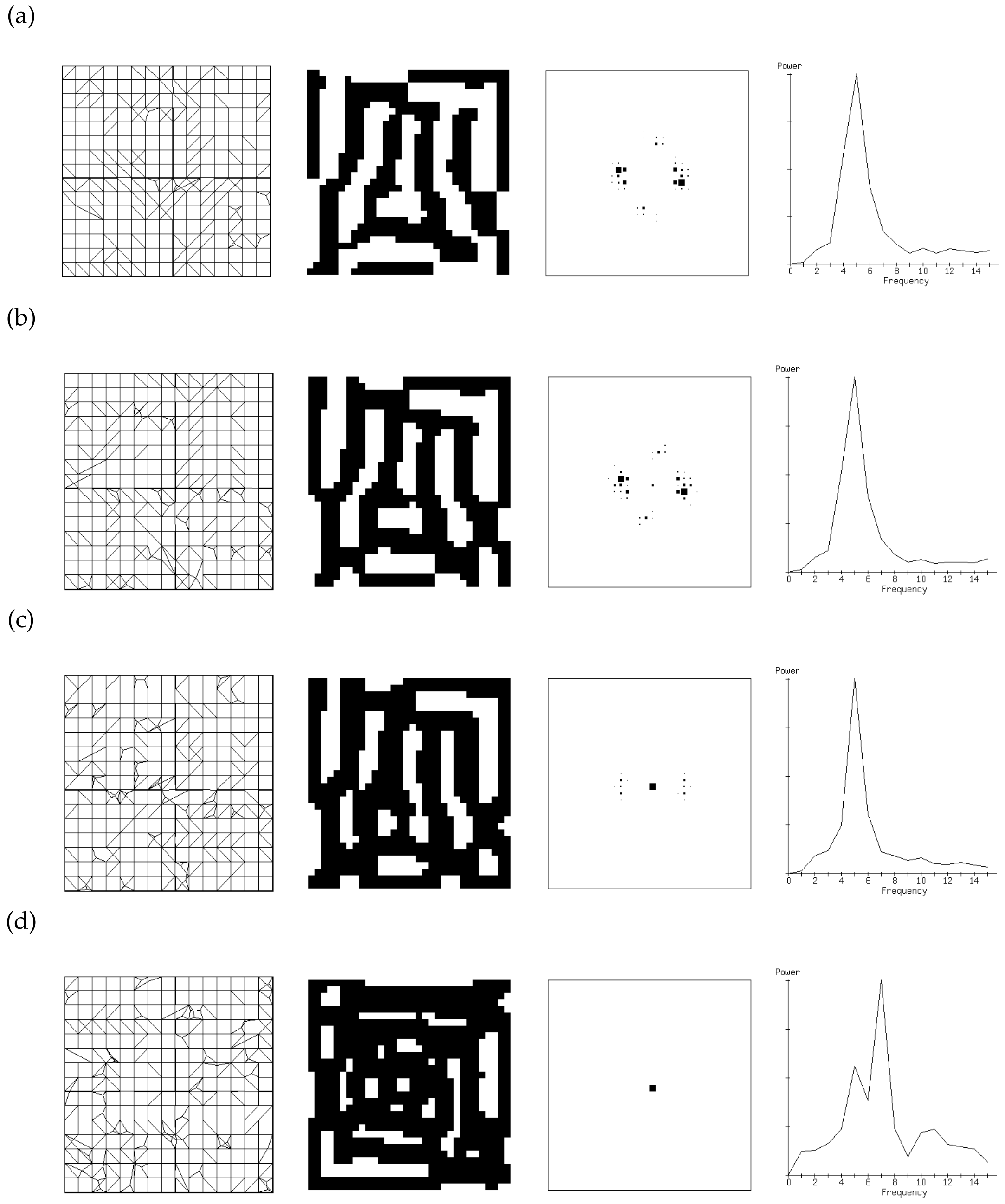


Figure 5.11: Effects of monocular deprivation - elastic net algorithm. In all cases slow annealing and a separation of retinae of 0.2 was used. (a) Normal case. (b) Deprivation parameter = 0.8. (c) Deprivation parameter = 0.6. (d) Deprivation parameter = 0.4.

