PROFESSOR: Today we are going to continue discussion about two very important issues. The first one is the understanding of so-called quarter wave plate. That may not mean anything to you in the beginning, but I hope after this lecture, you will know what does that mean and why that is actually interesting.

The second thing is that-- the second topic we want to talk about in the lecture today is, OK, we have been talking about electromagnetic waves for a long time already, since the last few lectures. But we have never touched the topic, how do we actually create electromagnetic wave, right? And we are going to answer that in the lecture today. So that's my plan about these two topics.

OK, so before we start, it's a reminder about why we have learned last time. So we have learned several situations related to polarization. So we have learned linearly polarized wave. What is linearly polarized wave?

If you plot the wave amplitude as a function of time as a function of space, it's going up and down, up and down, up and down. And the direction of the field doesn't change as a function of time. So that is actually called linearly polarized light. And we also learned circularly polarized light, right?

When you have two components, one is in the x direction, the other one is in the y direction, if the two components are out of phase, say, they differ by 90 degrees, for example, and they the same amplitude, then the superposition of these two waves will be a circularly polarized wave. So basically, the wave propagation looks like this. So basically, the pointy angle of the electric field is rotating as a function over time as a function of the distance it travels.

And the other case, which is also interesting, is that when you have-- OK, for example, different phase difference. Like delta phi different from 90 degree, or say you have different amplitude, although the phase difference is 90 degree but you have different amplitude in the x and the y direction. If that's the case, then you have a situation which not only the direction is changing, but also the amplitude is changing as a function of time. And that we call it elliptically polarized.

So that's actually the three situations we learned. And also we learned about how to make polarized light during the class. So usually, the light source we are talking about, or even present in this room, like the light from the light bulb, et cetera, those are unpolarized light, right? So that means you have a lot of electromagnetic wave emitted from the light with different initial time, emission time. And those are in slightly different angular frequency, slightly different pointing direction.

So you can have all kinds of different emissions. And the sum of all those emissions is unpolarized light, which is actually the light source I have here. And you can use so-called polarizer. So the polarizer can actually kill one of the direction, and only keep all the projection to the easy axis. And in this presentation, the easy axis is in the x direction.

And you can see that if you start with unpolarized light, and basically, you have that pass through a polarizer, then the resulting electric field will be pointing toward the so-called easy axis. So the easier axis to pass, right? So therefore, all the electric field perpendicular to the easy axis is illuminated. And what is left over is the electric field, which is actually parallel to the easy axis.

And of course, you can rotate this polarizer and you would see that, OK, if you have a linearly polarized wave passing through polarizer, because easy axis is actually now in line with the polarization, what is going to happen is, as I said, still only the component which is actually parallel to the easy axis will passed through. And the resulting electric field will still be pointing to the direction of the easy axis. So that's actually what we have learned last time.

OK, so that means we know how to generate linearly polarized wave, right? Because you just need a polarizer and put it in front of your light source, then you produce a linearly polarized save. But we didn't talk about how to produce a circularly polarized wave, right?

OK, so that is actually the topic which I would like to talk about. So let's take a look at the diagram here. So assuming that I have a single layer of sheet, which I call wave plate. This is actually the zoom in and zoom in of that sheet.

And the interesting property of this wave plate sheet is that the refraction index in the x direction and the refraction index for the linearly polarized wave in the y direction, they are different. That can happen, right? Because when we were discussing two-dimensional and three-dimensional waves, the dispersion relation can be dependent on the k vector, right?

So that should not surprise you by now. And it depends on the structure of the material you use to make this wave plate. So therefore, you can have different velocity when you have an incident where pointing in the x direction, and compared to an incident where pointing in the y direction.

So in short, we can actually summarize this kind of information, the dispersion relation, into two components. One is the velocity, the phase velocity in the x direction, which is denoted as nx. Just a reminder, the speed of the light will be equal to c divided by nx, right? So larger n means smaller speed of light in material.

And if that happens, if nx is different from ny, what is going to happen is that if you have an incident wave, when it passes through this wave plate, what is going to happen is that the x component, the delay in phase in the x component, would be different from the delay in phase in the y component. And that is, essentially, how we can actually make use of that to create elliptically polarized wave or circularly polarized wave, OK?

So let's take a look at this example together. So suppose I have incident light with angular frequency omega, OK? Since I give you already the omega, what I really need is the speed of light, then I can calculate the resulting wave number and wave lengths. So this is actually the incident wave angular frequency.

And the lens of the-- or say the thickness of the wave plate is called little l. And we can actually check immediately what would be the corresponding wave number in the median for the linearly polarized wave in the x direction and linearly polarized wave in the y direction. So we can actually calculate Kx will be equal to nx over c omega, because this is actually just omega divided by v, which is the phase velocity in the median for waves in the x direction. And that would be equal to 2 pi over lambda x.

Similarly, you can also conclude that Ky can be written as ny divided by c times omega, which is 2 pi over lambda u. Kx and Ky are the wave numbers inside for the progressing harmonic waves inside the median, OK? One is in the x direction. The other one's in the y direction.

So if we keep those in mind, you will see that, huh, if I have different nx and ny, when the same frequency light goes through this median, its x component will travel through different amount of period, where is a different amount of phase difference. Compared to the light polarized-- compared to the component in the y direction, as you can see from this demonstration.

