## PROBLEMS

**2.1.** A piece of infinitely permeable magnetic material completes the magnetic circuit in Fig. 2P.1 in such a way that it is free to move in the $x$- or $y$-direction. Under the assumption that the air gaps are short compared with their cross-sectional dimensions (i.e., that the fields are as shown), find $\lambda(x, y, i)$. For what range of $x$ and $y$ is this expression valid?

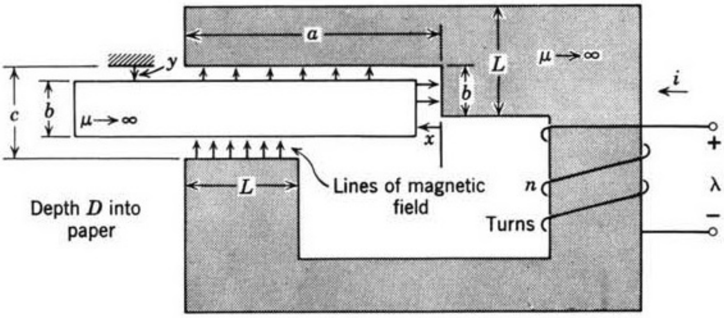


Fig. 2P.1

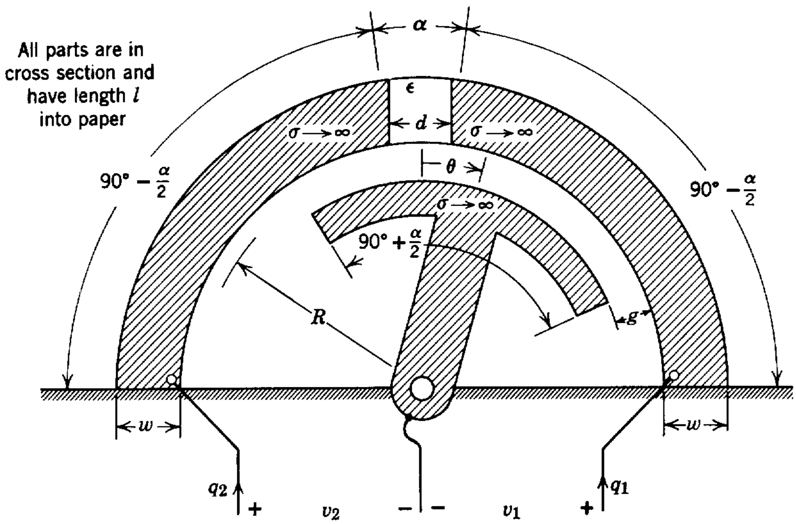


Fig. 2P.2

**2.2.** Three pieces of infinitely conducting material are arranged as shown in Fig. 2P.2. The two outer pieces are stationary and are separated by a block of insulating material of permittivity $\epsilon$. The inner piece is free to rotate an angle $\theta$. The gap $g$ is much less than the average radius $R$, which implies that the fields are approximately those of a plane-parallel geometry. Neglect the fringing fields. Find $q_1(v_1, v_2, \theta)$, $q_2(v_1, v_2, \theta)$.

**2.3.** The cross section of a cylindrical solenoid used to position the valve mechanism of a hydraulic control system is shown in Fig. 2P.3. When the currents $i_1$ and $i_2$ are equal, the plunger is centered horizontally ($x = 0$). When the coil currents are unbalanced, the plunger moves a distance $x$. The nonmagnetic sleeves keep the plunger centered radially. The

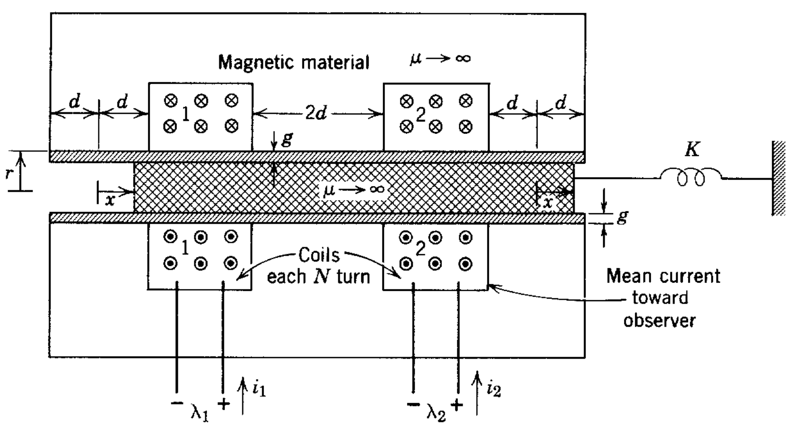


Fig. 2P.3

displacement $x$ is limited to the range $-d < x < d$. Show that the electrical terminal relations are

$$\lambda_1 = L_{11}i_1 + L_{12}i_2,$$
$$\lambda_2 = L_{12}i_1 + L_{22}i_2,$$

where

$$L_{11} = L_0\left[3 - 2\left(\frac{x}{d}\right) - \left(\frac{x}{d}\right)^2\right],$$

$$L_{22} = L_0\left[3 + 2\left(\frac{x}{d}\right) - \left(\frac{x}{d}\right)^2\right],$$

$$L_{12} = L_0\left[1 - \left(\frac{x}{d}\right)^2\right].$$

What is $L_0$ in terms of the system geometry?

**2.4.** (a) Write the differential equation governing the motion of mass $M$ acted on by the force source $f$ and the linear damper with coefficient $B$ (Fig. 2P.4).

