Massachusetts Institute of Technology

Department of Physics

Course:	8.701 – Introduction to Nuclear and Particle Physics
Term:	Fall 2020
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Discussion Problems

from recitation on **September 22nd, 2020**

Problem 1: Decay rate

Calculate the lifetime of A, the particle in our toy experiment.

- The lowest-order contribution to $A \to B + C$ is shown in Figure 1. There is no internal line and one vertex. Following the Feynman rules we find a factor $-ig$ and a delta function which we have to drop and replace by an i. Thus, we get $M = g$.

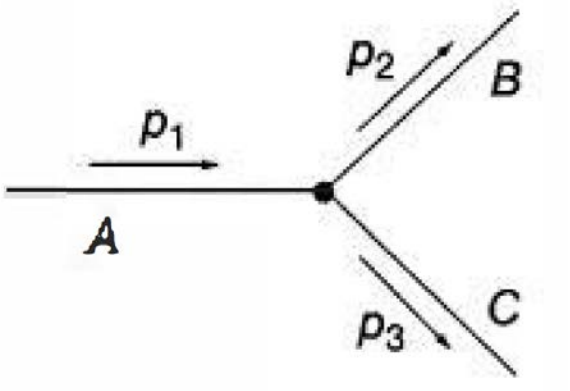

Figure 1: Lowest-order contribution to $A \to B + C$.

The decay is then $\Gamma = \frac{g^2|p|}{8\pi\hbar m_A^2 c}$ with the magnitude of the outgoing particles $|p| = \frac{c}{2m_A}\sqrt{m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}$. The lifetime for A is then $\tau = \frac{1}{\Gamma} = \frac{8\pi\hbar m_A^2 c}{g^2|p|}$

Problem 2: Scattering cross section

Calculate the differential cross section for the process $A + A \to B + B$

- The scattering process at lowest order is shown in Figure 2.

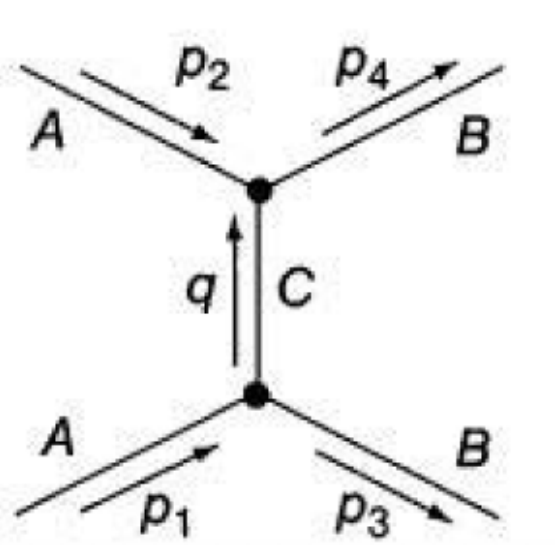

Figure 2: Lowest-order contribution to $A + A \to B + B$.

We have two vertices and one internal line with the propagator $\frac{i}{q^2 - m_C^2 C^2}$, two delta functions $2\pi\delta^4(p_1 - p_3 - q)$ and $2\pi\delta^4(p_2 + q - p_4)$ and have one integration $\frac{1}{(2\pi^4)} d^4q$. This yields $-i(2\pi)^4 g^2 \int \frac{1}{q^2 - m_C^2 C^2} \delta^4(p_1 - p_3 - q)\delta^4(p_2 + q - p_4) d^4q$.

Integrating and replacing the remaining delta function result in $M = \frac{g^2}{(p_4 - p_2)^2 - m_C^2 C^2}$. There is a second Feynman diagram (see Fig. 3) of the same order with p_3 and p_4 interchanged.

Looking at the specific example of $m_A = m_B = m$ and $m_C = 0$ we find $M = -\frac{g^2}{p^2 \sin^2\theta}$ and for the differential cross section $\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{\hbar c g^2}{16\pi E p^2 \sin^2\theta}\right)^2$.

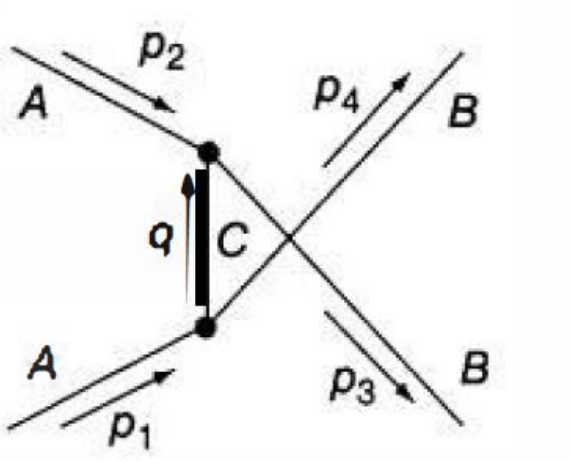

Figure 3: Lowest-order contribution to $A + A \to B + B$.

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