Therefore, we can actually conclude that there must be a phase difference between the x component and y component. And we can calculate that-- this is actually delta phi, the phase difference between the x direction and y direction will be equal to 2 pi times l divided by lambda x. So basically, it's the number of waves times 2 pi. l over lambda x is number of periods past. And the times 2 pi translates that to phase.

And we are taking the difference between the x and the y direction. And we can conclude that based on what we have written here. As you see that this is just nx minus ny divided by c times omega times l, OK? So this is actually how the wave plate works.

Suppose I have a linearly polarizer wave coming into this plate. And the direction of the polarization is not in the x direction or y direction. So they are positioned-- they are components in the x direction and y direction. For example, I can have an incoming polarizer like this. And this is actually the x direction. This is actually the y direction.

And I can now decompose this kind of linearly polarized wave into two components. And after this wave passes through the wave plate, x component will be-- the phase difference between x and y will be increased by delta phi. So if originally there were no phase difference because this is actually a linearly polarized wave, and after it passes through the wave plate, it will be increased. And then the phase difference between x and y will be delta phi.

All right, so that's really nice tour. And the so-called quarter wave plate is a device which we intentionally set the delta phi to be pi over 2. Why is that interesting? That is because initially you have Ex equal to E0 cosine omega t minus Kz, and this is actually the y component. If initially you have this kind of incident wave, now it corresponds to a polarization which is actually 45 degree with respect to the x-axis. So this is actually the x-axis. And this is actually the y-axis.

When I have this kind of linearly polarized wave pass through the quarter wave plate, what is going to happen? What is going to happen is that one of the components will be delayed by delta phi or pi over 2. That will make you a circularly polarized wave. Because that will become sine and cosine.

Therefore, if you plot the locus of the electric field in two-dimensional xy plane, you will see a circle. So that is actually why we want are interested in a special setup which delta phi is equal to pi over 2, OK? So let me go through a few examples so that actually you get some feelings about what is actually a quarter wave plate.

Usually, instead of drawing this complicated diagram, we actually simplify the presentation into a diagram like this. So basically, you have a fast axis, which is the axis with smaller phase shift. And you have slow axis, which is actually the axis with larger phase shift. So basically, we just reduce the whole complicated setup into a simple diagram like that.

So suppose I have an incident wave which is actually linearly polarized in this direction. In this direction, which I can call it x direction. And this is actually y direction. And I have that pass through a quarter wave plate, where the fast axis is in the x direction, and there's a slow axis in the y direction.

Can somebody tell me, what would be the resulting polarization after this electromagnetic wave passes through this quarter wave plate? Somebody want to try it? Yes.

STUDENT: It would be polarized in the y direction.

PROFESSOR: Polarized in what?

STUDENT: In the y direction.

PROFESSOR: No. Basically-- OK, maybe I didn't explain that clearly. So initially, in this example, all the electric field is in the x direction. Therefore, in the y direction, there's nothing there. So that's actually a linearly polarized wave. The direction in actually in the x direction.

And this quarter wave plate is going to slow down the y component by a phase of pi over 2. So what would be the resulting polarization? Yes.

STUDENT: Very similar to [INAUDIBLE].

PROFESSOR: Yes, that's right. So because we are dividing something which is zero. But zero is zero. So zero is zero is zero, right? So therefore, what you are going to get is this. It's still a linearly polarized wave, right? OK, doesn't surprise you after I explain to you more clearly.

And then you can see that if you have this-- OK, now I change the situation. So this is the x direction. This is the y direction. And I have something which is 45 degrees. And I have that pass through the same setup. Slow axis is in the y direction and the fast axis is in the x direction. What will we get? What kind of polarized light will we get after it passed through this quarter wave plate? Somebody can help me?

STUDENT: Circular.

PROFESSOR: Circular, yes. Thank you very much. So that's actually exactly what I was talking about in the beginning. The y component will be delayed by pi over 2. Therefore, it would become a circularly polarized wave.

How about I change this to 30 degrees? What is going to happen?

STUDENT: [INAUDIBLE].

PROFESSOR: Yeah, it will be elliptically polarized, right? Because now the projection to a different component is different. So therefore, it would be elliptically polarized wave. Very good. It seems to me that most of you actually understand what we are doing.

And now it's time to do some experiment to actually show you what we have done. Yes.

STUDENT: It's a little more complicated than that because the slope the refraction is [INAUDIBLE] be much, much slower than the fast axis.

PROFESSOR: Yeah. Yeah, that's right. You are absolutely right. So it depends on the delta phi, right? So if delta phi is not pi over 2, then it can be elliptically polarized. And in this setup, I say that this is actually a quarter wave plate, therefore, the delay is always delta phi equal to pi over 2. Yeah. So then-- thank you for that.

This is what we have been discussing is always quarter wave plate. Therefore, the delta phi between the slow and fast axis is always pi over 2, OK? So that everybody is on the same page. Yes.

STUDENT: How can you have a material that has a different refraction index for different directions?

PROFESSOR: For example, we were talking about materials-- or say the two-dimensional discrete case, right? So we can have little mass arranged in the x and y direction. But the space between mass in the x direction and y direction are different, then you have a dispersion relation which is actually different for the harmonic wave in the x direction compared to y direction. And that's just some random example. And that can be achieved by engineering the material we will use for the wave plate, OK? Good question.

