PROBLEMS

4.1. The object of this problem is to analyze a physical configuration that yields the electrical terminal relations of (4.1.6) and (4.1.7) almost exactly. The system of Fig. 4P.1 consists of two concentric cylinders of ferromagnetic material with infinite permeability and zero conductivity. Both cylinders have length l and are separated by the air gap g. As indicated in the figure, the rotor carries a winding of N_r turns distributed sinusoidally and

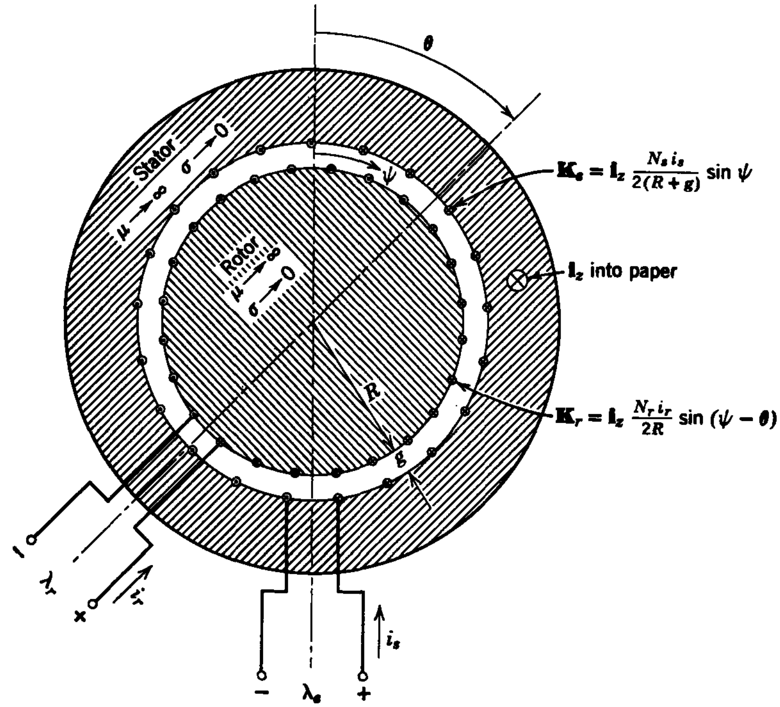

Fig. 4P.1

having negligible radial thickness. The stator carries a winding of N_s turns distributed sinusoidally and having negligible radial thickness. Current through these windings leads to sinusoidally distributed surface currents as indicated. In the analysis we neglect the effects of end turns and assume $g \ll R$ so that the radial variation of magnetic field can be neglected.

(a) Find the radial component of air-gap flux density due to stator current alone.
(b) Find the radial component of air-gap flux density due to rotor current alone.
(c) Use the flux densities found in parts (a) and (b) to find λ_s and λ_r in the form of (4.1.6) and (4.1.7). In particular, evaluate L_s, L_r, and M in terms of given data.

4.2. Rework Problem 4.1 with the more practical uniform winding distribution representable by surface current densities

$$\mathbf{K}_s = \begin{cases} \mathbf{i}_z \dfrac{N_s i_s}{\pi(R+g)}, & \text{for} \quad 0 < \psi < \pi, \\ -\mathbf{i}_z \dfrac{N_s i_s}{\pi(R+g)}; & \text{for} \quad \pi < \psi < 2\pi, \end{cases}$$

$$\mathbf{K}_r = \begin{cases} \mathbf{i}_z \dfrac{N_r i_r}{\pi R}, & \text{for} \quad 0 < (\psi - \theta) < \pi, \\ -\mathbf{i}_z \dfrac{N_r i_r}{\pi R}, & \text{for} \quad \pi < (\psi - \theta) < 2\pi. \end{cases}$$

In part (c) you will find the mutual inductance to be expressed as an infinite series like (4.1.4).

4.3. With reference to Problems 4.1 and 4.2, show that if either the rotor winding or the stator winding is sinusoidally distributed as in Problem 4.1, the mutual inductance contains only a space fundamental term, regardless of the winding distribution on the other member.

4.4. The machine represented schematically in Fig. 4P.4 has uniform winding distributions. As indicated by Problem 4.2, the electrical terminal relations are ideally

$$\lambda_s = L_s i_s + i_r \sum_{n\,\text{odd}} \frac{M_o}{n^4} \cos n\theta,$$

$$\lambda_r = L_r i_r + i_s \sum_{n\,\text{odd}} \frac{M_o}{n^4} \cos n\theta,$$

where L_s, L_r, and M_o are constants. We now constrain the machine as follows: $i_r = I =$ constant; $\theta = \omega t$, $\omega =$ constant, stator winding open-circuited $i_s = 0$.

