Massachusetts Institute of Technology

Department of Physics

Course: 8.701 — Introduction to Nuclear and Particle Physics
Term: Fall 2020
Instructor: Markus Klute

Problem Set 1

handed out September 10th, 2020

Problem 1: Particles [20 points]

Detailed information on particle properties (e.g., masses, decays, etc.) can be found in the Particle Data Group (PDG) collection: pdg.lbl.gov (see Particle Listings). This is a very useful resource. Let's start to get familiar with some common particles. Look up the following particles: $\pi^{\pm}$, π^0, $K^{\pm}$, K^0_S, K^0_L, $\mu^{\pm}$, $e^{\pm}$, proton, and neutron. For each particle, write down the particle's mass, lifetime, and main decay mode (the largest one). How far would each particle travel before decaying on average assuming its speed is c but ignoring time dilation?

- Particle Properties

	Mass [MeV/c^2]	τ [s]	Main Decay	cτ [m]
$\pi^{\pm}$	140	2.6×10^{-8}	$\pi \to \mu\mu_\nu$	8
π^0	135	8.5×10^{-17}	$\pi \to \gamma\gamma$	3×10^{-8}
$K^{\pm}$	494	1.2×10^{-8}	$K \to \mu\nu_\mu$	4
K_S	498	9×10^{-11}	$K_S \to \pi^+\pi^-$	0.03
K_L	498	5×10^{-8}	$K_L \to \pi e \nu_e$	15
$\mu^{\pm}$	106	2×10^{-6}	$\mu \to e\nu_e\nu_\mu$	660
$e^{\pm}$	0.511	Stable	Stable	Stable
p	938	Stable	Stable	Stable
n	940	880	$n \to pe\bar{\nu}_e$	3×10^{11}

Problem 2: Pion Decay [20 points]

Charged pions, π^+ (π^-), can decay to electrons as well as to muons and the corresponding neutrinos. In this decay, the parity violation of the weak interaction is maximal. All arguements hold for π^+ and π^-.

a)

Prepare a sketch of the pion decay at rest noting the momentum vectors as well as the spin of the involved particles. Using the sketch, discuss why you expect maximal parity violation of the weak interaction in pion decays. Following this discussion, which branching fraction is larger? Include your reasoning in the answer.

b)

Show the following relation for momentum and energy, assuming that neutrinos are massless:

$$p_\ell = \frac{m_\pi^2 - m_\ell^2}{2\,m_\pi}$$
$$E_\ell = \frac{m_\pi^2 + m_\ell^2}{2\,m_\pi}\,,$$

with $m_\pi = 139,57\,\mathrm{MeV}$ the mass of the pion and $m_\ell = 105,66\,(0.511)\,\mathrm{MeV}$ the mass of the muon (electron).

- **a)**

The decay, here for π^-, follows:

$$\pi^- \to \ell^- \bar{\nu}_\ell\,, \qquad \ell = \mu, \mathrm{e}$$

The W^- boson couples exclusively to left-handed particles and right-handed antiparticles. Since we can assume the neutrino to be massless, its chirality corresponds to its helicity. The momentum- and spin-configuration is shown in Fig 1.

b)

Using energy conservation, we find for pion at rest:

$$E_\pi = m_\pi = E_\nu + E_\ell\,, \qquad \text{mit: } p_\nu = p_\ell = p$$

For the momentum we find:

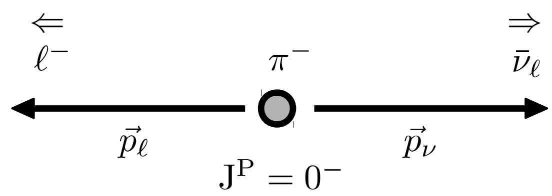

Figure 1: The decay of a π^- in a lepton (ℓ^-) and the corresponding anti-neutrino ($\bar{\nu}_\ell$). The simple arrows correspond to the momenta of the decay products and the double arrows to the spin orientation. The spin-configuration of the initial state is $J^{\mathrm{P}} = 0^-$. The spin of the decay products has to be opposite to each other. Since $\bar{\nu}_\ell$ is right-handed, ℓ^- has to be of positive helicity. As the ℓ^- is massive, this does not mean that it is right-handed.