All right, so we will go ahead and I'll show you some demonstration. We prepare. The first thing I have to do is to turn off the light to have some more excitement. I cannot even see where is my experiment now. Oh, right here, yeah. OK, woo.

OK, so look at what we have here. This is a projector. So what is the polarization of that light?

STUDENT: Unpolarized.

PROFESSOR: Unpolarized. Yeah, very good. OK, I'm very happy to hear that. All right, so now I have the polarizer and I put it on it. What is the polarization of this light?

STUDENT: [INAUDIBLE].

PROFESSOR: I couldn't hear you.

STUDENT: It's linear.

PROFESSOR: Linear, yeah. Linearly of-- don't be afraid. OK, you can say that. No, this is not linear. The edge is not linear, right? But it's OK. I'm talking about everything inside of this material. Very good.

So now what I'm going to do is to put two polarizers on top of each other. And of course, I can rotate such that the polarizer, the easy axis is perpendicular to each other, OK? So you see that ha, I almost black most of that light.

So the first thing which I have been doing is that I first turn this unpolarized light polarized. And it's actually oscillating in one direction. And I block it again with the second one. And then you see that it's black. It's consistent with what we expect.

So we are happy. We are not happy? No? Yes, we are happy. All right, so remember the discussion we had before.

So what I could do is to add a third one, a third polarizer. So I can have the first polarizer which actually makes the direction of the oscillation in this direction. Then I put a second polarizer, where the easy axis is in this direction. Then I actually-- I'm going to extract all the components which projected to this axis. Therefore, after passing the second polarizer, the oscillation of the wave will be in this direction.

Therefore, aha, now I put the third polarizer on, you can see that in the middle, because it changed the direction of the polarization by 45 degrees already by this polarizer, therefore you can see that there's some residual light survived. And then you can actually calculate what will be the intensity of the light surviving these three polarizers.

And you can see that the ones which didn't pass the second polarizer is actually completely blocked by the two polarizers, which their easy axes are perpendicular to each other. So now, the interesting thing is that now I have a quarter wave plate here. OK, it's here. Hope you can see it.

And I'm going to insert this into this experiment and see what is going to happen. Look at what we have here. Oh, this is actually much brighter, right? So basically, this water-- this quarter wave plate-- sorry, it's not water-- quarter wave plate, OK? Quarter wave plate actually turned the polarized light into a circularly polarized light.

And after this circularly polarized light continued and passed through the third polarizer, you can see that, huh, the light passed through this kind of combination is a lot more than this experiment which was three polarizers. And we can also calculate what would be the expected intensity.

And the good news is that we are not going to calculate that now, but in your P set. So you will be able to show that, indeed, the intensity you expect with quarter wave plate will be higher than why you expect with three polarizers. So that's actually the first experiment I would like to show you.

The second experiment is also very interesting. So I have here-- OK, first I need to see if I can turn on the light. I have to turn on this light. Very good.

So look at this tube. This tube is made of water and sugar. So we all love sugar. And I love it too much, so I add too much into this tube. So it's actually oversaturated sugar water. And so there is sugar inside and there are some animal or whatever living inside. But we don't care. We are not studying biology.

But what is actually interesting is that, OK, now I have a light source from the lamp inside emitting what kind of polarized light? Unpolarized light, right? And I have that pass through a polarizer, which is here. There's a polarizer here.

And therefore, what I want to say is that the incident light into this tube is polarized. That's the first thing I want to say. The second thing I want to say is that, OK, a linearly polarized light, due to superposition principle, actually you can decompose that into two circularly polarized light. Both of them are actually rotating in different directions.

You can actually work on the mass and you will see that, ha, indeed, it works. So our linearly polarized light you can always rewrite it as a superposition of two circularly polarized light, but rotating in different directions. The interesting thing is the material which we use in this demonstration is oversaturated sugar.

And we know that the molecule for the sugar and those kind of material is asymmetric under mirror transformation. It's asymmetric. It's a chiral material. OK, chiral is actually just some name, but it doesn't mean anything to you. But what is actually interesting is that this material is asymmetric.

If you have a mirror and this material is looking at the mirror, in the other side of the mirror, it looks different. It's like your hand, right? So in a mirror, it's asymmetric.

So what is interesting is that due to this kind of structure in the material that the light passed through, the circularly polarized light, counterclockwise polarized light, will have different refractive index compared to clockwise. Clockwise and counterclockwise light will have different refractive index. Therefore, you see that now you can see some kind of rotation or some kind of change in the polarization as a function of distance the light travels through.

So basically, this material would rotate the linearly polarized light, because the refractive index for the clockwise and counterclockwise are different. So if you accept that, I would like to add another complication.

In addition to that, the refractive index also depends on the frequency of the incident light. Therefore, you will have different amount of rotation for different color. So therefore, you can see that once I have incident light which is linearly polarized, all the colors are lined up. You can see that here.

What is the color here? It's kind of bluish or white, essentially, right? But if you move slightly more, then it becomes pretty blue. And then if you move more, because of the dependence of the refractive index as a function of wavelength, therefore, you can see that this whole thing is actually changing color.

