{jklm}$ relating the oscillators is defined as

$$
\sum_{l,m} C_{jklm} Z_{lm} = Z_{j(k+1)} + Z_{j(k-1)} + Z_{(j+1)k} + Z_{(j-1)k} - 4Z_{jk}\ .
$$

The parameters $\alpha$, $\beta$, $D$ and $\gamma$ are real-valued. $\alpha$ and $D$ reflect the coupling strength between units. The term $P_{jk}(t)$ represents the influence of the pacemaker $P$ on each oscillator of layer $I$. The boundary conditions are of the zero-flux type.

The information to be processed is sent to the device via a binary matrix $I_{jk}$. If $I_{jk} = 0$ then the connection between elements $jk$ of the two layers is nonexistant. However if $I_{jk} = 1$ then the two layers are connected via elements $jk$, and $\gamma$ describes the strength of the binding. Moreover in this model if $P_{jk}(t) \equiv 0$ then $\gamma = 0$. Thus it is only when the pacemaker $P$ is active, and therefore a periodic orbit has been stabilized in layer $I$, that there is entrainment of the second layer by the first layer. The entrainment can be total if all $I_{jk} = 1$, and it is partial if only some of the $I_{jk}$ are nonzero. Moreover the dynamics of the second layer is critically dependent on the parameter $\gamma$. If $\gamma = 0$ then the two layers have independent dynamics whereas large values of $\gamma$ with a large number of "on" links representing incoming information can synchronize the activities of the two layers.

If $P_{jk}(t) = 0$, $\gamma = 0$, $N = 9$, $\alpha = -10$, $\beta = 2$ and $D = 1.3$, then both layers follow spatiotemporal chaotic activity. The dynamics of each network separately could be viewed as evolving on a chaotic atractor embedded in a $2(N \times N)$ dimensional space. It is well known that such a strange attractor comprises an infinite number of unstable periodic orbits [9, 10, 11]. Recently with the help of a technique proposed by Ott, Grebogi and Yorke [13] one could stabilize four different orbits out of the global attractor of the network by applying micropulses to the system with the help of the pacemaker $P$ [14]. Figure 2 displays three orbits which exhibit spatiotemporal structure. The nature of the stabilized orbit is a function of the pacemaker $P$. More details can be found in ref. [14].

The orbit $C_0$ corresponds to the bulk oscillation of the network, where all the oscillators are in phase. The period of this oscillation is $T = \pi$. Although the

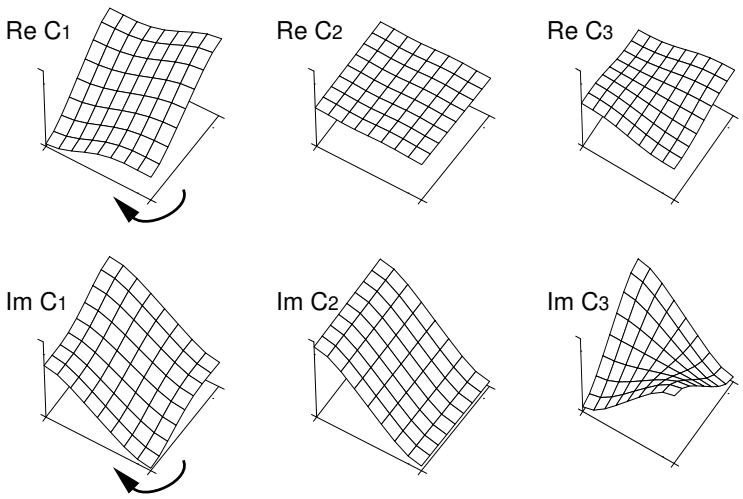


Figure 2: Snapshots of network activity $W$ corresponding to the unstable periodic orbits $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ with periods $T = 13.66$, 15.4 and 2.25 respectively. The vertical axes span the interval $[-1.2, 1.2]$. Parameter values are $N = 9$, $\alpha = -10$, $\beta = 2$ and $D = 1.3$. For orbit $\mathcal{C}_1$, arrows show the direction of rotation of the wave of activity (from ref. [14]).

