Massachusetts Institute of Technology

Department of Physics

Course:	8.701 – Introduction to Nuclear and Particle Physics
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Discussion Problems

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Problem 1: Binding energy of iron

The iron nuclide $^{56}_{26}$Fe lies near the top of the binding energy curve and is one of the most stable nuclides. What is the binding energy per nuclean (in MeV) for the nuclide $^{56}_{26}$Fe (atmic mass of 55.9349 amu)?

difference between the mass of 26 protons, 30 neutrons, and 26 electrons, and the observed mass of an ${}^{56}_{26}\mathrm{Fe}$ atom:

$$\mathrm{Mass\backslash;defect}\quad [(26\ \times\ 1.0073\ \mathrm{amu})\ +\ (30\ \times\ 1.0087\ \mathrm{amu})\ +\ (26\ \times\ 0.00055\ \mathrm{amu})]\ -\ 55.9349\ \mathrm{amu}$$

$$56.4651\ \mathrm{amu}\ -\ 55.9349\ \mathrm{amu}$$

$$0.5302\ \mathrm{amu}$$

We next calculate the binding energy for one nucleus from the mass defect using the mass-energy equivalence equation:

$$E\quad mc^2 = 0.5302\ \mathrm{amu}\ \times\ \frac{1.6605\ \times\ 10^{-27}\ \mathrm{kg}}{1\ \mathrm{amu}}\ \times\ (2.998\ \times\ 10^{8}\ \mathrm{m/s})^2$$

$$7.913\ \times\ 10^{-11}\ \mathrm{kg{\cdot}m/s^2}$$

$$7.913\ \times\ 10^{-11}\ \mathrm{J}$$

We then convert the binding energy in joules per nucleus into units of MeV per nuclide:

$$7.913\ \times\ 10^{-11}\ \mathrm{J}\ \times\ \frac{1\ \mathrm{MeV}}{1.602\ \times\ 10^{-13}\ \mathrm{J}} = 493.9\ \mathrm{MeV}$$

Finally, we determine the binding energy per nucleon by dividing the total nuclear binding energy by the number of nucleons in the atom:

$$\mathrm{Binding\backslash;energy\backslash;per\backslash;nucleon} = \frac{493.9\ \mathrm{MeV}}{56} = 8.820\ \mathrm{MeV/nucleon}$$

Note that this is almost 25% larger than the binding energy per nucleon for ${}^{4}_{2}\mathrm{He}$.

Problem 2: Carbon dating

You find a pottery shard containing 1g of carbon. Its activity is 0.0231 Bq (decays per second). How old is it? ^{14}C is radioactive and produced in the upper athmospere and we find in living things a ratio of $\frac{^{14}C}{^{12}C}$ of 1.2×10^{12}. The half-life of ^{14}C is 5730 years.

- Find the initial activity of the carbon sample. A sample of 12g ^{12}C has $N_a = 6.02 \times 10^{23}$ atoms and 1g has 5.02×10^{22} atoms. This means we had 6.02×10^{10} ^{14}C atoms initially. Now we need the decay constant $\lambda = \frac{ln2}{5730y} = \frac{0.693}{1.81 \times 10^{11} s} = 3.83 \times 10^{-12} s^{-1}$. The initial sctivity was $\lambda \cdot N_0 = 0.231$Bq. With $\ln \frac{R}{R_0} = -\lambda t$ we find 19063 years or 3.327 half-lifes.