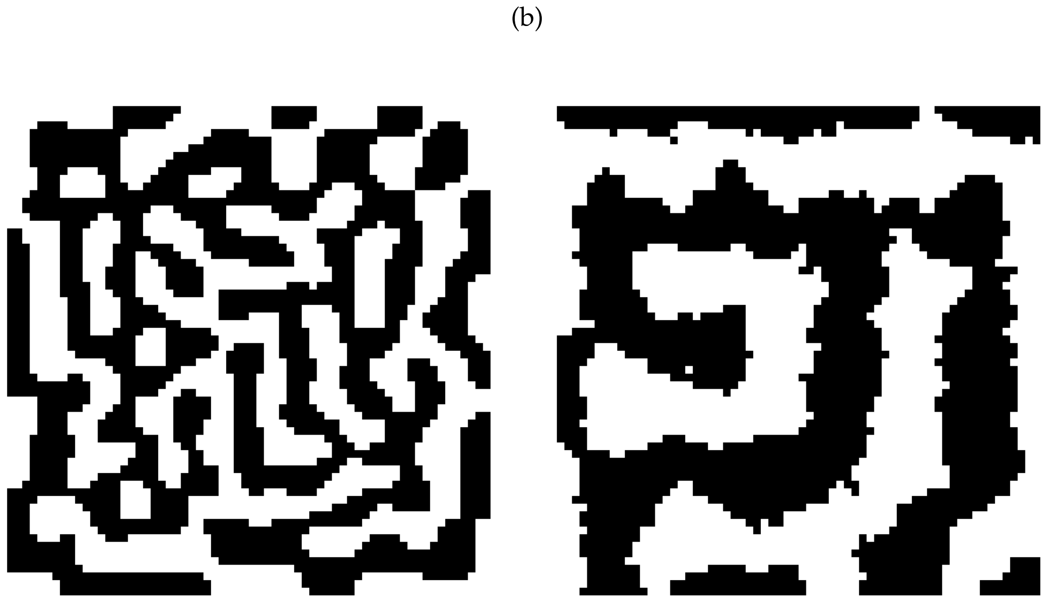


Figure 5.10: Results of one run of the (a) elastic net and (b) Kohonen-type algorithms for $2 \times 32 \times 32$ retinal units mapping to $64 \times 64$ cortical units, $2l = 0.15$, slow annealing.

stripe periodicity (i.e. sum of widths of neighbouring right- and left-eye stripes) remains the same as in the normal case, as shown by the constant position of the peak of the power spectrum.

### 5.6.4 Anisotropic boundary conditions

One interesting phenomenon of natural stripes is that they exhibit global order: stripes remain parallel over large distances, which might not be expected from the operation of purely local mechanisms as are present in most models. [LeVay et al 1985] provides a review of theoretical ideas regarding possible sources of an anistropic ordering influence, and hence how such global order might arise. Such ideas include elongation of geniculocortical arborizations [von der Malsburg 1979], anisotropic growth of the cortex [Swindale 1980], and differences in the strength of two orthogonal gradients of adhesiveness [Fraser 1985]. In [LeVay et al 1985] however it is suggested that the effect could be due to anisotropic boundary conditions:

> "The largest of the LGN layers are roughly circular in outline....whereas the cortex is approximately twice as long as it is wide, with the long axis oriented horizontally in our standard format. Thus, to a first approximation, it is reasonable to regard the geniculocortical mapping problem as one of mapping two circular discs, one for each eye, onto a horizontally elongated ellipse. From a global standpoint, the simplest way of achieving this is to slice each disc into thin vertical strips and then to interdigitate the left and right eye strips, thereby forming an appropriately elongated ellipse. If, instead, one started with discs cut into horizontal stripes that were stacked vertically, the strips would have to be stretched horizontally or shrunk vertically by 4-fold in order to obtain the requisite elongation. More generally, the only way to obtain elongation along one axis without anisotropic stretching of the inputs is to interdigitate stripes cut orthogonal to that axis." [LeVay et al 1985]

We tested this general hypothesis by performing experiments with the elastic net algorithm as above except that the cortical sheet was now rectangular. Two cases were considered: $32 \times 16$ and $32 \times 24$ cortical units, in order to test the amount of anisotropy required to produce global ordering. In both cases the separation between retinae was 0.2. Results are shown in figure 5.13.