orbits $C_1$, $C_2$ and $C_3$ are periodic at the level of the $2N^2$ dimensional dynamics, when viewed at the network level they exhibit spatiotemporal structures. Phase differences are seen between individual oscillators of the network. The orbit $C_1$ with $T$=13.66 shows a rotating wave activity of the amplitude of individual units around the central unit of the network. The amplitude of the latter is zero, that is we are in the presence of a "phase defect". In this case the rotation is clockwise. The orbits $C_2$ and $C_3$ show stationary waves of different symmetries. Orbit $C_2$ is antisymmetric and shows a polar structure. The activity is high on one side and low on the opposite side. The situation reverses with constant period $T$=15.4 . In orbit $C_3$ spatiotemporal phenomena show a different structure. The activity is high in units along one of the diagonals and near the corners of the network, whereas the oscillators on the other diagonal show lower activity close to the corners. Again as in $C_2$ the situation changes regularly with constant frequency. The period of $C_3$ is $T$=2.25 . In principle an infinite number of other periodic orbits could be stabilized in this system. We have not considered them in this paper.

Let us go back to our device as shown in Fig. 1 and describe how it can process information. The total input into the system is divided into two parts. The pacemaker $P$, which emits the appropriate micropulses, and the input, which must be processed. Thus, the information is captured on the first layer and on the links relating the latter to the second layer. The response of the

second layer defines the output of the system. The pacemaker $P$, according to the nature of the information to be processed, stabilizes the first layer into one of the orbits described in Fig. 2. In analogy with the human brain, where the first act in any cognitive process is to become attentive to an external input, we call these orbits the "attentive states" of the device. Now the device is ready for processing the input. The latter is inprinted in the links $I_{jk}$ which are nonzero only if they represent a part of the input. The attentive state, which represents well defined spatiotemporal structure, entrains the output layer according to the number and location of "on" links in the device. Thus in the second layer each input pattern generates its own specific spatiotemporal structure.

The dynamics of the ouput layer can remain chaotic, or be of a quasi-periodic or periodic nature. In order to discriminate between various inputs, we need to quantify the spatiotemporal activity of the response layer. Unfortunately so far there are no satisfactory methods for the quantification of spatiotemporal chaotic activity. However as we are interested in the differential change in the network coherence of layer $II$ as a result of the input, the evaluation of the global activity of that layer will be sufficient. Troughout this paper we shall use the average value of the squared amplitude of the forced layer $\langle |W|^2 \rangle(t)$ as the output function. The brackets $\langle \cdot \rangle$ denote a *space* average over the entire network.

We have also computed other quantities such as the mean value of the real part of $W$ over the network. The time evolution of the cross-correlation function between the two layers was also computed. It is defined as

$$C(t) = \mathrm{Re}\, \frac{\sum_{j,k} \overline{Z_{jk}}(t)\, W_{jk}(t)}{\left(\sum_{j,k} |Z_{jk}(t)|^2\right)^{1/2} \left(\sum_{j,k} |W_{jk}(t)|^2\right)^{1/2}} \quad .$$

In our simulations the function $\langle |W|^2 \rangle(t)$ seemed in general more appropriate for pattern and motion discrimination than the other monitored functions.

The chaotic categorizer of Fig. 1 can be used as a pattern recognition device as well as a motion detector. The ability of the device to perform a given task is a function of the attentive state that is generated by the device under the action of the input to be processed. With our categorizer the orbits $C_2$ and $C_3$ are suitable for pattern recognition and also for detection of linear motion, whereas $C_1$ is an orbit which leads to detection of clockwise and anticlockwise motion.