$$E_\pi = m_\pi = E_\nu + E_\ell\,, \qquad \text{mit: } p_\nu = p_\ell = p$$

$$\begin{aligned}
&E_\pi = m_\pi = p + \sqrt{p^2 + m_\ell^2} \\
&(m_\pi - p)^2 = p^2 + m_\ell^2 \\
&m_\pi^2 - 2\,m_\pi\,p + p^2 = p^2 + m_\ell^2
\end{aligned}$$

$$p = \frac{m_\pi^2 - m_\ell^2}{2\,m_\pi}$$

And for the energy:

$$\begin{aligned}
E &= \sqrt{p^2 + m_\ell^2} \\
&= \frac{1}{2\,m_\pi}\sqrt{m_\pi^2 - 2\,m_\pi^2 m_\ell^2 + m_\ell^4 + 4\,m_\pi^2 m_\ell^2} \\
&= \frac{1}{2\,m_\pi}\sqrt{(m_\pi^2 + m_\ell^2)^2}
\end{aligned}$$

$$E = \frac{m_\pi^2 + m_\ell^2}{2\,m_\pi}$$

Problem 3: Higgs Boson branching fractions [20 points]

The coupling of the Higgs boson to fermions is proportional to their masses. The partial width is then proportial to the coupling squared. Calculate Higgs boson branching fractions assuming (simplifying) that only decays to bottom and charm quarks as well as to taus and muons are possible.

The partial width for Higgs decay to fermion pairs is proportional to the fermion mass squared:

$$\Gamma_i \propto g_i^2 \propto m_i^2.$$

For quarks, there is a color factor of 3. So, for Higgs decay to charms (1.28 GeV), bottoms (4.18 GeV), taus (1.78 GeV) and muons (0.11 GeV), the branching ratios can be calculated by:

$$Br[\text{charm}] = \frac{\Gamma_c}{\Gamma_{tot}} = \frac{3 \times 1.28^2}{3 \times 1.28^2 + 3 \times 4.18^2 + 1.78^2 + 0.11^2} = 8.1\%$$

$$Br[\text{bottom}] = \frac{\Gamma_b}{\Gamma_{tot}} = \frac{3 \times 4.18^2}{3 \times 1.28^2 + 3 \times 4.18^2 + 1.78^2 + 0.11^2} = 86.6\%$$

$$Br[\text{tau}] = \frac{\Gamma_\tau}{\Gamma_{tot}} = \frac{1.78^2}{3 \times 1.28^2 + 3 \times 4.18^2 + 1.78^2 + 0.11^2} = 5.2\%$$

$$Br[\text{muon}] = \frac{\Gamma_\mu}{\Gamma_{tot}} = \frac{0.11^2}{3 \times 1.28^2 + 3 \times 4.18^2 + 1.78^2 + 0.11^2} = 0.020\%$$

Problem 4: Discrete symmetries [20 points]

Discuss how the following properties translate under parity (P) and time reversal (T) operations:

a)
Position vector, $\vec{r}$.

b)
Momentum vector, $\vec{p}$.

c)
Angular momentum vector, $\vec{L}$.

d)
Spin, $\vec{s}$.

e)
Static electric field, $\vec{E}$.

f)
Static magnetic Field, $\vec{B}$.

g)
Potential energy of an electric dipol from particle spin in a static electric field, $\vec{s} \cdot \vec{E}$.

h)
Potential energy of a magnetic dipol from particle spin in an static magnetic field, $\vec{s} \cdot \vec{B}$.

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Teilaufgabe	$\hat{O}$	$\hat{P}(\hat{O})$	$\hat{T}(\hat{O})$	Comment
a)	$\vec{r}$	$-\vec{r}$	$\vec{r}$	
b)	$\vec{p}$	$-\vec{p}$	$-\vec{p}$	Time derivative
c)	$\vec{L}$	$\vec{L}$	$-\vec{L}$	Axial-vector but time derivative
d)	$\vec{s}$	$\vec{s}$	$-\vec{s}$	see **c)**
e)	$\vec{E}$	$-\vec{E}$	$\vec{E}$	Vectorfield $(\nabla\phi)$
f)	$\vec{B}$	$\vec{B}$	$-\vec{B}$	see **c)** $(\vec{B} = e\,\vec{v} \times \vec{r})$
g)	$\vec{s} \cdot \vec{E}$	$-\vec{s} \cdot \vec{E}$	$-\vec{s} \cdot \vec{E}$	see **d)** and **e)**
h)	$\vec{s} \cdot \vec{B}$	$\vec{s} \cdot \vec{B}$	$\vec{s} \cdot \vec{B}$	see **d)** and **f)**

Problem 5: Conservation law and interaction [20 points]

For which of the following interactions (electromagnetic, weak, and strong) do the following conservation laws hold?

a)

Lepton number conservation.

b)

Strong isospin, T_3.

c)

Strangeness, S.

d)

Paritity conservation.

e)

CP-conservation.

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	E&M	Weak	Strong
Lepton number	✓	✓	✓
Strong isospin, T_3	✓	–	✓
Strangeness, S	✓	–	✓
Paritity	✓	–	✓
CP	✓	–	✓

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