Massachusetts Institute of Technology

Department of Physics

Course:	8.701 – Introduction to Nuclear and Particle Physics
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Instructor:	Markus Klute
TA :	Tianyu Justin Yang

Discussion Problems

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Problem 1: Triangle Group

Consider symmetries of the equilateral triangle, see Fig. 1. It is carried into itself by a clockwise rotation through 120 (R_+), and by a counterclockwise rotation through 120 (R_-), by flipping it about the vertical axis a (R_a) and axis b (R_b) and c (R_c). Construct a multiplication table for the triangle group, filling the blanks in Tab. 2. In row i, column j put the product R_iR_j. Is this an Abelian group? How can you tell, just by looking at the table?

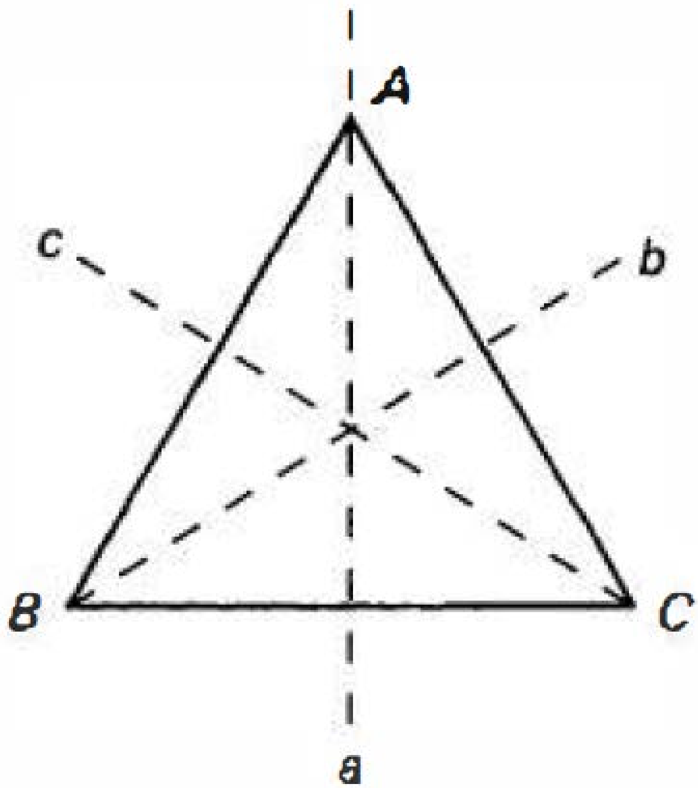

Figure 1: Equilateral triangle.

	I	R_+	R_-	R_a	R_b	R_c
I						
R_+						
R_-						
R_a						
R_b						
R_c						

Figure 2: Multiplication table for triangle group.

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	I	R_+	R_-	R_a	R_b	R_c
I	I	R_+	R_-	R_a	R_b	R_c
R_+	R_+	R_-	I	R_b	R_c	R_a
R_-	R_-	I	R_+	R_c	R_a	R_b
R_a	R_a	R_c	R_b	I	R_-	R_+
R_b	R_b	R_a	R_c	R_+	I	R_-
R_c	R_c	R_b	R_a	R_-	R_+	I

Figure 3: Multiplication table for triangle group.

The group is not Abelian; the multiplication table is not symmetrical across the main diagonal (for example, $R_+R_a = R_b$, but $R_aR_+ = R_c$).

Problem 2: Isosping - dynamic implications

Consider three nucleon-nucleon scattering processes

(a) $p + p \rightarrow d + \pi^+$

(b) $p + n \rightarrow d + \pi^0$

(c) $n + n \rightarrow d + \pi^-$

Figure 4: Nucleon-nucleon scattering processes.

The deuteron has isospin I=0 and the pion I=1. Isospin is conserved in the scattering process. Cross-sections go like the absolute square of the amplitude. What is the ratio of cross sections, $\sigma_a : \sigma_b : \sigma_c$?

- Since the deuteron carries $I = 0$, the isospin states on the right are $|1\ 1\rangle$, $|1\ 0\rangle$, and $|1 -1\rangle$, respectively, whereas those on the left are $pp = |11\rangle$, $nn = |1-1\rangle$, and $pn = (\frac{1}{\sqrt{2}})(|1\ 0\rangle + |0\ 0\rangle)$.* Only the $I = 1$ combination contributes (since the final state in each case is pure $I = 1$, and isospin is conserved), so the *scattering amplitudes* are in the ratio

$$\mathcal{M}_a : \mathcal{M}_b : \mathcal{M}_c = 1 : \left(\frac{1}{\sqrt{2}}\right) : 1 \qquad (4.42)$$

As we shall see,† the *cross section*, σ, goes like the absolute square of the amplitude; thus

$$\sigma_a : \sigma_b : \sigma_c = 2 : 1 : 2 \qquad (4.43)$$

Process (c) would be hard to set up in the laboratory, but (a) and (b) have been measured, and (when corrections are made for electromagnetic effects) they are found to be in the predicted ratio [7].

Figure 5: Answer.

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