# 3 Pattern Discrimination

All three attentive states brought about by orbits $C_1$, $C_2$ and $C_3$ are suitable in some degree for pattern discrimination. However because of symmetries inherent in orbits $C_2$ and $C_3$ these are more suitable than $C_1$ for pattern processing.

We start with the device in the attentive state $C_2$. In this state the stabilized orbit in the first layer shows a polarity that oscillates in time. At a given time one may see a high activity at the right hand side of the network while the activity

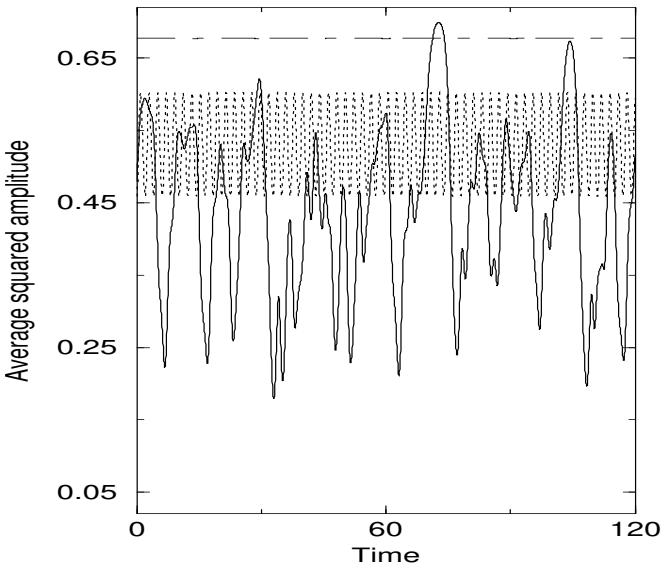


Figure 3: Space average of the squared amplitude $\langle |W|^2 \rangle$ measured from layer $II$. Orbit $\mathcal{C}_2$ is stabilized in layer $I$. Three patterns are presented to the device: a bar activating the nine middle links and parallel to the direction of polarity of $\mathcal{C}_2$ (dot-dashed line), a bar activating nine links perpendicularly to the direction of polarity (solid line), and a bar activating nine links along one of the diagonals of the network (dotted line) . $\gamma = 2.44$ and all other parameters are as in Fig. 2.

is at its lowest level at the left hand side. The situation reverses periodically. In this attentive state the device is presented with a bar which activates the nine middle links between the two layers and is parallel to the direction of polarity of the orbit $C_2$. When these links are "on" there follows an entrainment of the layer $II$ by the layer $I$. The value of the space average $\langle |W|^2 \rangle$ is a measure of this entrainment and is shown in Fig. 3. One sees a constant value of 0.68. In another experiment the bar is presented again to the middle links of the network but perpendicularly to the polarity of $C_2$. The response of the system as seen in Fig. 3 is irregular, with high amplitude and a decreased mean value. If the bar is presented along the diagonal, the response is periodic with a time averaged value of 0.53 (see also Fig. 3). Therefore our simple device when in the attentive state $C_2$ can discriminate between different orientations on the plane.

If the same type of experiment is conducted in the attentive state $C_3$, the device can only discriminate between a bar perpendicular to the side of the network and one presented along the diagonal. This property stems from the fact that orbit $C_3$ has a reflexion symmetry with respect to the two diagonals and therefore the two axes perpendicular to the side of the network entrain the

