Massachusetts Institute of Technology

Department of Physics

Course:	8.701 – Introduction to Nuclear and Particle Physics
Term:	Fall 2020
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Discussion Problems

from recitation on **September 3rd, 2020**

Problem 1: Gamma Factor for LEP Electron

At LEP @ CERN, electrons and positrons were accelerated to 100 GeV. How large was γ?

- We know that:

$$E_e = \gamma m_e c^2 = 100\,\text{GeV},$$

and:

$$m_e c^2 \approx 0.5\,\text{MeV} = 5 \times 10^{-4}\,\text{GeV}.$$

So,

$$\gamma = \frac{100\,\text{GeV}}{5 \times 10^{-4}\,\text{GeV}} = \boxed{2 \times 10^5}.$$

Problem 2: Splitting the Deuteron

How much energy do we need to split a proton and neutron (deuteron)?

- In the atomic mass unit, $1\,u = 1.66 \times 10^{-27}\,kg$, the masses of the relevant particles are:

$$m_d = 2.01355\,u,$$
$$m_p = 1.00728\,u,$$
$$m_n = 1.00867\,u,$$

So, we would need $(m_d - m_p - m_n)c^2 = 0.0025u\,c^2$ of energy to split the deuteron into a proton and a neutron. That is equivalent to $\boxed{2.23\,\text{MeV}}$ of energy. This nuclear process is written as:

$$n + p \longleftrightarrow d + 2.23\,\text{MeV}$$

Problem 3: Photon Reabsorption

An excited particle emits a photon. Under which condition can this photon be reabsorbed?

- Suppose the excited particle is initially at rest with mass m_0. After emitting a photon with energy Q, it has mass m_0' and is moving at speed v.

By conservation of energy and momentum, we have:

$$\begin{aligned} m_0c^2 &= Q + m_0'\gamma(v)c^2 \qquad (1) \\ 0 &= \frac{Q}{c} - m_0'\gamma(v)v, \end{aligned}$$

which means the energy and momentum of the particle, after emitting the photon, are:

$$\begin{aligned} E' &= m_0c^2 - Q \\ p' &= \frac{Q}{c}. \end{aligned}$$

Requiring the particle to be on-shell, it gives:

$$E'^2 - (p'c)^2 = (m_0c^2 - Q)^2 - Q^2 = (m_0c^2)^2 - 2m_0c^2Q = (m_0'c^2)^2.$$

So,

$$\boxed{Q = Q_0\left(1 - \frac{Q_0}{2m_0c^2}\right) < Q_0,}$$

where

$$Q_0 \equiv (m_0 - m_0')c^2.$$

The energy conservation in Equation 1 also requires:

$$\boxed{m_0' = \frac{m_0 - \frac{Q}{c^2}}{\gamma(v)}}$$

Problem 4: Fixed-Target $\bar{p}$ Production

What is the minimal beam energy in a proton on proton fixed target experiment to produce anti-protons?

- The process can be written as: $p + p \longrightarrow p + p + p + \bar{p}$.
  In the center of mass frame, $E = 2\gamma m_p c^2 = 4 m_p c^2 \Rightarrow \gamma = 2; \beta = 0.866$.
  Siwthing to the lab frame, $\beta = \frac{2*0.866}{1+0.866^2} = 0.990 \Rightarrow \gamma = 7$
  So, we need $E = \gamma m_p c^2 \approx \boxed{6.57\,\text{GeV}}$ of beam energy in a proton on proton fixed target experiment to produce anti-protons.

Problem 5: Pion Decays

Assume the decay of a pion at rest into an electron and positron. How fast are the decau products?

- By conservation of momentum, the electron and positron must fly out in equal and opposite directions, which means $\gamma_{e^-} = \gamma_{e^+}$, and $E_{e^-} = E_{e^+} = \frac{1}{2} m_\Pi c^2 = 67.5\,\text{MeV}$.
  $E_{e^-} = \gamma_{e^-} m_e c^2 = \gamma_{e^-}(0.511\,\text{MeV}) \Rightarrow \gamma_{e^-} = 132 \Rightarrow \boxed{v = 0.99997\,c}$.