In the first case almost entirely vertical stripes are formed, while in the second there is a strong bias towards vertical over horizontal, agreeing with LeVay et al's prediction (though for the squares-rectangles rather than circles-ellipses case). We suggest that this effect could perhaps provide another test of the adequacy of models

| Separation of layers | Elastic net | | | Kohonen-type algorithm | | |
|---|---|---|---|---|---|---|
| | $L_N$ | $L_C$ | Total | $L_N$ | $L_C$ | Total |
| 0.10 | 4215.7 | 336.4 | 4552.1 | 4510.7 | 355.5 | 4866.2 |
| 0.15 | 4248.4 | 362.9 | 4611.3 | 4670.6 | 612.9 | 5283.5 |
| 0.20 | 4378.5 | 546.7 | 4925.2 | 4847.0 | 791.6 | 5638.6 |
| 0.25 | 4654.5 | 823.4 | 5477.9 | 4846.6 | 927.1 | 5773.7 |
| 0.30 | 4887.1 | 1094.2 | 5981.3 | 4839.5 | 1010.7 | 5852.2 |

Table 5.3: Length of cortical wiring required for the slow annealing cases.

| Separation | Elastic net | | Kohonen-type | |
|---|---|---|---|---|
| | Fast | Slow | Fast | Slow |
| 0.10 | 4577.8 | 4552.1 | 5000.5 | 4866.2 |
| 0.15 | 4561.6 | 4611.3 | 5362.5 | 5283.5 |
| 0.20 | 5131.3 | 4925.2 | 5579.6 | 5638.6 |
| 0.25 | 5427.5 | 5477.9 | 5924.7 | 5773.7 |
| 0.30 | 5702.9 | 5981.3 | 6310.3 | 5852.2 |

Table 5.4: Comparison of total cortical wiring for fast and slow results for both algorithms.

This would appear to account for the qualitative difference between the two sets of results, thus indicating that better solutions to the optimization problem can indeed imply improved stripe morphology.

It should of course be noted that the comparisons we have made between the two algorithms have only been for certain sets of parameters. Indeed, we have shown that the Kohonen-type algorithm annealed slowly can produce better results than the elastic net annealed quickly. Our criteria of equal run-time is only a crude comparison of the two algorithms, being somewhat implementation-dependent. However, it is encouraging that our quantitative results are in general agreement with the comparisons that have been performed in the TSP case [Angeniol et al 1988, Peterson 1990, Simmen 1992].

**Larger scale**

One larger-scale simulation was performed for each algorithm (two $32 \times 32$ retinal sheets mapping to one $64 \times 64$ cortical sheet), and the results shown in figure 5.10. Similar conclusions apply to this case as to the smaller-scale cases: the stripes produced by the elastic algorithm are more uniform. Note the marked difference in the stripe width produced by the two algorithms.

### 5.6.3 Monocular deprivation

Monocular deprivation (reducing the competitive strength of one eye) was modelled in both algorithms by using a different value of $\alpha$ for each eye (this was first studied for these algorithms in [Goodhill 1990]). The amount of deprivation, or "deprivation parameter", was defined to be the ratio of $\alpha$ for the deprived eye to $\alpha$ for the normal eye. The variation of stripe pattern with the size of this deprivation parameter is shown for the elastic net algorithm in figure 5.11, and for the Kohonen-type algorithm in figure 5.12.

