14.5 Mechanical Energy and Conservation of Mechanical Energy

The total change in the ***mechanical energy*** *of the system is defined to be the sum of the changes of the kinetic and the potential energies,*

$$\Delta E_m = \Delta K_{\text{sys}} + \Delta U_{\text{sys}} . \tag{14.4.17}$$

For a closed system with only conservative internal forces, the total change in the mechanical energy is zero,

$$\Delta E_m = \Delta K_{\text{sys}} + \Delta U_{\text{sys}} = 0 . \tag{14.4.18}$$

Equation (14.4.18) is the symbolic statement of what is called ***conservation of mechanical energy.*** Recall that the work done by a conservative force in going around a closed path is zero (Equation (14.2.16)), therefore both the changes in kinetic energy and potential energy are zero when a closed system with only conservative internal forces returns to its initial state. Throughout the process, the kinetic energy may change into internal potential energy but if the system returns to its initial state, the kinetic energy is completely recoverable. We shall refer to a closed system in which processes take place in which only conservative forces act as ***completely reversible processes***.

14.5.1 Change in Gravitational potential Energy Near Surface of the Earth

Let's consider the example of an object of mass m_o falling near the surface of the earth (mass m_e). Choose our system to consist of the earth and the object. The gravitational force is now an internal conservative force acting inside the system. The initial and final states are specified by the distance separating the object and the center of mass of the earth, and the velocities of the earth and the object. The change in kinetic energy between the initial and final states for the system is

$$\Delta K_{\text{sys}} = \Delta K_e + \Delta K_o , \tag{14.4.19}$$

$$\Delta K_{\text{sys}} = \left(\frac{1}{2} m_{\text{e}} (v_{e,f})^2 - \frac{1}{2} m_{\text{e}} (v_{\text{e,i}})^2 \right) + \left(\frac{1}{2} m_o (v_{o,f})^2 - \frac{1}{2} m_o (v_{o,i})^2 \right). \qquad (14.4.20)$$

The change of kinetic energy of the earth due to the gravitational interaction between the earth and the object is negligible. The change in kinetic energy of the system is approximately equal to the change in kinetic energy of the object,

$$\Delta K_{\text{sys}} \cong \Delta K_o = \frac{1}{2} m_o (v_{o,f})^2 - \frac{1}{2} m_o (v_{o,i})^2 . \qquad (14.4.21)$$

We now define the mechanical energy function for the system

$$E_m = K + U^g = \frac{1}{2} m_o (v_b)^2 + m_o g y, \text{ with } U^g(0) = 0 , \qquad (14.4.22)$$

where K is the kinetic energy and U^g is the potential energy. The change in mechanical energy is then

$$\Delta E_m \equiv E_{m,f} - E_{m,i} = (K_f + U_f^g) - (K_i + U_i^g) . \qquad (14.4.23)$$

When the work done by the external forces is zero and there are no internal non-conservative forces, the total mechanical energy of the system is constant,

$$E_{m,f} = E_{m,i} , \qquad (14.4.24)$$

or equivalently

$$(K_f + U_f) = (K_i + U_i) . \qquad (14.4.25)$$