(a) Find the instantaneous stator voltage $v_s(t)$.
(b) Find the ratio of the amplitude of the nth harmonic stator voltage to the amplitude of the fundamental component of stator voltage.
(c) Plot one complete cycle of $v_s(t)$ found in (a).

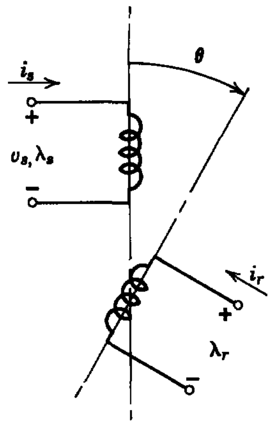

Fig. 4P.4

4.5. Calculate the electromagnetic torque T^e of (4.1.8) by using the electrical terminal relations (4.1.6) and (4.1.7) and the assumption that the coupling system is conservative.

4.6. A schematic representation of a rotating machine is shown in Fig. 4P.6. The rotor winding is superconducting and the rotor has moment of inertia J. The machine is constructed so that the electrical terminal relations are $\lambda_s = L_s i_s + M i_r \cos\theta$, $\lambda_r = M i_s \cos\theta + L_r i_r$. The machine is placed in operation as follows:

(a) With the rotor (r) terminals open-circuited and the rotor position at $\theta = 0$, the current i_s is raised to I_o.
(b) The rotor (r) terminals are short circuited to conserve the flux λ_r, regardless of $\theta(t)$ and $i_s(t)$.
(c) The current i_s is constrained by the independent current source $i(t)$.

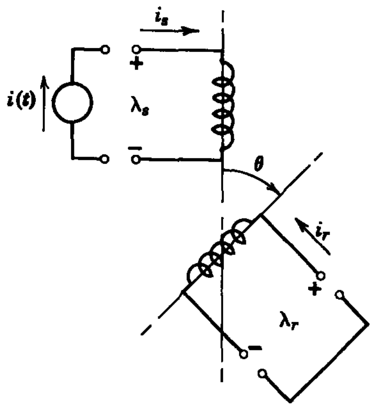

Fig. 4P.6

Write the equation of motion for the shaft with no external mechanical torque applied. Your answer should be one equation involving $\theta(t)$ as the only unknown. Damping may be ignored.

4.7. A smooth-air-gap machine with one winding on the rotor and one on the stator (see Fig. 4.1.1) has the electrical terminal relations of (4.1.1) and (4.1.2).

$$\lambda_s = L_s i_s + L_{sr}(\theta) i_r, \tag{4.1.1}$$

$$\lambda_r = L_{sr}(\theta) i_s + L_r i_r. \tag{4.1.2}$$

The mutual inductance $L_{sr}(\theta)$ contains two spatial harmonics, the fundamental and the third. Thus $L_{sr}(\theta) = M_1 \cos\theta + M_3 \cos 3\theta$, where M_1 and M_3 are constants.

(a) Find the torque of electric origin as a function of i_s, i_r, θ, M_1, and M_3.

(b) Constrain the machine with the current sources $i_s = I_s \sin \omega_s t$, $i_r = I_r \sin \omega_r t$ and the position source $\theta = \omega_m t + \gamma$, where I_s, I_r, ω_s, ω_r and γ are constants. Find the values of ω_m at which the machine can produce an average torque and find an expression for the average torque for each value of ω_m found.

4.8. The smooth-air-gap machine of Fig. 4.1.1 with the terminal relations given by (4.1.6) to (4.1.8) is constrained as follows: single-frequency rotor current, $i_r = I_r \sin \omega_r t$; stator current containing fundamental and third harmonic, $i_s = I_{s1} \sin \omega_s t + I_{s3} \sin 3\omega_s t$; and the position source $\theta = \omega_m t + \gamma$, where I_r, I_{s1}, I_{s3}, ω_r, ω_s, and γ are constants. Find the values of ω_m at which the machine can produce an average torque and give an expression for the average torque for each value of ω_m found.

4.9. Compute the torque T^e of (4.1.23) by using the electrical terminal relations of (4.1.19) to (4.1.22) and the assumption that the coupling system is conservative.

