MITOCW | Lec 1 | MIT 2.71 Optics, Spring 2009

GEORGE BARBASTATHIS: Hi, everyone. OK, Yeah, so let me tell you my vision of what SMA is and what we're supposed to do. Oh, before I do that, let me do one more introduction. This is Professor Colin Sheppard sitting on the other side. So he's the instructor on the Singapore side. So basically, the four of us are the team of instructors-- Professor Shepherd, Se Baek, Pepe, and myself.

And the class is not quite an SMA class. It is not quite part of the Singapore-MIT Alliance. It is part of something else called Singapore-MIT Alliance for Research and Technology, SMART. And there is a very complicated description of what SMART is and why we're teaching the class.

But the bottom line is that this is more or less the same class of optics that has been taught at MIT for the last 10 years. It's actually an improved version of the class. And it is being broadcast to Singapore at a very inconvenient time for everybody involved, both in Singapore and for us. So I think it is fair.

And so this is a summary, a little bit of what I just said about the instructors. We also have the two assistants. Primarily, you'll be dealing with Kate, those of you who are here. She's my assistant at MIT. And for those of you in Singapore, if you need sometimes to turn in assignments, or desperately get hold of me, or whatever, [? Deana ?] is your contact. She's in block S16.

But anyway, Singapore uses English as an official language, but as you will notice throughout the class, there's some differences. For example, in Singapore, we don't say "building." We say "block." So there's also some other subtle differences in Singapore English.

OK, we will seldom give any handouts in the class. Everything you need is in the website. I have listed there link over here. And when you log in there, you will hit two, three things, actually. You'll see the syllabus has been posted, a set of policies has been posted, which says some obvious things like don't copy, don't cheat on exams, and stuff like that. And also, the first lecture has also been posted. And I will tell you a little bit about how we deal with the posted lectures and so on.

And these are some administrative details here that I'm not sure if I want to spend any time on those. For those of you who are Course 2, if you're undergraduate, graduate, it meets your restricted elective requirement. If you're graduate, it prepares you for one of the qualifying exams offered in the department.

And a little bit about what the class covers. So the class is very introductory about light phenomena and how we design optical systems. And so this is a bunch of pictures I stole from various websites. It shows a rainbow. It shows the galaxy that we can capture with advanced telescopes. It shows-- do I have a pointer? I'd better just use this for now. Well, I guess I cannot use.

Anyway, so it shows a cell over here captured with a microscope-- which, I believe, is a confocal microscopes. So it is Professor Sheppard's expertise. And this is the picture of the esophagus of a person captured with an endoscopy. So they basically lower the fiber bundle inside, down someone's throat. And using a technique called optical coherence tomography-- it is an optical imaging technique-- they capture the esophagus.

And so finally, what you see here is, of course, an optical disk, if you're not familiar with those. And over here I believe that's a holographic setup. It is a diffusion screen, and this is supposed to be a setup for a three-dimensional display. When you look at it, it creates the illusion of a three-dimensional projection.

So if we had this, for example, in Singapore, then I would, in principle, appear as if I'm standing over there. This, of course, is still at the science fiction stage, but a lot of people are working on it. And in fact, nowadays, some companies, they offer a three dimensional television sets that you can buy for a small additional amount. I think it's a few hundred dollars. Samsung offers a 3D TV set.

Anyway, so the point of this slide is that there's many applications of optics that are interesting in engineering. And, of course, light phenomena are interesting in their own right. So the class will try to balance curiosity-based, more of a science approach, where we learn basic facts about how light behaves, how it propagates, how it interacts with matter, and we'll balance those with engineering applications that are, presumably, of interest to many of you.

And for issues that have to do with the expertise of both of us, both of the instructors, we will concentrate most of the applications on imaging. So we'll be dealing a lot with things like microscopes and telescopes primarily. These the major imaging instruments. But we'll also cover some other things for fun.

For example, we'll talk about the human eye. We'll describe its structure, its biology, how it works-- not in great detail, because this is, after all, not a biology class. But just for fun, because of the human eye turns out to have some very interesting optics inside it. And also, we'll discuss very briefly the eyes of insects, which, as you can see from the picture and you may already know, they're very different than the human eye. And we'll discuss a little bit why they're different and how each one of them operates.

And finally, some other optical imaging systems that look very surprising. This picture over here is an instrument called the Very Large Array telescope, or VLA. It is located in Socorro, New Mexico, here in the States. And it is composed of 27 antennas.

Each one of these little white things is an antenna. It's about 27 meters tall, and the diameter of the instrument, this branch over here, is approximately-- well, it varies. I don't know what it was when this picture was taken. But it can between 3 miles-- the diameter that is the size of Cambridge, Massachusetts-- and 15 miles-- that is the size of Washington, DC.

