Massachusetts Institute of Technology

Department of Physics

Course: 8.701 – Introduction to Nuclear and Particle Physics
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Instructor: Markus Klute
TA : Tianyu Justin Yang

Discussion Problems

from recitation on **October 15th, 2020**

Problem 1: Color Transformations

Color SU(3) transformations relable 'red', 'blue', and 'green' according to the transformation rule $c \to c' = Uc$, where U is any unitary $(UU\dagger = 1) 3 \times 3$ matrix of determinant 1, and c is a three-element column vector. See below for example. would

$$U = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

take $r \to g$, $g \to b$, and $b \to r$. Show that $|3\rangle$ and $|8\rangle$ go into linear combinations of one another: $|3'\rangle = \alpha\,|3\rangle + \beta\,|8\rangle$, $|8'\rangle = \gamma\,|3\rangle + \delta\,|8\rangle$
Find numbers for α, β, γ, and δ.

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$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

so, under the action of U,

$$r \rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = g$$

$$b \rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = r$$

$$g \rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = b.$$

$$|1\rangle = (r\bar{b} + b\bar{r})/\sqrt{2} \qquad |5\rangle = -i(r\bar{g} - g\bar{r})/\sqrt{2}$$

$$|2\rangle = -i(r\bar{b} - b\bar{r})/\sqrt{2} \qquad |6\rangle = (b\bar{g} + g\bar{b})/\sqrt{2}$$

$$|3\rangle = (r\bar{r} - b\bar{b})/\sqrt{2} \qquad |7\rangle = -i(b\bar{g} - g\bar{b})/\sqrt{2}$$

$$|4\rangle = (r\bar{g} + g\bar{r})/\sqrt{2} \qquad |8\rangle = (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}$$

$$\begin{aligned} |3\rangle \rightarrow |3'\rangle &= (g\bar{g} - r\bar{r})/\sqrt{2} = \alpha|3\rangle + \beta|8\rangle \\ &= \alpha(r\bar{r} - b\bar{b})/\sqrt{2} + \beta(r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6} \\ &= g\bar{g}\left(-2\frac{\beta}{\sqrt{6}}\right) + r\bar{r}\left(\frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{6}}\right) + b\bar{b}\left(-\frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{6}}\right). \end{aligned}$$

Evidently

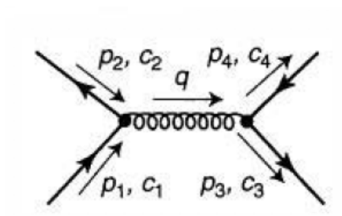
$$\frac{1}{\sqrt{2}} = -2\frac{\beta}{\sqrt{6}}, \quad \boxed{\beta = -\frac{\sqrt{3}}{2}}$$

$$-\frac{1}{\sqrt{2}} = \left(\frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{6}}\right) = \frac{\alpha}{\sqrt{2}} - \frac{1}{2\sqrt{2}}, \quad \boxed{\alpha = -\frac{1}{2}}$$

$$0 = \left(-\frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{6}}\right) = \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{6}} = 0. \quad \checkmark$$

Problem 2: QCD Amplitude

Find the amplitude M for the diagram below. What is the color factor in this case? Evaluate f in the color singlet configuration. Can you explain this result?



- Applying the Feynman rules, $\mathcal{M} =$

$$i\left[\bar{v}(2)c_2^\dagger\left(-\frac{ig_s}{2}\lambda^\alpha\gamma^\mu\right)c_1u(1)\right]\left(\frac{-ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}\right)\left[\bar{u}(3)c_3^\dagger\left(-\frac{ig_s}{2}\lambda^\beta\gamma^\nu\right)c_4v(4)\right],$$

with $q = p_1 + p_2 = p_3 + p_4$. Or,

$$\mathcal{M} = -\frac{g_s^2}{4q^2}\left[\bar{v}(2)\gamma^\mu u(1)\right]\left[\bar{u}(3)\gamma_\mu v(4)\right]\left(c_2^\dagger\lambda^\alpha c_1\right)\left(c_3^\dagger\lambda^\alpha c_4\right).$$

$$f = \boxed{\frac{1}{4}\left(c_2^\dagger\lambda^\alpha c_1\right)\left(c_3^\dagger\lambda^\alpha c_4\right)}.$$

In the singlet configuration,

$$f = \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left[(1\;0\;0)\,\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix} + (0\;1\;0)\,\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix} + (0\;0\;1)\,\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$$
$$\times\left[(1\;0\;0)\,\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix} + (0\;1\;0)\,\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix} + (0\;0\;1)\,\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$$
$$= \frac{1}{12}(\mathrm{Tr}\lambda^\alpha)(\mathrm{Tr}\lambda^\alpha) = \boxed{0},$$

since the lambda matrices are all traceless (Eq. 8.34)—a color singlet cannot couple to a color octet (the gluon).

It's zero. A singlet cannot couple to an octet (gluon).

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