4.10. A smooth-air-gap machine has a two-phase set of stator windings, each with a total of N turns. The windings are distributed sinusoidally and currents in them produce surface current densities as indicated in Fig. 4P.10. When $g \ll R$, the radial flux density produced in the air gap by each winding (see Problem 4.1), is

$$B_{ra} = \frac{\mu_0 N i_a}{2g} \cos \psi,$$

$$B_{rb} = \frac{\mu_0 N i_b}{2g} \sin \psi.$$

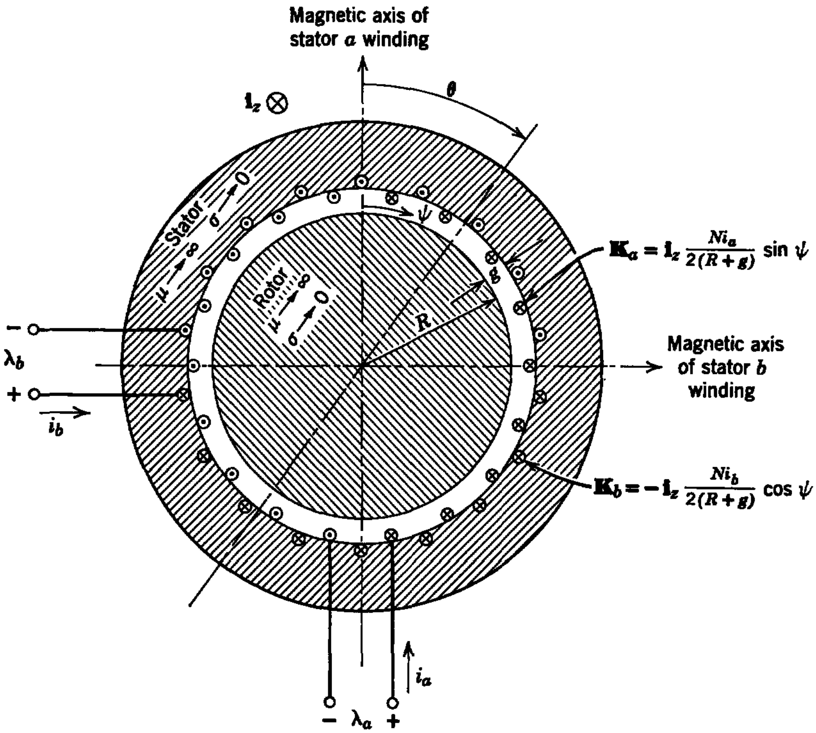

Fig. 4P.10

(a) For the two-phase excitation $i_a = I_a \cos \omega t$, $i_b = I_b \sin \omega t$, which is unbalanced in amplitude, find the total radial flux density.

(b) Express the answer to part (a) as a sum of two traveling waves. Identify the *forward* and *backward* components and show that their respective angular velocities are $\omega_f = \omega$ and $\omega_b = -\omega$.

(c) Evaluate the ratio of the amplitudes of backward and forward waves. Show that the ratio $\to 0$ for a balanced excitation (i.e., consider the limit for $I_b \to I_a$).

(d) Discuss how to achieve a constant amplitude backward wave only. This is the method used to reverse the direction of rotation of an ac machine.

4.11. Rework Problem 4.10 and replace the excitation of part (a) with $i_a = I \cos \omega t$, $i_b = I \sin(\omega t + \beta)$. This is a two-phase set of currents, balanced in amplitude but unbalanced in phase. For part (c) balanced excitation occurs when $\beta \to 0$.

4.12. Use (4.1.53) as the starting point to show that for steady-state operation the electrical power into a two-phase synchronous machine is equal to the mechanical power delivered, as expressed by (4.1.54).

4.13. The two-phase equivalent of a large turbogenerator of the type now being used to generate power is as follows:

2-phase
60 Hz, 2-pole
Rated terminal voltage, 17,000 V rms

Rated terminal current, 21,300 A rms
Rating, 724×10^6 VA
Rated power factor, 0.85
Armature inductance, $L_s = 4.4 \times 10^{-3}$ H
Maximum value of armature-field mutual inductance, $M = 0.030$ H
Rated field current, $I_f = 6100$ A

Calculate and plot a family of V-curves for this generator. The V-curves are plots of armature current versus field current at constant power and constant terminal voltage (see Fig. 4.1.15). Your family of curves should be bounded by rated armature and field current, zero-power-factor, and 90° torque angle. Indicate which of these limits the curves. Also indicate on your plot the 0 and 0.85 power factors, both leading and lagging, and the unity power factor. Plot curves for 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and full rated load of 615 MW and for rated armature voltage. It will be convenient to normalize armature current to the rated value and field current to that value necessary to produce rated terminal voltage with the armature open-circuited.

4.14. It is customary to define the complex power produced by an alternator as $P + jQ$, where P is real power and Q is reactive power. For a two-phase machine with balanced currents and voltages and a phase angle ϕ

$$v_a = \text{Re}\,(\hat{V}e^{j\omega t}), \qquad i_a = \text{Re}\,(\hat{I}e^{j\omega t}),$$
$$v_b = \text{Re}\,(-j\hat{V}e^{j\omega t}), \qquad i_b = \text{Re}\,(-j\hat{I}e^{j\omega t}),$$

where $\hat{V} = V$ and $\hat{I} = Ie^{-j\phi}$. The complex power supplied by both phases is $P + jQ = \hat{V}\hat{I}^* = VI\cos\phi + jVI\sin\phi$. By convention $Q > 0$ when $\hat{I}$ lags $\hat{V}$ (the load is inductive).