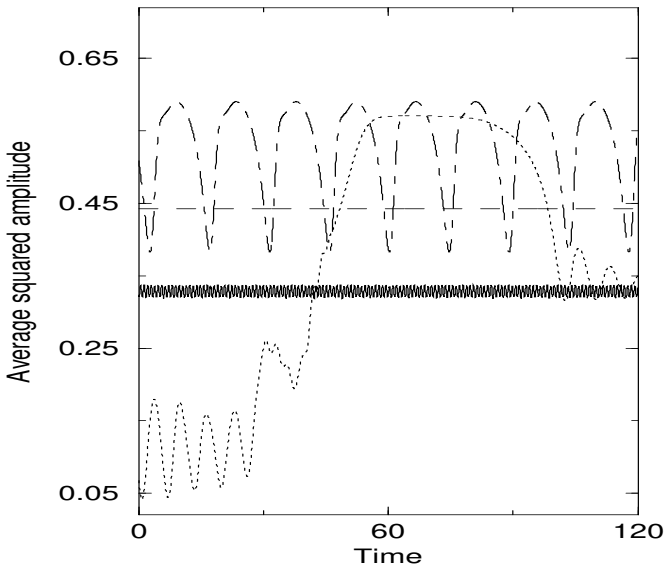


Figure 4: Responses $\langle |W|^2 \rangle(t)$ of the output layer in the presence of the patterns "+" and "×". With orbit $\mathcal{C}_2$ the response is shown as a dot-dashed line for pattern "+" and a dotted line for pattern "×". When orbit $\mathcal{C}_3$ is used, the response corresponding to the pattern "+" is shown as a dashed line, whereas the solid line is obtained with the pattern "×". Parameter values are as in Fig. 3.

layer $II$ in the same manner. Because of the quasi-circular symmetry of orbit $C_1$, the corresponding attentive state is not suitable for the type of pattern processing mentioned above.

Figure 4 shows the output of layer $II$ when two patterns "+" and "×" are presented to the system when it is in the attentive state $C_2$. We see that the two patterns are discriminated by the system. The same is true for the attentive state $C_3$ (see Fig. 4). However we notice that the form and the amplitude of responses for a given pattern are not identical for the two attentive states.

Let us consider another pair of patterns "N" and "Z". The first letter can be recovered by a 90° rotation of the second. From symmetry considerations and the result of experiments with single bars we expect that the attentive state $C_2$ will discriminate between patterns "N" and "Z" whereas the state $C_3$ will give the same answer for both patterns. Our simulations confirmed these conjectures.

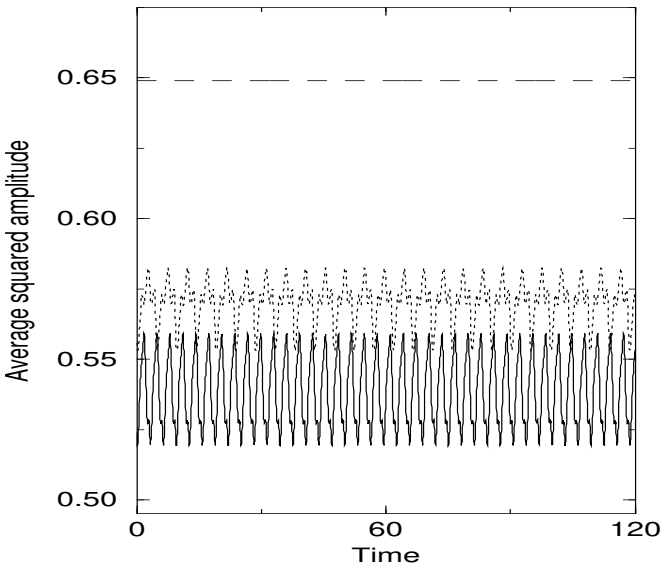


Figure 5: $\langle |W|^2 \rangle(t)$ computed in the attentive state $C_1$ for a clockwise (dashed line) and an anticlockwise (dotted line) motion with period of rotation $T$=18.96. If the object rotates in the anticlockwise direction with a period $T$=12.48 (solid line), a higher amplitude response with a lower time average value is seen. $\gamma = 30$ and the other parameters are as in Fig. 2.

# 4 Motion Detection

The device of Fig. 1 is able to discriminate between clockwise and counter-clockwise rotation. To this end we assume that the attentive state of the system is achieved by the stabilization of the first layer into orbit $C_1$ which shows a phase rotation of period $T$=13.66 (see Fig. 2). As we have stated already, in this example the phase motion of the stabilized orbit is clockwise. Because of the phase rotation of the orbit, the attentive state $C_1$ is able to discriminate between clockwise and anticlockwise motion.

