Massachusetts Institute of Technology

Department of Physics

Course: 8.701 – Introduction to Nuclear and Particle Physics
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Discussion Problems

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Problem 1: Tau Decay

The liftetime of the muon can be calculated to

$$\tau = \frac{1}{\Gamma} = \left(\frac{M_W}{m_\mu g_w}\right)^4 \frac{12\hbar(8\pi)^3}{m_\mu c^2}$$

Use this equation to calculate the lifetime of the tau and compare with the experimental values. Discuss the result.

(Note that $m_\mu/m_\tau = 0.059$, so it is reasonable to assume $m_\mu \ll m_\tau$.) The rate goes as the fifth power of the decaying particle's mass. But the τ can also go to $e + \bar{\nu}_e + \nu_\tau$, with the same rate. So

$$\frac{\tau(\tau \to \text{leptons})}{\tau(\mu \to \text{leptons})} = \frac{m_\mu^5}{2m_\tau^5}, \quad \text{or} \quad \boxed{\tau_\tau = \frac{1}{2}\left(\frac{m_\mu}{m_\tau}\right)^5 \tau_\mu}.$$

Numerically:

$$\tau_\tau = \frac{1}{2}\left(\frac{105.66}{1777}\right)^5 (2.197 \times 10^{-6}\text{ s}) = \boxed{8.16 \times 10^{-13}\text{ s}}.$$

The *actual* lifetime is 2.91×10^{-13} s. The discrepancy is due to the fact that the τ has many *hadronic* decay modes, in addition to the μ and e routes. In fact, the experimental branching ratios are 17.4% (μ) and 17.8% (e), for a total leptonic branching ratio of 35.2%, so $\tau = (0.352)\tau(\text{leptons}) = (0.352)(8.16 \times 10^{-13}\text{ s}) = 2.87 \times 10^{-13}$ s, which is quite close to the experimental result.

Problem 2: Kaon Decay

Calculate the ratio of the decay rates $K^- \to e^- + \bar{\nu}_e$ and $K^- \to \mu^- + \bar{\nu}_\mu$. Compare the observed branching ratios.

- We can follow the discussion of the pion decay and replace the pion mass with the kaon mass.

$$\frac{\Gamma(K^- \longrightarrow e^- + \bar{\nu}_e)}{\Gamma(K^- \longrightarrow \mu^- + \bar{\nu}_\mu)} = \frac{m_e^2(m_K^2 - m_e^2)^2}{m_\mu^2(m_K^2 - m_\mu^2)^2}$$

$$= \left[\frac{(0.510999)\{(493.667)^2 - (0.510999)^2\}}{(105.6584)\{(493.667)^2 - (105.6584)^2\}}\right]^2 = \boxed{2.57 \times 10^{-5}}$$

The experimental ratio (using data from the *Particle Physics Booklet*) is

$$\frac{1.55 \times 10^{-5}}{0.6344} = \boxed{2.44 \times 10^{-5}}.$$

The agreement is quite good.

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