A capability curve for an alternator is a plot of P versus Q for constant armature voltage and for maximum allowable operating conditions defined by rated armature current, rated field current, or steady-state stability (torque angle δ approaching a critical value which we take to be 90°). Plot the capability curve for the alternator described in Problem 4.13 for operation at rated voltage. Indicate on your plot the limit that determines that part of the curve. It is useful to normalize both P and Q to the rating of the alternator.

4.15. An automobile speedometer consists of a permanent magnet mounted on a rotating shaft connected to the automobile transmission. An aluminum "drag cup" with a pointer mounted on it is placed around this rotating magnet. The cup is free to rotate through an angle ψ but is restrained by a torsion spring that provides a torque $T_s = -K\psi$. The angular position of the cup can be used to determine the angular velocity of the shaft connected to the magnet and therefore the speed of the automobile. The model to be used in analyzing the speedometer is illustrated in Fig. 4P.15. The permanent magnet is represented by a coil excited by a constant-current source. The drag cup is simulated by two coils shunted by resistances. These coils are attached to a rotatable frame, which in turn is restrained by the torsion spring. An appropriate electrical model of the coupling field is

$$\lambda_1 = Mi_3\cos(\phi - \psi) + Li_1,$$
$$\lambda_2 = Mi_3\sin(\phi - \psi) + Li_2,$$
$$\lambda_3 = L_3 i_3 + Mi_1\cos(\phi - \psi) + Mi_2\sin(\phi - \psi).$$

Assuming that the rotational velocity of the shaft is constant (i.e., the speed of the car is constant), find the deflection of the rotatable frame (of the speedometer pointer) as a function of the shaft rotational velocity $\dot{\phi}$. You may assume that the device is designed in such a way that

$$\left|L\frac{di}{dt}\right| \ll |Ri|.$$

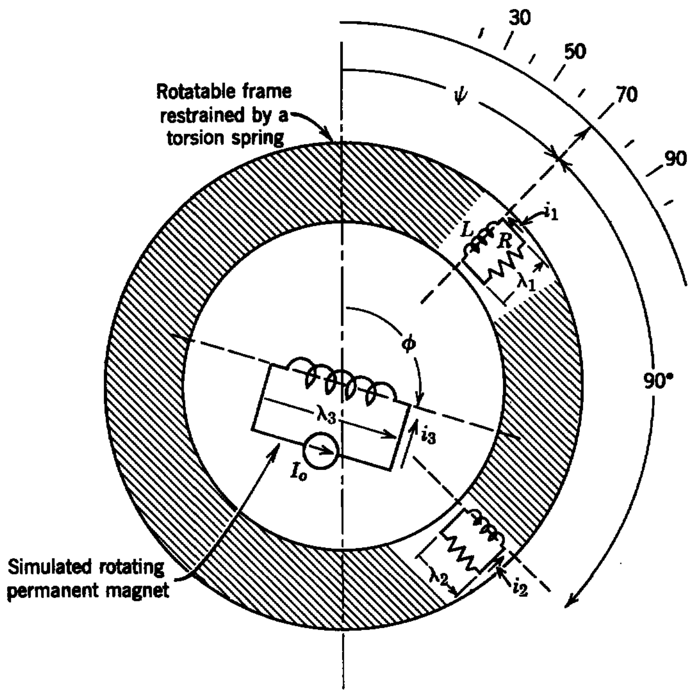

Fig. 4P.15

4.16. For nomenclature, refer to Fig. 4.1.17. The two-phase equivalent of a large, two-pole, polyphase, 60-Hz induction motor has the following parameters for operation at 60 Hz: $R_r = 0.100$ ohm, $\omega_s M = 4.50$ ohms, and $\omega_s(L_s - M) = \omega_s(L_r - M) = 0.300$ ohm. Neglect armature resistance. For operation at a constant amplitude of armature voltage $V_s = \sqrt{2}\,500$ V peak, calculate and plot torque, armature current, volt-ampere input, electrical power input, and mechanical power output as functions of mechanical speed for the range $0 < \omega_m < \omega_s = 120\pi$ rad/sec.

4.17. The induction motor of Problem 4.16 is driving a fan load with the torque speed characteristic $T_m = -B\omega_m^3$, where $B = 7.50 \times 10^{-6}$ N-M sec^3/rad^3. Assume steady-state operation.

