Massachusetts Institute of Technology

Department of Physics

Course:	8.20 —Special Relativity
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Problem Set 2

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Problem 1: Stellar Aberration and Parallax [20 pts]

Suppose I have a star located at a distance R from the center of the solar system which makes an angle θ with respect to the Earth orbital plane (ecliptic). You observe the star from Earth at various times throughout the year. Take the Earth-Sun distance as r_e and the angular velocity of the Earth's orbit as ω, such that the orbital velocity is $v = \omega r_e$. In this problem, $R \gg r_e$ and $c \gg v$.

(a) Find the maximum and minimum change in the observed angle of the star in the sky due to the change in the Earth's position over any six-month period (parallax).
[*Hint*: For this problem, work in a coordinate system with the Sun at the origin, and the Earth's orbit in the xy plane. Calculate the time evolution of the Earth-star vector (vector pointing from the Earth to the star), and derive the change in the star's observed angle from here. To better visualize changes in the observed angle, consider cases where there are only changes in the elevation and azimuthal angles first.]

(b) Perform the same analysis except look at the effect due to the Earth's velocity (aberration). For this problem, you may assume that the starlight coming to the Earth is parallel to that coming to the Sun.
[*Hint*: Make an analogy between this problem and part(a). Specifically, think about the vectors of interest for this problem.]

(c) Which effect would dominate for most observations? Take as an example one of our closest stars, Alpha Centauri, about 4.4 light years away.

Problem 2: Binary Stars (from Resnik, Ch 1) [20 pts]

Consider one star in a binary system moving in a uniform circular motion with speed v. Consider two positions: (I) the star is moving *away* from the Earth along a line connecting them, and (II) the star is moving *toward* the Earth along the line connecting them. Let the star's period of motion be T and it's distance be L. Assume that L is large enough that positions (I) and (II) are a half-orbit apart. Assume that you will be testing a set of *emission theories*, whereby the motion of the source is imparted to the velocity of the emission.

1. Find the time it would take for the star to appear to move from position I to position II, and from position II to position I.
2. Show that the star would appear *both* at position I and II if $T/2 = \frac{2lv}{c^2-v^2}$.

Problem 3: Interferometers [20 pts]

Consider a laser interferometer which emits a monochromatic beam of wavelength λ, arranged as in a Michelson-Morley configuration with two perpendicular arms. The length of each arm is given by $l_1 = l$ and $l_2 = l + \delta l$. We wish to attempt to measure the velocity of the apparatus with respect to an ether "wind" which is arbitrarily pointed in some direction with respect to the first arm (call that angle ϕ). Throughout this problem, keep your answer only to the first non-vanishing order of β.

(a) Assume for the moment that the two arm lengths are the same (i.e. $\delta l \to 0$). Compute the change in the number of fringes detected under this more general case where the velocity is not aligned with the detector. Remember that one rotates a interferometer with perpendicular arms by 90° to check for changes in the fringe pattern.

(b) Now take the more general case where the two lengths are not equal. What is the change in the number of fringes with an interferometer of unequal arm lengths?

(c) Show that in the limit $\delta l \to 0$ and the velocity is aligned, you reproduce the MM result.

(d) Suppose you simply wanted to show whether the ether effect existed (i.e. to detect an change in the number of fringes). Would it matter if the lengths were not exactly equal? What about if the apparatus was properly aligned? What would you need to be careful in order not to fool yourself you measured a positive result? [Note: No equations here. Just for you to think about.]

Problem 4: Getting All Your Clocks in a Row... [20 pts]

Suppose there are 5 clocks sitting on a train platform, all syncronized with each and each clock is 1 meter apart from the next. A passing train, moving at velocity $v = c/3$, has the identical setup. Two observers, one on the train (Zak) and one on the platform (Jill), have the confusing assignment of recording the time registered on each of these clocks (a UROP project that went horribly wrong...). Both Zak and Jill are located next to the center clock. They pass each other at $t = t' = 0$.

(a) Suppose Jill records the status of the clocks on the train when all her clocks read t=0. What does she observe? Draw what she sees.

(b) Suppose Zak records the status of the clocks on the platform when all his clocks register t'=0. What does he observe? Draw what he sees.

(c) How do you reconcile their observations?

Problem 5: Review: Space-Time Diagrams [20 pts]

Although events (occurrences in space-time) do not care what coordinate system one uses, in practical terms we always employ some coordinate system to describe the location of an event with respect to some other.

Let us start by looking at a more familiar way to represent coordinates, the x-y plane. Consider a point in this plane at some distance from the origin (say, p=(x_0, y_0)). Now, imagine that I decide to rotate my coordinate system by some angle, ϕ. My new coordinate system would be described as follows...

$$x' = x\cos\phi + y\sin\phi$$
$$y' = -x\sin\phi + y\cos\phi$$

...or, in matrix notation...

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

(a) Begin by drawing the x and y axis for both the non-rotated and rotated frames.

(b) Draw the point p from above and project where the point lands on each of the axes.

(c) Show that the *distance* from the origin remains unchanged in the x'-y' coordinate system.

(d) Show that $R^T R$ is unitary, where R is the above rotation matrix.

Lorentz transformations can also be considered a type of rotation, although here the rotation is between the spatial and time coordinates. Consider then a Lorentz boost along the x-axis (for simplicity, we consider just the x-t axis).

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

where $\tanh\phi = v/c$.

(e) Draw the x and ct axis for both the boosted and non-boosted frames.

(f) Now consider a point $q = (ct_0, x_0)$. Draw this point and project where the point lands on each of the axes.

(g) What strikes you as different about boost versus rotation?

(h) Let us now consider the "distance" of q from the origin. Show that if the distance in the (t, x) plane were to be defined in the same way as that in the (x, y) plane (i.e. $d^2 = (ct^2) + x^2$), then it loses invariance between the unboosted and boosted coordinate systems. How would you modify the definition of this "distance" to keep it invariant under boosts?
[Hint: Use the identity $\sinh^2\phi - \cosh^2\phi = -1$.]

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