And in the end, I have another polarizer which filter one of the directions. And I can change the direction, and you will see that I can filter out different colors. Which color do you like? Now it's red.

And of course, I can rotate this polarizer, and I am sampling different color. Because at the time different color of light passes through this material, they are rotated by different amount of degree. Therefore, I can filter out and create all kinds of different color on the wall.

The other thing which is interesting which I can do is that I can now change that direction of the incident light, or the direction of the polarization of the incident light, by rotating this one. You can see that the whole tube is changing color, like why you see in the barber shop, right?

OK, so maybe this is a fancy way to make that kind of tube. No? A physics barber shop. Maybe we should do that. OK, so I hope you enjoyed this demonstration. And we will take a five minute break to take questions. And the next topic we are going to talk about is how do we actually create electromagnetic wave at all. So let's come back in 15.

OK, so I hope you can hear me. All right, so welcome back from the break. So we are going to talk about the second topic we would like to cover in the lecture today. The question we are asking is, how do we actually create electromagnetic waves and so-called radiation?

So this is actually a picture from Hubble telescope. And you can see that light can travel through billions, or tens of billions, of light years and arrive at Earth. And you can actually measure them and see you what is actually going on in the past.

And that means if you have a source and you have some kind of radiation, and this source is going to emit energy towards somewhere, which is actually really, really far away toward the edge of the universe. So that is actually what we call electromagnetic wave and radiation. But the what is actually requirement for that to happen? What is the requirement for us to be able to see the stars which are so far away? That's the question.

So let me actually make a simple argument here. Suppose I have some kind of a light source. It's a source in the center. And we have learned about pointing vector, right? So what this actually pointing vector?

It's not really the pointing vector, right? So it's rate of energy transfer per unit area. So it's kind of pointing, but it's pointing to the direction of the energy transfer. So this is a vector. And it's actually highly related to the direction of the electric field and the magnetic field.

And now, if I-- since this is essentially the energy transfer per area, I can now capture the average pointing vector times area. And what is going to happen is that if I do this calculation at this surface-- this is actually a sphere which is covering this source. I can do this at sphere number 1. And I can actually also do that in the sphere number 2.

Since there are absolutely no other source-- I'm assuming that there's only one source here. There's only one light source in the universe, which is kind of lonely. Apparently it's not my universe, but somebody else's problem.

And then I will conclude that since there's nothing outside, I will conclude that S times A, if I evaluate that in the first surface, will be equal to S times A in the second surface. That's equal to power, OK? So that should not surprise anybody.

So that means the pointing vector will be proportional to 1/A, which is the surface area. And that means, based on simple mathematics, that would be proportional to 1 over r squared for this constant power transfer to happen. So this means that there's a source. And if I integrate all the energy transfer from some kind of surface, it's going to be a constant, no matter what surface you are choosing.

So that means if I look at the structure of the S vector, the pointing vector, we can conclude that at least the electric field and magnetic field has to be proportional to 1/r, which is the distance with respect to the source. Otherwise, it's going to be decaying faster, or reducing faster than 1/r, then the total power will approach zero when you increase r enough.

Then that means if you have that happen, you will not see anything if you are far enough. So if that's actually the case, we can now come back and discuss two situations which we are very familiar with. For example, you can say, how about I have a stationary charge?

So I can have a stationary charge and see what will happen. And apparently, if I have a charge here without actually moving it, it's going to emit-- basically, it is going to have an electric field around this. But electric field, based on what we learned from 8.02, is going to be 2 divided by 4 pi epsilon zero r squared r hat. It's going to be proportional to 1 over r squared.

It's already not very good news, because it's proportional to 1 over r squared. And it's hitted by this. The magnetic field is zero. If I have something times zero it's zero. Then there will be no energy transfer if you have a stationary charge just sitting there.

So apparently, this is not a good way to create electromagnetic wave, based on our argument. The pointing vector is actually equal to zero. So now you can say, OK, this is actually too boring, so let's introduce some excitement. How about we make this charge moving at a constant speed?

What we can do is like this. Basically, if you have a positively charged particle, you can actually make it move at a constant speed, velocity equal to v. And what you are going to see is that, oh, indeed, there will be some changes in the electric field and the magnetic field. And I'm not going to go through the calculation of this kind of situation. And I will leave that as an exercise.

But I would like to tell you what would be the conclusion. So if you have a single charge, which is essentially moving at a constant speed, and what is going to happen is that the electric field density, or the field line density will change. And you will be more concentrated in the direction, which is essentially perpendicular to the direction of the motion of this charge.

And we can actually calculate what would be the electric field. The electric field will be equal to q divided by 4 pi epsilon0 r squared, 1 minus beta squared. I will define bet in a moment. 1 minus beta squared sine squared theta 3/2 in the r direction. And where the beta is actually defined as u/c, which is u is actually the velocity of this little charge.

And of course, you can also calculate what would be the corresponding B, right? The magnetic field will be actually equal to u plus E divided by c squared. And that is actually proportional to 1 over r squared.

As you can see from here, the bad news is that, OK, you indeed now have both electric field and the magnetic field. There is some improvement. But the problem is that the reduction of the electric field and the magnetic field is a function of distance. It's too large.