(a) For operation with balanced armature voltage of $V_s = \sqrt{2}\,500$ V peak calculate the steady-state slip, mechanical power into the fan, electrical power input, and power factor.

(b) Calculate and plot the quantities of part (a) as functions of armature voltage for a range $\sqrt{2}\,450 < V_s < \sqrt{2}\,550$ V peak.

4.18. This problem is a version of the machine analysis in Problem 4.1 but with a three-phase winding on the stator. The geometry is illustrated in Fig. 4P.18; N_s is the total number of turns on each stator phase and N_r is the total number of turns in the rotor winding. The

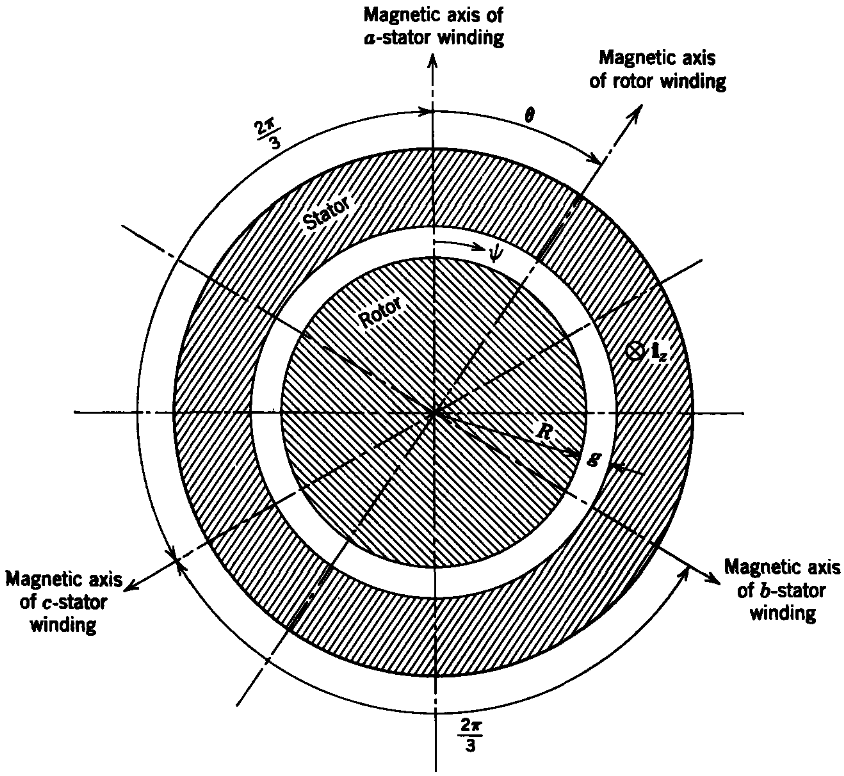

Fig. 4P.18

surface current densities produced by the three armature currents on the surface at $R + g$ are

$$\mathbf{K}_a = \mathbf{i}_z \frac{N_s i_a}{2(R+g)} \sin \psi,$$

$$\mathbf{K}_b = \mathbf{i}_z \frac{N_s i_b}{2(R+g)} \sin \left(\psi - \frac{2\pi}{3}\right),$$

$$\mathbf{K}_c = \mathbf{i}_z \frac{N_s i_c}{2(R+g)} \sin \left(\psi - \frac{4\pi}{3}\right).$$

The surface current density due to rotor current on the surface at R is

$$\mathbf{K}_r = \mathbf{i}_z \frac{N_r i_r}{2R} \sin (\psi - \theta).$$

Assume $g \ll R$ so that there is no appreciable variation in the radial component of magnetic field across the air gap.

(a) Find the radial flux density due to current in each winding.
(b) Find the mutual inductance between the a and b windings on the stator.
(c) Write the electrical terminal relations for the machine.
(d) Find the torque T^e of electrical origin.

4.19. Consider the machine in Problem 4.18 with the stator excitations

$$i_a = I_a \cos \omega t,$$

$$i_b = I_b \cos\left(\omega t - \frac{2\pi}{3}\right),$$

$$i_c = I_c \cos\left(\omega t - \frac{4\pi}{3}\right).$$

(a) Show that the radial component of air-gap flux density is expressible as a combination of two constant-amplitude waves, one rotating in the positive θ-direction with the speed ω and the other rotating in the negative θ-direction with speed ω.
(b) Show that when $I_a = I_b = I_c$ the amplitude of the wave traveling in the negative θ-direction goes to zero.