To see this let us consider the motion of a small object which activates three links at a time. In this experiment $\gamma = 30$. As the object moves, only the next neighboring link is activated and one of the previous "on" links is deactivated. Thus the successive activation and deactivation of links represents a circular motion. In our example the diameter of the circular trajectory spans over five network units and the rotation period of the object varies from $T$=6 to $T$=25 .

In a first experiment, when in the attentive state $C_1$ the device perceives the clockwise motion of the object which is imprinted in the links. The response of the layer $II$ is shown in Fig. 5 when the period of rotation of the object is $T = 18.96$. The value of the space average of the squared amplitude $\langle |W|^2 \rangle$=0.65

is almost constant in time. However, at very fine resolution small amplitude oscillations are seen around this value (not apparent in Fig. 5). The same response is seen for all values of the rotation periods considered. Thus in the attentive state $C_1$ the device is not sensitive to the speed of clockwise motion of the object.

Presently we reverse the direction of rotation of the object and keep all other conditions as above. The response of layer $II$ to anticlockwise motion is very different and is sensitive to the rotation speed. For $T$=6 the motion generates a chaotic response $\langle|W|^2\rangle$ around a time averaged value of 0.5 . As the speed decreases, the time behaviour of the response becomes less and less chaotic and gradually a time periodic output function appears. Figure 5 shows the responses associated with the anticlockwise motions of periods $T$=12.48 and $T$=18.96 . The corresponding time averaged values of $\langle|W|^2\rangle$ are 0.54 and 0.57 respectively. For the range of $T$ considered the value of $\langle|W|^2\rangle$ corresponding to anticlockwise motion is always smaller than the one seen for clockwise motion. The time averaged value and the shape of the response in the case of anticlockwise motion are sensitive to the speed of rotation. If we restrict the range of $T$ to $12<T<25$, we observe that the time averaged value of $\langle|W|^2\rangle$ is an increasing function of $T$. Thus in this range not only our device discriminates between clockwise and anticlockwise motion, but it also evaluates the speed of anticlockwise motion. For slow motions, $T>26$, and very fast motions, $T<0.5$, the response to clockwise and anticlockwise rotation is practically identical. A difference may be seen only in the fine structure of the $\langle|W|^2\rangle$ output function. Thus for these velocities the device is "blind" with respect to the direction of rotation. The responses in these ranges are similar to the response to clockwise rotation for $6<T<24$ (see Fig. 5). For values of $0.5<T<6$ the response of the system does not follow the smooth change that was described above. A static object could be considered as rotating with period $T\rightarrow\infty$ and thus it is perceived in the same manner as other objects rotating with long periods, that is $T>26$ .

Motion detection is not limited to the orbit $C_1$. The attentive states corresponding to orbits $C_2$ and $C_3$ are able to perceive a moving object and discriminate between "perpendicular" and "diagonal" motions.

Let us consider the attentive state $C_3$ and a moving object which activates only one link at a time. The motion starts from the middle of the boundary of the device and continues along a straight line perpendicular to that boundary. Each link is activated during eleven time units. The response of layer $II$ is monitored in the usual way and is markedly different from the one when the object moves along the diagonal. However opposite directions of motion along anyone of these paths cannot be distinguished. With orbit $C_3$ the two perpendicular orientations cannot be distinguished, and the same happens regarding the two diagonal ones. The distinction between the two perpendicular orientations is possible only if the device is in the attentive state $C_2$. The latter is also unable to discriminate between the two opposite directions along anyone of the two diagonals as well as along each of the two perpendicular axis.