Both of them are proportional to 1 over r squared, proportional to 1 over r squared. Therefore, the magnitude of S will be proportional 1 over r to the fourth. So if you are far enough, you can conclude that the total power will approach zero, even if you integrate over the whole surface surrounding this moving charge.

So apparently, that's actually not the solution we are seeking. Therefore, we have to do something more aggressive to accelerate the charge. So you can now have a charge moving at a constant speed. We see that it didn't do anything. Therefore, we have to make the velocity increase and see what'll happen.

So what I am going to do now requires concentration. So I will hope that you don't take notes. Just follow me so that you get what I am trying to argue. And of course, if you are really good in mathematics, you can actually also go through page 356 to 360 in George's book. There are some really mathematical deviations of the radiation from an accelerated charge.

So let's try to see how can we actually understand an accelerated charge and what is actually the associated electromagnetic field. So my goal is to have some kind of acceleration. So I would like to set up the stage.

So let's take a look at the slide here. At t equal to zero, time equal to zero, before I introduce any excitement, I have a charged particle initially at rest. And it's sitting there. What I'm going to do is that at some point, at t equal to zero, I try to accelerate this charge until t equal to delta t.

The original position of that charged particle is at a. And I try to accelerate this charge by acceleration a. And that only happened in a very small amount of time, which is delta t.

So what is going to happen is that this charge will get accelerated. And you can see that the velocity of the charge-- you can see velocity as a function of time here-- is increasing linearly in this period, and reaching maxima, which is a delta t. So after that, I stop the acceleration.

So originally, the charge is at rest. Then I accelerate it for some period of time. And I stop the acceleration at A prime, or t equal to delta t. And what is going to happen afterwards to the charge? Everybody is following?

You will be moving at constant velocity. Very good. So that's actually what you see here. And the wave can actually-- this information can propagate as a function of time. So that's actually the whole setup, which I would like to discuss.

Before that, I would like to bring your attention to the graph I was trying to draw here. So you can see that originally there is a line which is pointing up, like 45 degree with respect to this charge. So that's one of the field lines I was drawing here. That's actually the electric field line.

And as you can see that as I manipulate this charged particle, this is a sphere-- or a circle I should-- on this slide, which is actually telling you where this information already propagated in the space. So for example, if I am sitting here in the position of my little mouse here. Can you see that? No, you cannot see it.

If I'm sitting in the upper right corner of the slide, and I try to-- then the experiment starts and I move the charge, the observer at the upper right corner would not feel anything. Because it takes time for the field, or for the changes, or for the information to be sent from the position A to the observer, which are far away from the charged particle.

And the surface which-- the surface is actually where the information has propagated. So this information that my charge is accelerated, this information has already propagated to a sphere, which is actually far away by c times delta t away from the center, which is the location of the charged particle. And you can see that as time goes on, this black circle is actually becoming larger and larger, which contains the information that, OK, I accelerated the charge.

This is actually where you can see that out of this circle is as if the charge is stationary. So you can see the field line is still linear. And passing through this line, or say, this surface, the information is already propagated. If you standing inside this line, like for example, next to the question mark, if you are there, you feel, aha, now I observe the acceleration to the charged particle.

Finally, if I go toward the charged particle even more, and I will see, aha, if I am now inside the green circle, I know that this charged particle already stopped the acceleration. It's now moving at constant speed. So that's the meaning of these two little circles.

And now I am looking at the situation at time equal to t where the charge is at position B. And I should see something really interesting. As I mentioned before, if you have a constantly propagating charge, the field line is actually still a straight line, actually, right there in the equation.

If you have a stationary charge, it's also a linear straight line. And you can see that you have two straight lines, but in between, there's a kink which connects these two lines. So between these two lines, basically this is actually what we have here. So we have the original particle.

And this is actually where the particle have the field line as a moving charge. And there's another surface, which actually out of the surface, it's like there is no acceleration at all. The charge is still stationary at A. You can see that these two field lines are linear, and also essentially in the radial direction.

But the excitement is that since the field line has to be continuous, the excitement is that I have successfully created a kink, which is actually propagating in the radial direction. And this kink is going to be our electromagnetic wave because it has a component which is perpendicular to the direction of propagation.

Just a reminder, what is actually an electromagnetic field looks like, it looks like this, right? So basically, you have the electric field oscillating up and down in one of the directions, the polarization-- linearly polarized electromagnetic wave. And the whole wave is actually propagating toward the right-hand side of the board. And the electric field is in the perpendicular direction of the direction of propagation.

And this kink is actually what we are looking for, OK? And that really becomes the electromagnetic wave right there from the point source. Any questions so far? Everybody's following?

OK, so now, that's good. We have managed to create this situation. And I would like to be more concrete about several settings. The first one is actually we have a constant acceleration a, and this delta t is really small. Very small delta t, very small acceleration. Therefore, I would assume that u defined as delta t, the resulting velocity is much, much smaller than the speed of light.

So that's actually the setup which I would like to use. Then the question now is, how do we actually evaluate what will be the magnitude of this so-called kink electric field? So for this, it's actually also pretty easy.

So now I would like to copy the geometry which I have there. I am trying to draw a copy of that to my board here. So basically, originally the charge is stationary at A. And it's emitting an electric field, which is actually only in the radial direction.