4.20. A four-pole smooth-air-gap machine has a two-phase set of stator windings, each with a total of N turns. The windings are distributed sinusoidally and currents in them produce surface current densities as indicated in Fig. 4P.20. When $g \ll R$, the radial flux density produced in the air gap by each winding is (see Problems 4.1 and 4.10)

$$B_{ra} = \frac{\mu_0 N i_a}{2g} \cos 2\psi,$$

$$B_{rb} = \frac{\mu_0 N i_b}{2g} \sin 2\psi.$$

(a) For the two-phase excitation, $i_a = I_a \cos \omega t$, $i_b = I_b \sin \omega t$, which is unbalanced in amplitude, find the total radial flux density.
(b) Express the answer to (a) as a sum of two constant-amplitude traveling waves.

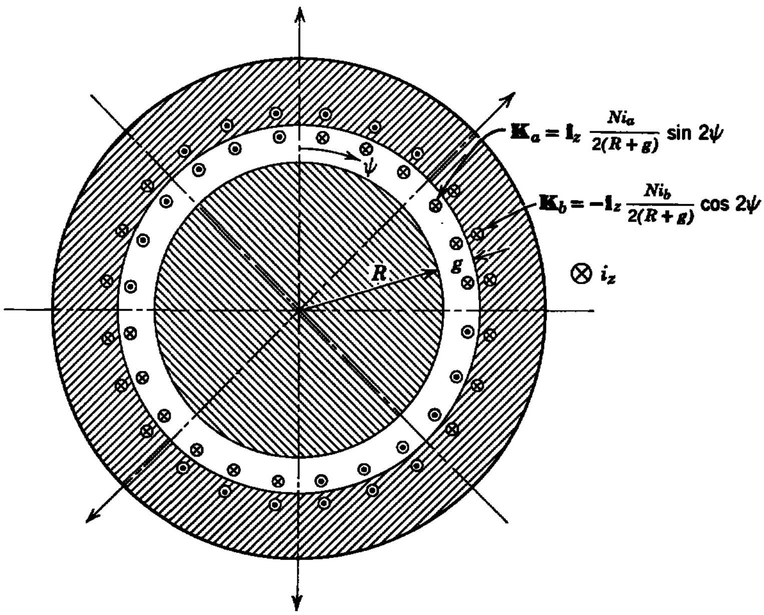

Fig. 4P.20

Identify the forward and backward components and show that their respective angular velocities are $\omega_f = \omega/2$ and $\omega_b = -\omega/2$.

(c) Show that the amplitude of the backward wave goes to zero when $I_b = I_a$ and that the amplitude of the forward wave goes to zero when $I_b = -I_a$.

4.21. Rework Problem 4.20 for a p-pole-pair machine for which the component radial air-gap flux densities are

$$B_{ra} = \frac{\mu_0 N i_a}{2g} \cos p\psi,$$

$$B_{rb} = \frac{\mu_0 N i_b}{2g} \sin p\psi.$$

Assume the same excitation as in part (a) of Problem 4.20. In part (*b*) the forward and backward waves have angular velocities $\omega_f = \omega/p$ and $\omega_b = -\omega/p$.

4.22. Derive the electromagnetic torque of (4.2.9), starting with the electrical terminal relations of (4.2.7) and (4.2.8) and the assumption that the coupling system is conservative.

4.23. The salient-pole, synchronous machine of Fig. 4P.23 is electrically linear and lossless and has a terminal inductance expressed as

$$L = \frac{L_o}{(1 - 0.25\cos 4\theta - 0.25 \cos 8\theta)},$$

where L_o is a positive constant. This is an alternative mathematical representation to the form given by (4.2.3).

(a) Describe briefly why the dependence of this inductance on θ is physically reasonable.

(b) Find the torque of electric origin T^e as a function of flux linkage λ, angle θ, and the constants of the system.

(c) As shown in Fig. 4P.23, the terminals are excited by a sinusoidal voltage source such that the flux λ is given by $\lambda(t) = \Lambda_o \cos \omega t$, where Λ_o and ω are positive constants. The rotor is driven by a constant-angular-velocity source such that $\theta(t) = \Omega t + \delta$, where Ω and δ are constants. Find the values of Ω, in terms of the electrical frequency ω, at which time-average power can be converted by the machine between the electrical and mechanical systems.