Firstly, we see the desired result that for both algorithms the deprived eye takes over less of the cortex than the normal eye: this effect is more marked for the elastic net than the Kohonen-type algorithm. However, there appears to be a significant difference in the way that this occurs for the two algorithms. For the elastic net, the stripes become thinner but remain continuous. For the Kohonen-type algorithm, the mapping from the deprived eye appears rather to break up into a series of blobs, each blob being of roughly the same size as the original stripe thickness. The degree of deprivation appears to exert a stronger effect on the elastic net than on the Kohonen-type algorithm. Both sets of results also reflect the biological data that for the deprived case

| Separation of layers | Elastic net | | Kohonen-type | |
|---|---|---|---|---|
| | Fast | Slow | Fast | Slow |
| 0.10 | 111.2 | 114.3 | 121.4 | 119.7 |
| 0.15 | 135.6 | 133.7 | 151.7 | 137.5 |
| 0.20 | 155.7 | 155.8 | 176.9 | 148.5 |
| 0.25 | 165.3 | 161.4 | 198.6 | 160.3 |
| 0.30 | 180.1 | 165.3 | 215.0 | 168.1 |

Table 5.1: Distance D for the results shown in figures 5.6 to 5.9.

| Separation of layers | Elastic net | | | Kohonen-type algorithm | | |
|---|---|---|---|---|---|---|
| | $L_N$ | $L_C$ | Total | $L_N$ | $L_C$ | Total |
| 0.10 | 4236.9 | 340.9 | 4577.8 | 4624.3 | 376.2 | 5000.5 |
| 0.15 | 4211.1 | 350.4 | 4561.6 | 4866.8 | 495.7 | 5362.5 |
| 0.20 | 4512.3 | 619.0 | 5131.3 | 4986.4 | 593.2 | 5579.6 |
| 0.25 | 4622.1 | 805.4 | 5427.5 | 5225.0 | 699.7 | 5924.7 |
| 0.30 | 4788.1 | 914.8 | 5702.9 | 5528.0 | 782.3 | 6310.3 |

Table 5.2: Length of cortical wiring required for the fast annealing cases.

Regarding stripe morphology, we see that there are general differences between the two algorithms. The elastic net produces patterns that are more regular, with longer stripes of a more uniform width. The Kohonen-type algorithm has a greater tendency to form blobs, especially for fast annealing. These qualitative impressions are confirmed by the fourier transforms of the stripe patterns and their power spectra: the spectra tend to show sharper peaks for the elastic net. We also note that, for a given separation, stripes produced by the Kohonen algorithm tend to be wider than those produced by the elastic net.

**Topography**

Qualitatively, it appears that the elastic net produces a more ordered topography for the map than the Kohonen-type algorithm. We now attempt to measure this difference quantitatively in two different ways. Firstly, the total distance $D$ as measured in feature space between neighbouring cortical units was calculated:

$$D = \sum_{i} \sum_{j \in N(i)} \|y_j - y_i\|,$$

where $N(i)$ refers to the four topological neighbours of cortical unit $i$. Values of $D$ for all separations and both annealing rates for the two algorithms are shown in table 5.1. $D$ inevitably increases as the separation of the retinae increases: however, for each separation, $D$ is usually smaller for the elastic net than for the Kohonen-type algorithm. We also see that, as expected, a smaller $D$ is usually produced by annealing more slowly.

Secondly, we implemented a simple cost function to measure length of wiring as in the minimal wiring principle: the total length $L$ ($p = 1$ in the terminology of page 55) of wire required to join up all the cortical units that represent neighbouring or corresponding retinal units was calculated. The results are shown for fast annealing in table 5.2 and slow annealing in table 5.3, where besides the total length we also give the length of the neighbourhood, which we now refer to as $L_N$, and correspondence, $L_C$, terms separately. An inevitable effect of increasing stripe width is that the correspondence term increases. Although as discussed earlier it is not clear how the neighbourhood and correspondence terms should be weighted, they are each usually bigger for the Kohonen algorithm than the elastic net for each separation of the two retinae. Just the total wiring length is compared for both algorithms and both annealing rates in 5.4. Again it is clear that both quantitatively and qualitatively slower annealing usually gives better solutions, as expected.

Thus it appears that in general the elastic net is optimizing more effectively than the Kohonen-type algorithm.