And it got accelerated by a really small time. I'm exaggerating in that figure, OK? So it got accelerated a really small amount of time. And after that, it reached a prime, which is the exaggerated version is actually probably there. And A and A prime is, in fact, very, very close to each other, because this is actually just a very, very small delta t.

I can have delta t goes to zero. Then A and A prime would be very, very similar. And now I let the time go on, and now this charged particle is now at point B. It's moved to point B. And I can connect B to A and A prime.

And I can actually conclude that, OK, since the resulting velocity of the charged particle up to a prime is actually equal to u, defined as a times delta t, and we are now at time equal to t. Therefore, the distance these charged particles pass through, or travel through, is actually u times t. Doesn't surprise you, right? So that's velocity times t.

And also, we can actually calculate this lens. This lens is actually-- I call it this point D here, which is the intersection between the second surface and the original field line. And I call this one E, which is the intersection of the field line from the moving charge and the second surface. And finally, I also have the intersection, which I call it F, which is actually where the field line and the surface actually join, which is actually the information about the charge has moved is actually the surface, which within that surface, people know the charge is actually already moved.

So once I have all these, I can now evaluate what will be at D and F. D and F are actually pretty straightforward as well, because all those surfaces are traveling at the speed of what? Light, right. So what is actually the delta t between these two surfaces? It's delta t, right? Because I actually stopped the acceleration at delta t, therefore, the distance between D point and F is actually just c times delta t.

And of course, now I have this. I can connect E and D. And roughly, because a and a prime are very, very close to each other, and also t is very large, therefore the BE, this line, is roughly parallel to these AF line. So these two lines are actually roughly parallel to each other.

Therefore, I can now evaluate what will be this line, D and E-- what would be the size of the distance between D and E. And that can be evaluated. And it's actually just u perpendicular times t-- perpendicular to the direction of the field. And I can copy that here. The distance between D and E is just u perpendicular times t.

And of course, I can approximate that is actually just a line. And I have a theta angle which is actually DEF. So now I can actually try to use this information, this geometrical argument information, to figure out what will be the electric field, this kink. So now I can have the electric field, the same triangle here, this is angle theta. And this is the electric field parallel to the AF line.

And I can have also E perp, which is actually the perpendicular to AF, this line. And the kink, E kink, is actually what we would like to figure out as well. And basically, this E kink is what we want to figure out. And the E has the following two components. One is the E parallel. The other one is the E perp, which is the perpendicular and the parallel components to the AF line.

And we can already make use of the similarity of these two triangles, right? Basically, this field line is actually pure geometrical, therefore, I know what is actually theta from this geometrical argument. So what is actually theta, basically, you can get that from the information of c delta t. And then u perp times t.

So therefore, I can conclude that the magnitude of E perp divided by magnitude of E parallel will be equal to u perp t divided by c delta t. And this E kink is like this. It actually has a direction. However, you can see that, wait a second, you have this ratio, right? But the E kink is actually pointing to this direction.

And this ut is pointing up to upward direction. Therefore, if you take this ratio, the E kink will be pointing to the upper left direction. Therefore, you really need a minus sign here, right? Therefore, the E perp would be pointing downward. Therefore, that's actually how you get this minus sign there.

From this pure geometrical argument, you can actually conclude what would be the ratio between E perp and the E parallel, which is actually equal to that. And I can write it down explicitly. Basically, that's going to be equal to a delta t times t divided by c delta t. Remember, u is equal to a times delta t.

Therefore, I can now cancel delta t. Then basically, what I get is minus a perp t divided by c. And now this is actually equal to minus a perp r divided by c squared, where r is actually just c times t. r is actually the distance between the position you are evaluating this field and the origin, which is A, OK?

So you can now conclude that-- based on this geometrical argument, you can conclude that E perp is highly related to the E parallel. The E perp is equal to minus a perp r divided by c squared E parallel. Any questions so far? Yes.

STUDENT: How'd you get r real quick?

PROFESSOR: R is actually-- yeah, so r is actually just c times t. So it's the whole distance is the r. Cool. All right, so you can see that right now all of those things are purely geometrical, right? So this is really no magic. And no even integration.

So now we are going to do some integration. So now we are almost there. I would like to figure out what would be the E kink. And I am especially interested in E perp, because E perp is the direction which is actually perpendicular to the direction of propagation. It's really cool.

So that's actually related to the magnitude of the electromagnetic field radiating. So I would like to know E perp, but I don't know what is E parallel. So what we could do is to use Gauss' law in this example.

So now what I could do is that I can draw a pillbox, which is actually through the surface number 1. This is actually surface number 1. What I could do is I can draw a pillbox which is actually passing through the surface number 1.

Out of surface number 1, we know the physics very well, which is actually the electric field of a single stationary charge. So therefore, I know what is actually the electric field outside. Which is actually pointing outward in the radial direction.

And the E parallel is actually what we are stuck with. So we don't know what is actually the magnitude of E parallel. That's the electric field inside the surface number 1. Makes sense?

So now we also have the component which is actually perpendicular to the direction of propagation. So this is actually the contribution of the E perp and the contribution of E perp, which they go from the side to the site. Go in from the side, go out from the side of this pillbox.

So I can now immediately conclude that the total contribution of this surface integral will be equal to 0, because of Gauss' law. There's no charge in my pillbox. Therefore, all those things should cancel. Apparently, these will cancel, because side in, side out, the same magnitude, which is E perp.