# 5 Discussion

We have shown that a simple device, made of two interconnected layers of oscillators and featuring spatiotemporal chaotic dynamics, is capable of pattern selection and motion detection. This capability is intrinsic to the system and is unsupervised. As a result of algorithmic complexities inherent in the OGY method, in this simple example only three orbits with spatiotemporal structure and the bulk oscillation were stabilized in the first layer. However in principle there is an infinite number of unstable orbits in a chaotic attractor that could be stabilized. With the stabilization of higher order orbits one would expect more intricate symmetries or more complex spatiotemporal phenomena, specially if larger networks are considered. Thus one would think that such a chaotic device could process a great variety of information relative to static as well as moving objects.

Spatio-temporal chaos may arise also in artificial neural networks. Usually one tries to avoid such situations in classical computational techniques, as chaos is perceived generally as a nuisance. Instead of avoiding chaos, we believe that more elaborate devices, based on principles underlined in this paper, could make chaotic dynamics a desireable tool for unsupervised computations. Chaotic attractors are loci of an infinite number of unstable periodic orbits. Therefore even small networks may exhibit high capacity.

Another important aspect of our model is that it is a paradigm for understanding some aspects of cerebral cortical activity. The system described in this paper embodies several key features of cortical architecture which consists in interconnected layers of neurons, each layer with its own specific input-output pathways. On the other hand, measurements from EEGs as well as model systems and the nature of cortical tissue suggest that the activity at the cortical layers could be of spatiotemporal chaotic nature [1, 2, 3, 4, 5, 6, 7]. Moreover all information processing becomes possible if one is in a state of attentiveness. This state can be reached by external or internal cues.

Building on the properties of our device let us speculate on the nature of the state of attentiveness in cerebral cortex. We propose that the first effect of any sensory cue is to stabilize one or several unstable periodic orbits in one of the cortical layers which we shall call the layer of attentiveness. Any sensory cue may be thought of as a sum of different elements each representing a particular feature of incoming information and thus stabilizing a specific orbit. This fact is plausible in view of the layered structure of the cortex. For example a given sensory cue can stabilize all three orbits of Fig. 2, each in one sheet of neurons of the attentiveness layer. This in turn implies that in the presence of a sensory cue the attentiveness layer switches from a chaotic or turbulent state, to a more coherent state which is a superposition of well defined oscillatory modes. Overall, one may see a much more synchronized activity in one of the layers of the cortex during the attentive state. Only when such a coherent state is reached may other cortical areas become active and process the relevant information.

We believe the theory we propose could be tested experimentally in the following manner. During *in vivo* animal experiments a vertical battery of

electrodes could be implemented chronically in a well defined area of the sensory cortex, for example the visual cortex. The multi-electrodes span several layers and sublayers of the cortex. One records from all electrodes simultaneously in the absence of sensory cues. The field potentials could then be analysed by several techniques of nonlinear time series analysis [1, 2, 3, 4]. The values of dynamical parameters such as entropies, Lyapunov exponents and dimensions characterize the dynamics of the signal. A second measurement is performed by presenting a sensory cue to the animal, and the same type of quantitative analysis is repeated. If our conjecture is correct then the comparison of data from each layer will show an increase in coherence, that is a more synchronized dynamics, in one of the layers. Such an experiment will show the existence of an attentiveness layer and can determine its location in the cortex.

Another consequence of our conjecture is that the incoming information is processed by other layers of the cortex in the form of specific spatiotemporal activities such as limit cycles, coherent propagation phenomena and spatiotemporal chaos with various degrees of coherence. This is in sharp contrast with the manner in which computation is performed in classical artificial neural networks. Experimental data pointing in this direction has been reported in the literature [16, 17, 18].

Recently we could show that stabilization of unstable periodic orbits could be achieved in model systems describing more adequately the cortical tissue [19]. This model takes account of propagation delays in neuronal transmission between excitatory and inhibitory neurons, which are not oscillating units. The ideas expressed above must presently be tested with these more brainlike networks. Work in this direction is in progress.

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