(b) Calculate and make a dimensioned sketch of $dx/dt$ and $x$ as functions of time for $t > 0$ when the force source is the impulse ($u_0$ = unit impulse) $f = I_0u_0(t)$. (This is like hitting the mass with a hammer.)

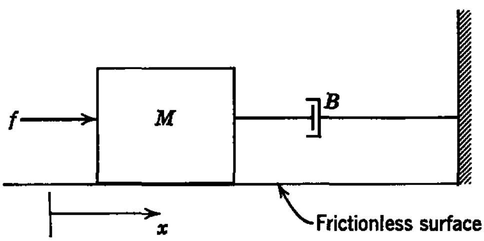


**Fig. 2P.4**

**2.5.** (a) Find the response $x(t)$ of the system shown in Fig. 2P.5$a$ to a driving force $f(t)$ which is

(1) an impulse $\qquad f(t) = I_0u_0(t)$,

(2) a step $\qquad f(t) = F_0u_{-1}(t)$.

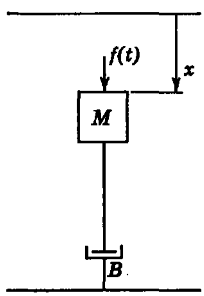


**Fig. 2P.5$a$**

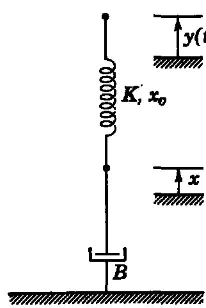


**Fig. 2P.5$b$**

(b) Find the response $x(t)$ of the system shown in Fig. 2P.5*b* to a driving displacement $y(t)$ which is

(1) $$y(t) = Au_0(t),$$

(2) $$y(t) = Y_0 u_{-1}(t).$$

**2.6.** The mechanical system shown in Fig. 2P.6 is set into motion by a forcing function $f(t)$. This motion is translational only. The masses $M_2$ and $M_3$ slip inside the cans as shown. Note that the upper can is attached to the mass $M_1$.

(a) Draw the mechanical circuit with nodes and parameters designated.

(b) Write three differential equations in $x_1$, $x_2$, and $x_3$ to describe the motion.

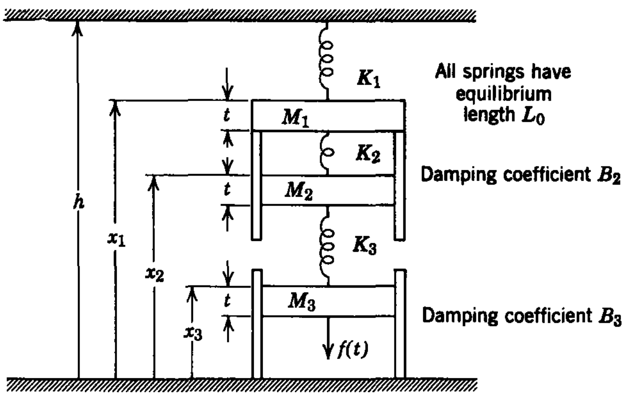


Fig. 2P.6

**2.7.** In the system in Fig. 2P.7 the two springs have zero force when both $x_1$ and $x_2$ are zero. A mechanical force $f$ is applied to node 2 in the direction shown. Write the equations governing the motion of the nodes 1 and 2. What are the natural frequencies involved?

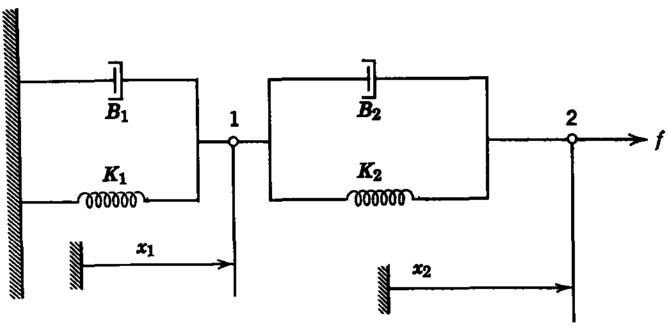


Fig. 2P.7

**2.8.** The velocity of the point $P$ shown in Fig. 2P.8 is

$$\mathbf{v} = \mathbf{i}_r \frac{dr}{dt} + \mathbf{i}_\theta r \frac{d\theta}{dt}.$$

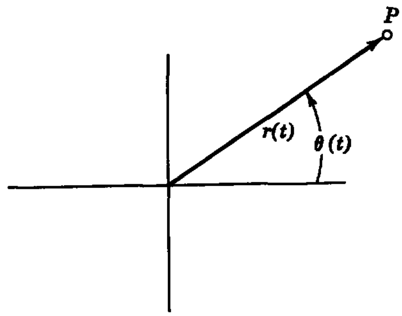


Fig. 2P.8

Show that the acceleration is

$$\frac{d\mathbf{v}}{dt} = \mathbf{i}_r\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right] + \mathbf{i}_\theta\left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right),$$

where

$$-r\left(\frac{d\theta}{dt}\right)^2 = \text{centripetal acceleration,}$$

$$2\frac{dr}{dt}\frac{d\theta}{dt} = \text{Coriolis acceleration.}$$

*Hint.* Remember in carrying out the time derivatives that $\mathbf{i}_r$ and $\mathbf{i}_\theta$ are functions of time. In fact, you will wish to show that

$$\frac{d\mathbf{i}_r}{dt} = \mathbf{i}_\theta\frac{d\theta}{dt}, \qquad \frac{d\mathbf{i}_\theta}{dt} = -\mathbf{i}_r\frac{d\theta}{dt}.$$