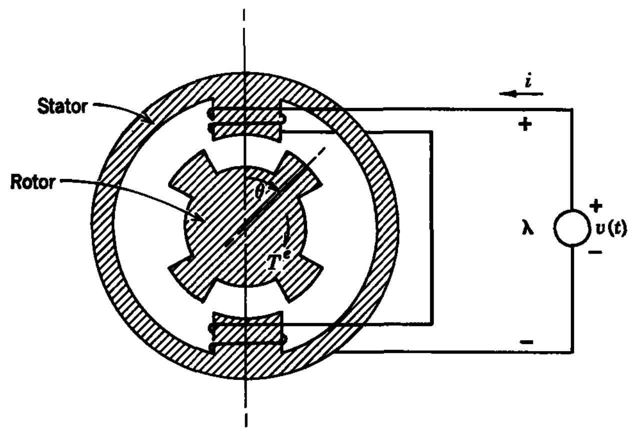

Fig. 4P.23

4.24. The two-phase equivalent of a salient-pole, synchronous motor has the following parameter values and ratings [see (4.2.28) to (4.2.30) for definitions]

2-phase	60 Hz
Rated output power,	6000 hp
Power factor,	0.8 leading
Rated armature voltage,	3000 V rms
Voltage coefficient,	$\omega M = 350$ V/A
Direct axis reactance,	$\omega(L_0 + L_2) = 4$ ohms
Quadrature axis reactance,	$\omega(L_0 - L_2) = 2.2$ ohms

(a) Find the field current necessary to give maximum rated conditions at rated voltage. This is rated field current.

(b) Calculate and plot a family of V-curves for loads of 6000, 3000, and zero hp and rated voltage; V-curves are plots of armature current as a function of field current for constant load power (see Problem 4.13). Indicate the factor that limits the extent of the plot: rated armature current, rated field current, or steady-state stability (pull-out torque is approached).

4.25. As discussed at the end of Section 4.1.6a, synchronous condensers are essentially synchronous machines operating with no shaft torque. They are used for power-factor correction and they are conventionally of the salient-pole type of construction. Start with (4.2.41), assume zero shaft torque [$\gamma = 0$ from (4.2.37)] and operation at constant armature voltage amplitude, and construct vector diagrams to show the machine appearing capacitive and inductive.

4.26. This is a problem that involves the use of a synchronous condenser to correct power factor in a power system. The correction is actually achieved by using the synchronous condenser to regulate voltage. We consider one phase of a balanced two-phase system. In Fig. 4P.26a a power system feeds a steady-state load which has admittance $Ye^{-j\phi}$ as shown.

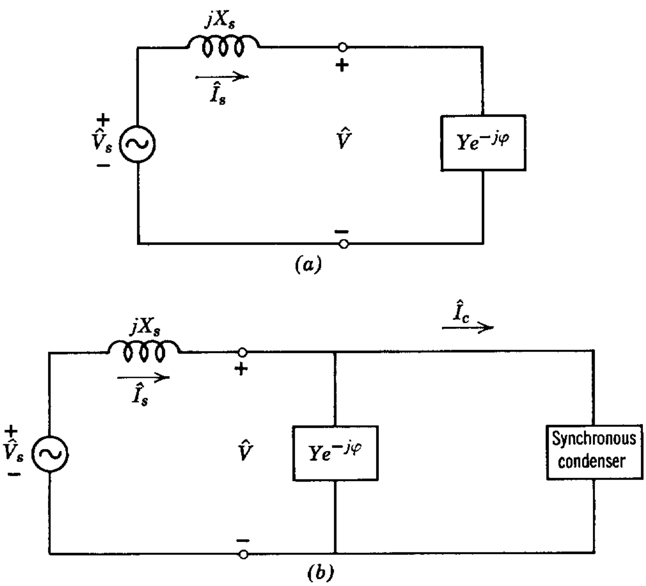

Fig. 4P.26

The Thevenin equivalent circuit of the system, as viewed from the load, is the source $\hat{V}_s$ in series with the inductive reactance jX_s. To fix ideas assume the following parameters and excitations: $V_s = \sqrt{2}\,100{,}000$ V peak, $X_s = 10$ ohms, $Y = 0.01$ mho.

(a) Find the ratio of the magnitudes of the load voltage V and the source voltage V_s for $\phi = 0$ and $\phi = 45$ degrees.
(b) Now a synchronous condenser is connected across the load as shown in Fig. 4P.26b and draws current $\hat{I}_c$. Find the volt-ampere rating required for the synchronous condenser to make the ratio $|\hat{V}|/|\hat{V}_s|$ equal to unity for each case in part (a). Compare each with the real power drawn by the load.

4.27. A two-phase, 60-Hz, salient-pole, 2-pole, synchronous motor has the following ratings and constants:

Rated output power,	1000 hp
Rated armature volts,	$\sqrt{2}$ 1000 V peak
Rated power factor,	unity
Direct axis reactance,	$\omega(L_0 + L_2) = 3.0$ ohms
Quadrature axis reactance,	$\omega(L_0 - L_2) = 2.0$ ohms
Speed voltage coefficient,	$\omega M = 150$ V/A

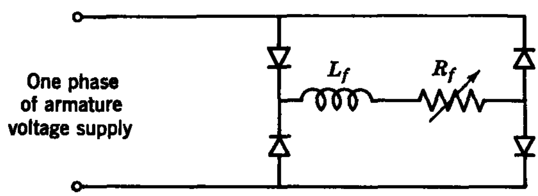

Fig. 4P.27

(a) The field winding of the motor is supplied from one phase of the supply by a full-wave bridge rectifier as shown in Fig. 4P.27. The field winding inductance is large enough that only the dc component of field voltage need be considered. Calculate the total field circuit resistance R_f necessary to achieve unity-power-factor operation at rated voltage with 1000 hp load.
(b) Calculate and plot the torque angle δ as a function of armature supply voltage from 10 per cent above rating down to the value at which the motor can no longer carry the load.

