Massachusetts Institute of Technology

Department of Physics

Course:	8.701 – Introduction to Nuclear and Particle Physics
Term:	Fall 2020
Instructor:	Markus Klute
TA :	Tianyu Justin Yang

Discussion Problems

from recitation on **September 10th, 2020**

Problem 1: W boson branching fraction

The W boson decays via the weak interaction to leptons and neutrinos $[(e^+\nu_e)$, $(\mu^+\nu_\mu)$, $(\tau^+\nu_\tau)]$, or pairs of quarks $[(u,\bar{d}),(c,\bar{s})]$ - why not $(t,\bar{b})$? While we will study the weak interaction in much more details, we can already calculate branching fractions assuming that all fermions have the same weak charge. What is the branching fraction $\mathcal{B}(W^+ \to \mu^+\nu_\mu)$?

- $W^+ \to t^+\bar{b}$ are not possible because the top quark mass (175 GeV) is more than twice that of a W boson (80 GeV).

To first order, all possible decays of the W are equally likely. The problem therefore reduced to counting all possible decays. The $\mathcal{B}(W^+ \to \mu^+\nu_\mu)$ will then be one over the number of possible decays.

So how many possibilities are there. We find 3 leptonic decays and 6 to quark-antiquark decays, as each quark comes in three colors.

Therefore, we find $\mathcal{B}(W^+ \to \mu^+\nu_\mu) = \frac{1}{9}$.

Comparing this to the measured values as reported by the particle data group, see Fig. 1.

Problem 2: Cross section and impact paramater

A beam of small balls (mass m and radius r) scatters elastically of a larger ball (mass M and radius R) with $M \gg m$ and $R \gg r$

W^+ DECAY MODES

W^- modes are charge conjugates of the modes below.

	Mode		Fraction (Γ_i/Γ)	Confidence level
Γ_1	$\ell^+\nu$	[a]	(10.86 ± 0.09) %	
Γ_2	$e^+\nu$		(10.71 ± 0.16) %	
Γ_3	$\mu^+\nu$		(10.63 ± 0.15) %	
Γ_4	$\tau^+\nu$		(11.38 ± 0.21) %	
Γ_5	hadrons		(67.41 ± 0.27) %	

Figure 1: Leading W decay modes.

a)

Calculate the differential and total geometrical cross section. Hint: use $\sin(x+y) = \sin(x)\,\cos(y) + \cos(x)\,\sin(y)$).

b)

How large is the fraction of small balls scattering under a scattering angle of $\theta \leq 30°$?

c)

The total cross-section for electron-proton scattering at HERA ($\sqrt{} =$) is about 10 mb. Compare this to the geometrical cross-section of a proton assuming a radius of 0, 862 fm. What does the result mean?

- **a)**

For the geometry of the scattering see Fig. 2. For the geometrical cross section you find:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = -\frac{b}{\sin\theta}\frac{\mathrm{d}b}{\mathrm{d}\theta}$$

The physical model of the scattering makes the connection between b and θ. For the scattering on a hard ball we find:

$$\frac{b}{R} = \cos\theta/2$$

$$b = R\,\cos\theta/2 \qquad ; \qquad \frac{\mathrm{d}b}{\mathrm{d}\theta} = -\frac{R}{2}\sin\theta/2$$

It follows ($\sin\theta = \sin(\theta/2 + \theta/2) = 2\cos\theta/2 \cdot \sin\theta/2$):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{R\,\cos\theta/2}{\sin\theta}\right)\left(\frac{R}{2}\sin\theta/2\right) = \frac{R^2}{2}\frac{\cos\theta/2 \cdot \sin\theta/2}{\sin\theta} = \frac{R^2}{4}$$

In this case, the differential cross section does not explicitely depend on the scattering angle θ. The total cross section follow from integration:

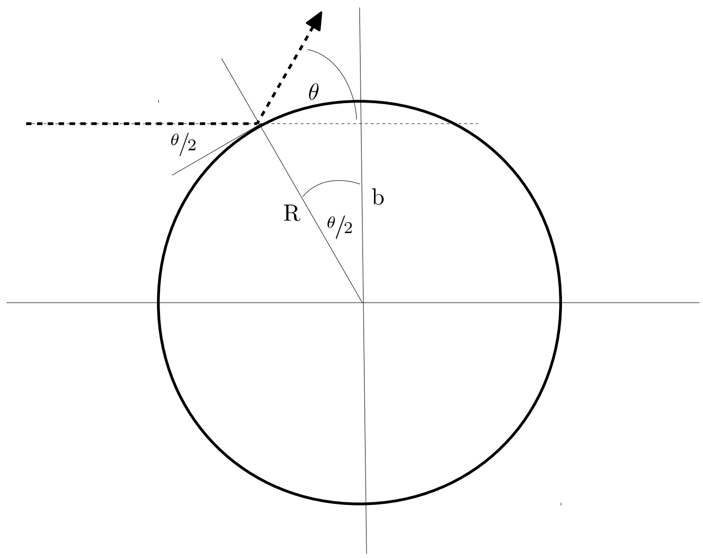

Figure 2: Geometry of the scattering of a small ball and a large ball with raduis R und a scattering angle θ, considered in the rest-frame of the large ball.

$$\sigma_{\text{tot}} = \int_{\Omega} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \mathrm{d}\Omega = \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} \mathrm{d}\cos\theta \, \frac{R^2}{4} = \pi \, R^2$$

The result is as expected: the cross section for which we find scattering is the area of the large ball, hence the geometrical cross section. From here you can also derive the total cross section not using $r \ll R$.

b)

Calculate the solid angle from the integral:

$$\Delta\Omega = \int_{\partial\Omega} \mathrm{d}\Omega = \int_0^{2\,\pi} \mathrm{d}\phi \int_0^{\pi/6} \sin\theta \, \mathrm{d}\theta = 2\,\pi \Big[-\cos\theta \Big]_0^{\pi/6} = 0.27\,\pi$$

The part of $\Delta\Omega$ scattered small balls follows from the ratio:

$$\frac{\Delta N}{N} = \Delta\Omega \, \frac{\mathrm{d}\sigma/\mathrm{d}\Omega}{\sigma_{\text{tot}}} = 0.27\,\pi \, \frac{R^2/4}{\pi R^2} = \frac{0.27\pi}{4\,\pi} = 0.067$$

c)

A proton radius of $r = 0.88\,\mathrm{fm}$ (charge radius) leads to a geometrical cross section of

$$\sigma_{\mathrm{tot}}^{\mathrm{geo}} = \pi\, r^2 = 2.33\,\mathrm{fm}^2$$

$$\text{mit } 1\,\mathrm{b} = 100\,\mathrm{fm}^2 \text{ergibt sich:}$$

$$\sigma_{\mathrm{tot}}^{\mathrm{phys}} = 10\,\mathrm{mb} = 1\,\mathrm{fm}^2$$

$$\frac{\sigma_{\mathrm{tot}}^{\mathrm{phys}}}{\sigma_{\mathrm{tot}}^{\mathrm{geo}}} = \frac{1}{2.33} = 0.43$$

The interpretation is that not all electron with impact parameter within the radius of the charge distribution are actually scattered.