## 10.2 Momentum (Quantity of Motion) and Average Impulse

Consider a point-like object (particle) of mass $m$ that is moving with velocity $\vec{\mathbf{v}}$ with respect to some fixed reference frame. The quantity of motion or the **momentum**, $\vec{\mathbf{p}}$, of the object is defined to be the product of the mass and velocity

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} . \tag{10.2.1}$$

Momentum is a reference frame dependent vector quantity, with direction and magnitude. The direction of momentum is the same as the direction of the velocity. The magnitude of the momentum is the product of the mass and the instantaneous speed.

**Units:** In the SI system of units, momentum has units of $[\mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-1}]$. There is no special name for this combination of units.

During a time interval $\Delta t$, a non-uniform force $\vec{\mathbf{F}}$ is applied to the particle. Because we are assuming that the mass of the point-like object does not change, Newton's Second Law is then

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt} = \frac{d(m\vec{\mathbf{v}})}{dt}. \qquad (10.2.2)$$

Because we are assuming that the mass of the point-like object does not change, the Second Law can be written as

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}. \qquad (10.2.3)$$

The **impulse** of a force acting on a particle during a time interval $[t, t+\Delta t]$ is defined as the definite integral of the force from $t$ to $t+\Delta t$,

$$\vec{\mathbf{I}} = \int_{t'=t}^{t'=t+\Delta t} \vec{\mathbf{F}}(t')\,dt'. \qquad (10.2.4)$$

The SI units of impulse are $[\mathrm{N}\cdot\mathrm{m}] = [\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-1}]$ which are the same units as the units of momentum.

Apply Newton's Second Law in Eq. (10.2.4) yielding

$$\vec{\mathbf{I}} = \int_{t'=t}^{t'=t+\Delta t} \vec{\mathbf{F}}(t')\,dt' = \int_{t'=t}^{t'=t+\Delta t} \frac{d\vec{\mathbf{p}}}{dt'}\,dt' = \int_{\vec{\mathbf{p}}'=\vec{\mathbf{p}}(t)}^{\vec{\mathbf{p}}'=\vec{\mathbf{p}}(t+\Delta t)} d\vec{\mathbf{p}}' = \vec{\mathbf{p}}(t+\Delta t) - \vec{\mathbf{p}}(t) = \Delta\vec{\mathbf{p}}. \qquad (10.2.5)$$

Eq. (10.2.5) represents the integral version of Newton's Second Law: the impulse applied by a force during the time interval $[t, t+\Delta t]$ is equal to the change in momentum of the particle during that time interval.

The average value of that force during the time interval $\Delta t$ is given by the integral expression

$$\vec{\mathbf{F}}_{\text{ave}} = \frac{1}{\Delta t}\int_{t'=t}^{t'=t+\Delta t} \vec{\mathbf{F}}(t')\,dt'. \qquad (10.2.6)$$

The product of the average force acting on an object and the time interval over which it is applied is called the ***average impulse***,

$$\vec{\mathbf{I}}_{\text{ave}} = \vec{\mathbf{F}}_{\text{ave}}\,\Delta t. \qquad (10.2.7)$$

Multiply each side of Eq. (10.2.6) by $\Delta t$ resulting in the statement that the average impulse applied to a particle during the time interval $[t, t+\Delta t]$ is equal to the change in momentum of the particle during that time interval,

$$\vec{\mathbf{I}}_{\text{ave}} = \Delta\vec{\mathbf{p}}. \qquad (10.2.8)$$

**Example 10.1 Impulse for a Non-Constant Force**

Suppose you push an object for a time $\Delta t = 1.0\text{ s}$ in the $+x$-direction. For the first half of the interval, you push with a force that increases linearly with time according to

$$\vec{\mathbf{F}}(t) = bt\,\hat{\mathbf{i}}, \quad 0 \le t \le 0.5\text{s} \text{ with } b = 2.0\times 10^{1}\,\text{N}\cdot\text{s}^{-1}. \qquad (10.2.9)$$

Then for the second half of the interval, you push with a linearly decreasing force,

$$\vec{\mathbf{F}}(t) = (d - bt)\hat{\mathbf{i}}, \quad 0.5\text{s} \le t \le 1.0\text{s} \text{ with } d = 2.0\times 10^{1}\,\text{N} \qquad (10.2.10)$$

The force vs. time graph is shown in Figure 10.3. What is the impulse applied to the object?

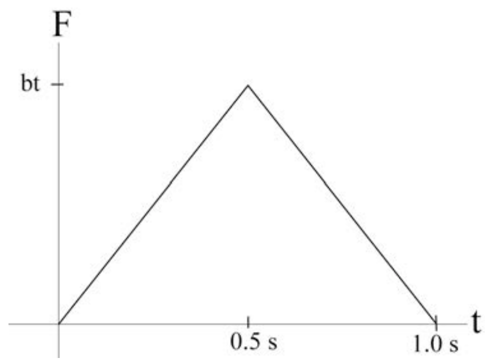


**Figure 10.3** Graph of force vs. time

**Solution:** We can find the impulse by calculating the area under the force vs. time curve. Since the force vs. time graph consists of two triangles, the area under the curve is easy to calculate and is given by

$$\begin{aligned}\vec{\mathbf{I}} &= \left[\frac{1}{2}(b\Delta t/2)(\Delta t/2) + \frac{1}{2}(b\Delta t/2)(\Delta t/2)\right]\hat{\mathbf{i}} \\ &= \frac{1}{4}b(\Delta t)^2\,\hat{\mathbf{i}} = \frac{1}{4}(2.0\times 10^{1}\,\text{N}\cdot\text{s}^{-1})(1.0\,\text{s})^2\,\hat{\mathbf{i}} = (5.0\,\text{N}\cdot\text{s})\hat{\mathbf{i}}.\end{aligned} \qquad (10.2.11)$$

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