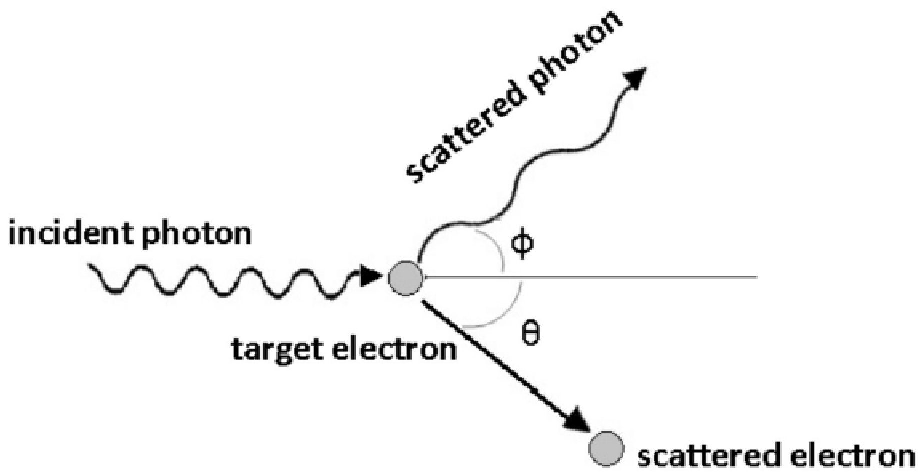
Problem 6: Fixed-Target Pion Production

What is the minimal beam energy of a proton colliding with a proton at rest to produce a $p + n + \Pi^+$?

- The process can be written as: $p + p \longrightarrow p + n + \Pi^+$.
  In the center of mass frame, $E = 2\gamma m_p c^2 = (m_p + m_n + m_{\Pi^+})c^2 \Rightarrow$
  $\gamma = 938 + 940 = 1402 * 938 = 1.08; \beta = 0.37$.
  Siwthing to the lab frame, $\beta = \frac{2*0.37}{1+0.37^2} = 0.65 \Rightarrow \gamma = 1.32$
  So, we need $E = \gamma m_p c^2 \approx \boxed{1.24\,\text{GeV}}$ of beam energy in a proton on proton fixed target experiment to produce a p+n+Π^+.

Problem 7: Compton Effect

The energy of a photon is $E = h\nu = \frac{h}{\lambda}$. Calculate the change in the photon's wavelength.



- Conservation of 4-momenta:

$$\begin{aligned} p_\gamma + p_e &= p'_\gamma + p'_e \\ (p_\gamma - p'_\gamma)^2 &= (p'_e - p_e)^2 \\ p_\gamma^2 + {p'_\gamma}^2 - 2p_\gamma \cdot p'_\gamma &= p_e^2 + {p'_e}^2 - 2p_e \cdot p'_e \end{aligned}$$

We let $c = 1$ in the following derivation, and use the fact that $m_\gamma = p_\gamma^2 = 0$, $\vec{p_e} = 0$ and $p_\gamma = E_\gamma(1, \hat{r})$:

$$\begin{aligned} -2p_\gamma \cdot p'_\gamma &= 2m_e^2 - 2p_e \cdot p'_e \\ -p_\gamma \cdot p'_\gamma &= m_e^2 - p_e \cdot p'_e \\ -p_\gamma \cdot p'_\gamma &= m_e^2 - m_e E'_e \\ -(E_\gamma E'_\gamma - \vec{p_\gamma} \cdot \vec{p'_\gamma}) &= m_e(m_e - E'_e) \\ E_\gamma E'_\gamma(1 - \cos\phi) &= m_e(E'_e - m_e) \\ E_\gamma E'_\gamma(1 - \cos\phi) &= m_e(E_\gamma - E'_\gamma) \end{aligned}$$

Since $E_\gamma = \frac{h}{\lambda}$,

$$\begin{aligned} \frac{h^2}{\lambda\lambda'}(1 - \cos\phi) &= m_e h(\frac{1}{\lambda} - \frac{1}{\lambda'}) \\ \Delta\lambda = \lambda' - \lambda &= \boxed{\frac{h}{m_e}(1 - \cos\phi)} \end{aligned}$$

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