

# Massachusetts Institute of Technology

## Department of Physics

Course: 8.701 – Introduction to Nuclear and Particle Physics  
Term: Fall 2020  
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### Discussion Problems

from recitation on September 8rd, 2020

#### Problem 1: Cosmic rays

Greisen, Zatespin, and Kuzmin (GZK) predicted that there is a maximum energy for which any cosmic ray (which are predominantly protons) will be observed. Their argument was that if energetic enough, cosmic rays will lose energy via the process  $p\gamma_{CMB} \rightarrow p\pi^0$ , where  $\gamma_{CMB}$  is a cosmic microwave background photon, whose energies are  $E_{CMB} \approx 3 \times 10^{-4}\text{eV}$ . Therefore, the threshold energy for which this process can occur sets the maximum expected energy a cosmic ray should be observed to have (called the GZK cutoff). Calculate the GZK cutoff energy in eV, and compare your result to the observed cosmic ray energy spectrum (see Fig. 29.8 of the PDG review on Cosmic Rays or below). [Hint: You can take the energy to be much larger than the proton mass and assume that the frame in which we want to know this energy is the one where the proton and CMB photon collide head on.]

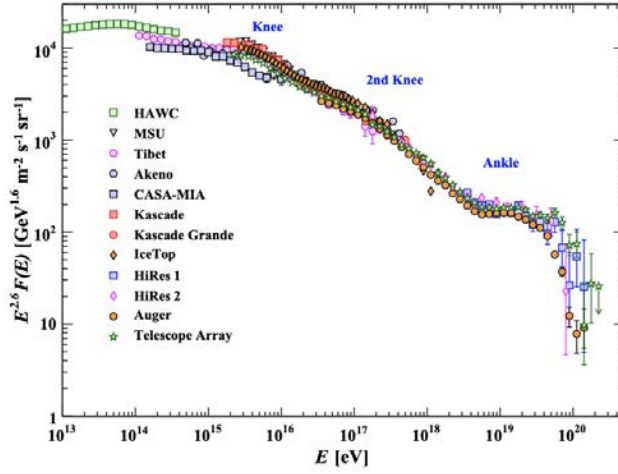


Figure 29.8: The all-particle spectrum as a function of  $E$  (energy-per-nucleus) from air shower measurements [95–105]

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## Problem 2: Mandelstam variables

Consider the reaction between two particles with 4-momenta  $p_1^\mu$  and  $p_2^\mu$ . The outgoing particles have 4-momenta  $p_3^\mu$  and  $p_4^\mu$ . Discuss the variables  $s$ ,  $t$ , and  $u$  for the center-of-mass frame and a fixed-target frame where the second particle is at rest. Assume that the masses involved are much smaller than the energies ( $m_i \ll E_i$ ).

$$s = (p_1^\mu + p_2^\mu)^2 = (p_3^\mu + p_4^\mu)^2 \quad 4 E_1^{*2} \quad 2 m_2 E_1$$

$$t = (p_1^\mu - p_3^\mu)^2 = (p_4^\mu - p_2^\mu)^2 \quad \frac{s}{2} (\cos \theta^* - 1) \quad 2 m_2 (E_3 - E_1)$$

$$u = (p_1^\mu - p_4^\mu)^2 = (p_3^\mu - p_2^\mu)^2 \quad -\frac{s}{2} (\cos \theta^* + 1) \quad -2 m_2 E_3$$

$$s + t + u = \sum_{i=1}^4 m_i^2$$

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