Therefore, that cancel is trivial. And the interesting thing is that we can also figure out that e parallel will have to be equal to E out. So that the sum of all the integral will be equal to 0, because of Gauss' law. That's actually a very big amount of information, because I know how to write down E out.

So E parallel will be equal to E out. We learned from 8.02 this is actually just q divided by 4 pi epsilon0 r squared. Does that surprise you? Should not, right, because out of the surface, people think nothing actually really happened to the charged particle. So it's actually still stationary sitting there.

So therefore, I have the information of E parallel, therefore, I can now conclude what would be the E perp. Now, E perp will be equal to minus q a perp divided by 4 pi epsilon0 c squared r, because this is actually just a perp minus a perp r divided by c squared times E parallel.

Look at what we have achieved. Look at this. This is actually proportional to what? 1/r, right? So that means the decaying speed of this E perp is really slow compared to the electric field from a stationary charge. So that's actually very encouraging.

And of course, you can also write down what will be the resulting magnetic field. And it's going to be also proportional to 1/r. So what we can actually conclude is that the E rad is a function of direction of the-- evaluating this E radiated electric field is a function of t. And we can actually-- based on this exercise, this will be minus q.

a is a vector, but now I only take the perpendicular direction. And this thing is actually evaluated at t minus r over c divided by 4 pi epsilon0 c squared r. Let's take a look at this formula closely together, since we have spent a lot of time trying to get this result.

So look at this structure. So basically, the radiated energy has a minus sign in front of q and a perp, because the E kink is actually pointing in the opposite direction compared to the directional acceleration, as you can see from here. The E kink and the E perpendicular is pointing to the opposite direction of the acceleration. Therefore, we have this minus sign there.

And only the perpendicular direction motion, acceleration, works. And there's this little component here, t minus r/c. This is actually-- now multiplying this factor is evaluated at the t equal to t minus r/c. It's evaluated at that time.

So this is actually evaluated at retarded time. So that means I am really slow. I need to wait for the information to arrive my detector so that I know there are acceleration happening. Finally, I can now also conclude what will be the magnetic field. The magnetic field rad, as I mentioned, would be proportional to 1/r.

And of course, I also give you the explicit formula in the lecture notes. And now we can actually conclude that s will be proportional to 1 over r squared. So that means I can now send energy to the edge of the universe, because of all this hard work we have been doing here. Any questions?

All right, before the end of the lecture today, I'm going to show you an experiment here. So here I have an antenna, which you can have electron going back and forth, oscillating harmonically really, really fast like this. Therefore, there will be acceleration, because of this harmonic oscillation. And I'm going to turn off the light.

Also probably hide the image. OK, this is good. But I have to be able to see the button. Can I see it? No. Oh, I'm in trouble. Ah, here.

OK, here I have a receiver. It's also a metal rod. And I have a light bulb in between, which is actually trying to receive the information from-- or say that it receives the electromagnetic wave emitted from that source. Which you have electrons going back and forth in that direction.

So now, first, I am trying to align my setup in this direction so that it's really-- what would be the polarization of an electromagnetic wave? The polarization is going to be in a horizontal direction. Yes, very good. Therefore, if I have this set up like this, it's actually perpendicular to the direction of the polarization, therefore, I see nothing here,

It is also possible that the light bulb is actually broken, but let's see. So now what I'm going to do is to change the direction. You see that? I am moving also closer really carefully. Now you can see what happened. You can see that now I receive the signal from this machine.

The emitted light is actually polarized in the horizontal direction. And now I have also the electron going back and forth, and that actually can light up the light bulb. Now, if I change the direction, you can see that this is actually gone.

And I can do this again. And I can go farther away from the source. You can see that now the light is actually disappearing. Why? That is because you get the 1/r term. Therefore, it's actually disappearing. And if I move closer to the source, it's reappearing.

So now I need an assistant to hold this thing for me. Who can volunteer? And I would like to rotate that. I can actually also rotate my setup. Can you help? Yes.

OK, be careful. And I hope you can survive this. So now what am I going to do-- OK, so stay there. And what I'm going to do is I can rotate the whole setup, the same concept. If I rotate the setup, I have to be careful so that I am not touching this more. I want to survive.

And you can see now what is actually the direction of the emission. It's actually in this direction, right? The direction of the polarization is in the back and forth direction. And you see that that the light bulb is actually turned off. And now I can turn it back on. And you see that it's still there.

OK, thank you very much. You survived. Not everybody actually survives this.

[LAUGHTER]

So you can see it now I can move really close to this thing. And what is going to happen? The amount of energy will be too high, and probably this light bulb will explode or broken. Do you want to see that?

STUDENT: [INAUDIBLE].

PROFESSOR: Oh, my god. Let's see. Ooh, [INAUDIBLE].

[LAUGHTER]

OK, very good. So now this experiment is dead. And then we can-- it's a very good time to close the lecture today. And thank you very much for attending the lecture today. And I hope now you understand how we actually create light. And enjoy the homework, because you will be able to figure out why the quarter wave plate combination will give you a higher light intensity.

OK, so if you have any questions, I will be here and just standing up here.

