

PROFESSOR: Scattering, so where did we get to last time? We did the following things. This is a recap.

We wrote the solution, a scattering solution, or I'll use r . And this solution had a wave that represented the incoming wave, and it had what we call the scattered wave. So there were two pieces to the solution.

We understood that setting up a scattering problem with a central potential. We had a wave that came in. And then we had the spherical wave, and that was an outgoing spherical wave.

I've added the subscript k to represent the wave number or the k of the wave. This is an energy eigenstate we're calculating. And therefore, the energy is $\hbar^2 k^2$ over $2m$. And we're writing the solution with that energy. That's an energy eigenstate.

And this solution is, as written, not an exact solution of the Schrodinger equation. But it's approximately exact for r much greater than a , where a is the range of your potential. So this is only valid in those cases.

It's not valid near the scattering. For r near zero, that's not true. And it's not an exact solution.

This would have been an exact solution of the wave of the time independent Schrodinger equation. And this without the f would have been an exact solution. But with the f is an approximate solution. But that's a solution that represents the physics of scattering.

Then we also showed that the differential cross section was in fact given by this function f . So this function f is really what we're after. And if we know f , we know the differential cross section, which is something we measure experimentally. We're scattering particles, and we detect them. And the differential cross section tells us about our ability and the number of particles that each detector picks up.

Then we've restricted ourselves to the case of central potentials. And for those central potentials, v of r with some function of just the scalar distance. In such cases, the cross section would not have a fine dependence.

You can imagine here is the object. It's vertically symmetric, and you're shooting waves. And therefore, the cross section and the amplitude will be independent of the angle ϕ . There's already a direction picked up by the incoming wave. But otherwise, it's just that.

So we wrote solutions in those cases. If we are going to solve, as we will today, some of these problems, you need to write complete solutions. And spherical solutions are of this form.

Although, the relevant ones that we will be using will have no m . We'll be focusing in solutions that have m equals zero. For us, m will be equal to zero. But these are the general solutions.

So maybe I might as well constrain ourselves to our case, already central potential. So we will have solutions of this kind. And these are solutions of the radial equation.

And we reviewed those and mentioned that ψ of r would be given by $A J_l$ of kr plus $B N_l$ of kr times Y_{l0} of ω . This will be our solutions. These are the spherical Bessel functions. And those were solutions that are valid as long as you are away from the scattering center. So this is valid for r greater than a .

Indeed, you know that this could not be valid all the way to the center of the part of where this scattering is happening, because a is divergent as r goes to zero. So this is not the solution. This is just the solution for r greater than a , where a is the range of the potential.

Imagine a potential that totally becomes zero after some distance a , then that is your solution for r greater than a . That is the most general solution given the spherical symmetry of this equation.

Then the last thing we discussed was that one part of our solution, e to the ikz , could be written in that way, because it's a solution. So it must admit an expansion of this form. So it is like this.

You have a sum over all else with some funny coefficients, including i to the l , Y_{l0} of ω . I might as well no put θ , because there's no ϕ dependence in Y_{l0} . J_l of kr . So this is a pretty remarkable expression, we commented last time, that represents your plane waves as spherical waves.

Last, but not least, we have an expansion that is useful for a large argument of the spherical Bessel functions. They both fall off, like one over the argument, with sines and cosines. And you see a constant shift there in this sines and cosines of $l\pi$ over two. And this is for x much greater than one, not exact either, but approximate.

OK, so these are some of our ingredients are ready. That's how far we got. And now, we're

going to try to get more information and learn how to solve for f_k .

We need to calculate f_k . If you have a given problem, you want the current section, you need f_k . Now the thing we're going to do is a little intricate, a lot of funny formulas, so let's try to keep the ideas very clear about it. So--