PROFESSOR: So so far even though these things look maybe interesting or a little familiar, we have not yet stated clearly how they apply to physics. We've been talking about vector spaces, V, for a particle. Then V tensor N. And we've looked at states there. We've looked at permutation operators there. Symmetric states there. Antisymmetric states there. What is missing is something that connects it to quantum mechanics. And that is given by the so-called symmetrization postulate.

So it's a postulate. It's something that you technically can't derive. Therefore, you postulate. You can say it's an extra axiom even, quantum mechanics. You would say also that probably there's no other way to do things. So in some sense, it's forced. But I think it's more honest to admit it's an extra postulate.

So here it goes. I'll read it first. Then I'll write it. So it says the following. If you have a system of N identical particles, arbitrary states in V tensor N are not physical states. The physical states are the states that are totally symmetric. In that case, the particles are called bosons. Or totally antisymmetric, in which case the particles are called fermions. That's basically it.

The arbitrary state that is neither symmetric nor antisymmetric, and all that is not a physical state of a system of identical particles. So it's stated here. In a system with N identical particles, physical states are not arbitrary states in V tensor N. Rather, they are totally symmetric. In parentheses, they belong to SymN of V. In which case, the particles are said to be bosons or they are totally antisymmetric. Are totally antisymmetric. They belong to anti NV. In which case, they are said to be fermions.

All right. So this is really pretty fundamental. It's a statement that we began our discussion by saying we have an electron at this up and another electron at this down. Is it the plus minus state or is it the minus plus state? And we said it's neither one. You cannot declare those two to be equivalent. And that's what this says. It's not arbitrary states in this. But they have to become totally symmetric, if they're bosons or totally antisymmetric if they are fermions.

So we have to make a few comments. And that's what will keep us busy for the next 20, 25 minutes. And to understand this and see what it implies and how we use it.

So here it is. The statement we have made is a statement of fact in three dimensions. There's no particles that we like and we study. There are further possibilities in worlds of lower dimension two, three dimensions, space, time, or so two dimensional space, that allows for further kinds of statistics that are interesting. But this is the general statement.

So the first comment is this is the general statement. But further statistics, other types of space, exist in two dimensions, not in three, where we live. We say three in the Galilean ways. 3 plus 1 time. But this is spatial dimensions. This is the statistical definition of bosons and fermions.

But then comes the spin statistics theorem. That applies then you understand with quantum mechanics. And that's a deep theorem that ends up telling you that particles with spin 0, 1, 2 integer are bosons. And particles with spin 1/2, 3/2 [INAUDIBLE] are fermions.

So it's a great achievement of quantum field theory associating the statistical properties with the spin. It's a deep connection. It's valid for elementary particles or composite particles. The list of particles is not that big. Particles of spin 1, we know a lot of them. A photon is spin 1. Gluons are spin 1. The w's, the c's are spin 1 particles.

Particles spin 0. We didn't know an elementary one for a while. We finally know the [INAUDIBLE] is there, spin 0. Particles of spin 2. Just [INAUDIBLE] along. Higher spin unknown. It may exist. String theory has them. All kinds of theories postulate them. They may exist. They may not.

Spin of 1/2, fractional spin. Well, all the matter particles, quarks, muons, leptons, neutrinos are spin 1/2. Spin 3/2 is hypothetical as an elementary particle. If it exists, it would be called the gravitino. There are composite particles. You could [INAUDIBLE] with quarks, or with other particles, mesons, baryons. You can get several spins. Not with elementary particles. Many time unstable particles.

So anyway, those are very interesting things. Now next fact is that you can go from elementary particles to composite particles. So if you have, for example, the hydrogen atom. That this has a proton and an electron. And you want to figure out if it's a boson or a fermion.

Well, you think of another hydrogen atom here, p and e. And you write a wave function that involves the protons, the first proton, the second proton, the first electron, and the second electron. That's a little bit of a funny notation. This would correspond to maybe the coordinates of the first proton, the second proton, the first electron, the second electron.

Since protons are identical particles that are fermions, the wave function must change sign if P1 and P2 are exchanged. There would be a minus sign. It's antisymmetric. Since the electrons are fermions, it should antisymmetric by exchanging this 2. So it's also antisymmetric under this exchange.

So finally, if you exchange this item with this item, you must exchange the two protons and the two electrons at the same time. Two minuses and you get a plus. Therefore, the hydrogen atom is a boson. So these statistical properties build up. So an object built with a number of bosons and a number of fermions will be a boson or a fermion, depending on those numbers. So that's a nice thing. So the hydrogen atom is a boson.