OK, hello everybody. Today I'm going to show you a demonstration, which actually demonstrates the effect of polarizer and quarter wave plate. Here is the setup.

I have a projector here, which emits our polarizer light. And if I put a sheet of polarizer on top of it with easy axis in the vertical direction, like what my finger-- in the direction oscillation along the direction of my finger, then basically, you will see that the intensity of the light is reduced. Because for the unpolarized light, light component which is actually oscillating along the easy axis can pass through the polarizer, but the component which is actually oscillating perpendicular to the easy axis, like this, is not going to pass the polarizer.

Therefore, a large fraction of unpolarized light is actually filtered out, and you will see a reduction in the intensity on the screen. So what I'm going to do now is to place another polarizer on top of the first one. So now we have two sheets.

And you can see that after adding the second sheet, you see some change in the intensity. But if I rotate this sheet so that now the easy axis of the first and the second sheet are perpendicular to each other, you can see on the screen that all the light which are emitted from the projector is actually filtered out.

Why is that? That is because now the first sheet actually filters out all that light which is actually oscillating in the direction perpendicular to the easy axis. If I actually introduce another filter which has easy axis now perpendicular to the one from the first sheet, then I'm going to filter out both directions. Therefore, all the light are filtered out due to this putting perpendicular setup.

Now, if I introduce a third sheet, insert that between the two existing sheets, but now I am trying to actually insert that such that the direction of the easy axis is actually 45 degrees with respect to the easy axis of the first sheet. According to our calculation in class, also in your homework, you should see some light which will pass through this setup.

And let's take a look at the experimental result. You can see that, indeed, after you insert a third sheet, you see that now the easy axis is actually 45 degrees with respect to the first sheet. And you do see the intensity of the light becomes larger, or you see a brighter light output passing through these three polarizers.

And if I rotate it so that actually the easy axis of the second sheet is actually changing, you can see that it reached maxima at roughly 45 degrees, which is actually consistent with what we predicted from your homework. And then the other thing which you predicted from the homework is that if we insert a quarter wave plate between these two sheets, you are going to see a brighter light passing through this setup.

So let's actually take a look at what will happen by inserting the quarter wave plate between these two sheets. And this is actually the result. You can see that, indeed, the intensity is higher compared to the three polarizer experiment. And also, the intensity actually reached maxima when the fast axis of the polarizer is 45 degrees with respect to the easy axis, as we predicted from your homework.

And we can actually put both experimental results side by side. Indeed, the results from the-- so now I am inserting the polarizer also between the two sheets. And you can see that, indeed, the light passing through three polarizers, the intensity is actually lower than two polarizers and the one quarter wave plate setup.

Hello, everybody. So today, we are going to show you a demonstration of dipole radiation.

Here is the setup. So we have a radiator here with two antenna. And when I turn it down, there will be current going back and forth through these two antenna. And therefore, this setup is going to emit polarized electromagnetic wave.

And we are able to detect those electromagnetic waves by using a detector here, which consists of two antenna and one light bulb here. When there are current on this antenna, you will see the light emitting from the light bulb. And the intensity of the light bulb actually can help us to understand the structure of the radiation from the dipole radiator.

So what I am going to do now is to turn this setup on. You can see now, the setup is on and the light is on. And there will be current going back and forth through these two antenna.

So since the oscillation of the charge will generate electromagnetic wave-- since the direction of oscillation is in the horizontal direction, therefore, the electric field of the electromagnetic wave is going to be in the horizontal direction. So this can be actually verified by using the detector here.

When my detector-- the direction of the antenna is actually perpendicular to the direction of the oscillation, you basically don't see any light emitting from the light bulb. Now I'm going to rotate my detector. You can see that as we actually rotate so that the antenna is parallel to the direction of the oscillation, then you will see that, huh, we will see a large intensity of light emitted from the light bulb.

But on the other hand, if we actually rotate such that the direction of the antenna is perpendicular to the direction of oscillation of the charges, then you will not see any light emitted from the light bulb. This can also be demonstrated from on the other side of the experiment.

So now, instead of standing in front of the setup, I'm going to go to the side of this dipole radiator. So here is actually roughly 90 degrees with respect to where I was standing. And you can see that no matter which angle-- no matter which angle of my detector is in, basically, you will never see light emitted from the light bulb.

That is because the direction of the oscillation is in this direction. And according to our formula, our prediction is that there will be no electromagnetic wave traveling in this direction. And therefore, no matter which angle you are actually trying to detect the emitted light, the light bulb will never light up. So that's essentially consistent with our declaration.

The second thing which I would like to actually show you is that we can also detect the nodal point of the emitted electromagnetic wave by moving this detector around in the classroom. For example, if now I move farther away from the dipole, now I am here and you can see that the intensity goes to zero at this point, because we are actually in one of the nodes of the electromagnetic radiation.

And if I now move further away from the setup, you can see now the light is actually emitting again. And also, the intensity increases. And again, if I move farther and farther away from the setup, you can see that the light becomes dimmer and dimmer, and disappears again.

Here is actually another node in the classroom. And also, you can see that as a function of distance, the maxima intensity emitted by the light bulb is also decreasing because of the larger and larger distance with respect to the source. So this demonstration actually shows that we can understand the dipole radiation. And the other experimental results are consistent with the calculation we have done in class.