4.28. The two-phase equivalent of a large, salient-pole, 72-pole, water-wheel generator of the type now being used has the following constants and ratings:

Rating,	200×10^6 V-A
Frequency,	60 Hz
Power factor,	0.85 lagging
Rated terminal voltage,	10,000 V rms
Rated armature current,	10,000 A rms
Armature inductance,	$L_0 = 2.65 \times 10^{-3}$ H
	$L_2 = 0.53 \times 10^{-3}$ H
Maximum armature-field mutual inductance,	$M = 0.125$ H

(a) Calculate the field current necessary to achieve rated conditions of armature voltage, current, and power factor.
(b) Plot a capability curve for this generator. See Problem 4.14 for a description of a capability curve. In this case the stability limit of maximum steady-state torque will occur for $\delta < 90°$ (see Fig. 4.2.6).

4.29. Figure 4P.29 shows a pair of grounded conductors that form the rotor of a proposed rotating device. Two pairs of fixed conductors form the stator; one pair is at the potential

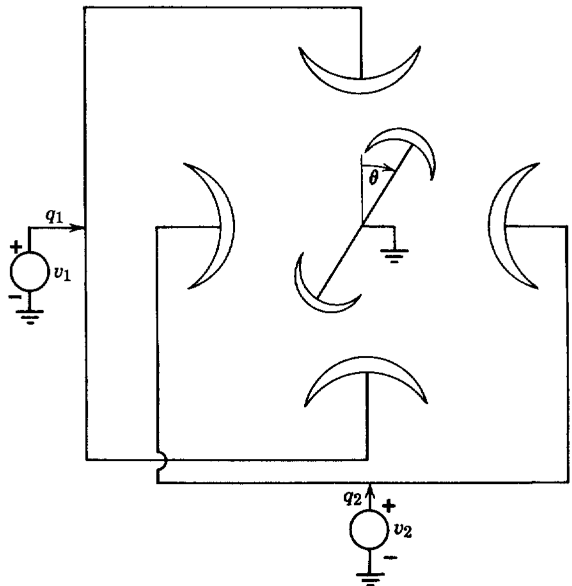

Fig. 4P.29

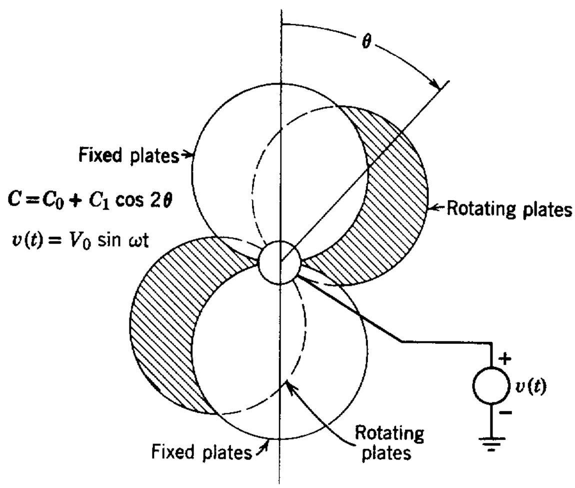

Fig. 4P.30

v_1 and supports a total charge q_1; the other is at the potential v_2 and supports the total charge q_2. Given that $q_1 = C_o(1 + \cos 2\theta)v_1$, $q_2 = C_o(1 + \sin 2\theta)v_2$, where C_o is a given positive constant,

(a) what is the electrical torque exerted on the rotor in the θ direction?
(b) The voltages v_1 and v_2 are now constrained to be $v_1 = V_o \cos \omega t$, $v_2 = V_o \sin \omega t$. Under what condition(s) will the device produce a time-average torque?
(c) Under the condition(s) of (b), what is the time-average torque?

4.30. A pair of capacitor plates is attached to a rotating shaft in such a way that when θ is zero they are directly opposite a pair of fixed plates. It is assumed that the variation in capacitance can be approximately described by the relation $C = C_0 + C_1 \cos 2\theta$. If a potential difference $v(t) = V_o \sin \omega t$ is applied to the plates through a slip ring, what are the shaft rotational velocities at which the device can behave like a motor?