So that is an instrument. An optical instrument, if you can believe it, has the size of an entire state in the US. And it is used to observe very remote galaxies with high resolution. So it falls a little bit outside the scope of the class, because it uses it uses statistical optics, which we don't cover over here, but we may mention it in passing a little bit later.

So this is, of course, among other things that you may have seen. But actually, it cannot be done. This is Luke Skywalker. I'm old enough to have been around when this movie came out, *Star Wars.* And I just want to point out that a lightsaber can actually not be made. It is one of these impossible things that you can see in science fiction, but it violates some physical principles. So unfortunately, it cannot be made, but anyway, it is still fun to think about.

OK, so I think I covered the class objectives. And we have to balance here physical intuition with engineering understanding and design. So we'll cover the fundamentals in pretty good detail, and we'll also cover some applications in pretty good detail to the degree that we can within the scope of a semester. So we'll try to be to be very careful about that, how we balance the two. Sometimes the two can compete, right? So we'll try to balance the competition between basics and application. And also, we'll cover some applications.

And I mentioned, primarily, we'll deal with microscopy and, also, some topics such as telescopes. There are some other topics that I will not really cover in class, like optical data storage. But as I will mention in a second, there is class projects that are like mini research projects that you will do later in the semester. So you're welcome to pick topics out of those if you are interested.

Some basics about the sort of prerequisites. This is the basic math and physics that you need. I think most of you have covered these topics in your-- at least on the MIT side, you covered those in places like 18.03, and 2.004, and 8.02. So I assume that all of you who are undergraduates have taken those classes. And there is it two textbooks. So the class is a little bit expensive, sorry about that. There's two textbooks, [? Hecht ?] and Goodman. We'll be using, throughout the first half of the semester, mostly [? Hecht, ?] and then, throughout the second half, mostly Goodman.

But the textbooks-- we did not really cover the sequence of the textbooks. You should use them primarily as a reference. All of this stuff is posted, by the way, on the website. So you don't need to write down the book names. But anyway, so the textbooks are reference. Your primary sources are the notes and what you do in class. And then you can go back and read them. And there some other texts that are useful if you have access to them through the library, and so on.

Some other administrative details. This is the grade distribution for the undergraduate class, 2.71-- 30 homeworks, 30 quizzes, 40 final. We have eight homeworks. And the homeworks are due-- actually, they are due nine days after they're posted. So you have plenty of time to work on them.

And also, Pepe will strategically schedule his office hours so that you can ask questions before the homework. And we'll see how to do the office hours for the Singapore students. We'll work something out when I get there. Anyway, the first homework is not due until February 18, so there's no need to panic about that yet. And the homeworks will also be posted on their website.

And the 7.10 is very similar, but the 7.10, the graduate version, also has a project. So the project also counts for a significant portion of the grade. And I think I already mentioned-- this is like a mini research project where you either give a short lecture on a hot topic in current optics research, or you pick your own topic, and you do a little bit of calculation or some simple thinking. It's supposed to be a class topic. It's not a [INAUDIBLE] or anything, but like a mini research topic.

In fact, there are a couple of people already asking me if they can do something related to their current research. And that's perfectly fine, but you have to sell it, because this is a team project, right? So if you want to do something related to you research, you need to recruit two, three, four, other colleagues from the class and form a team around it. So it's up to you. And I certainly encourage, actually, research-related projects in the class.

So this is only for 710. If you are enrolled in 701 and would like to be involved in this, you are welcome to do it, but to be fair, you cannot get credit. However, you can be undergraduate and enroll in the graduate version of the class. So for example, if you are planning to stay on at MIT for graduate school, that's not a bad idea, because the class is H-level, so it will already count you towards some of your credits later. So anyway, that's something that we can discuss separately, if you like.

And finally, the ugly side, we do have quizzes, and exams, and all this stuff. I hate them myself. I hated them when I was a student. But it's a little bit like the dentist, right? In You hate it, but you have to do it. So this is the distribution of the quizzes and the final.

One important thing I would like to emphasize, which is, I always deliver a small sermon when I start a class, and that has to do with asking the question. I really think that you get the most benefit not from listening to me while I lecture, especially at 7:00 AM, you know, when, including me, you're all sleepy. And I will be sleepy in Singapore, too, because it's going to be late.

Anyway, so you don't get the most benefit out of that. You don't get the most benefit by reading your book alone at home in your bed and so on. The most benefit you get is actually from participating in discussions in the classroom-- with your peers, with the instructors, everybody.

So I would like to encourage you to not hesitate for any reason whatsoever. If there's something that bothers you, some question, something you're not understanding, something you're uncertain, or whatever, please do ask. There's no reason to be shy. Very often people-- myself included-- if you're in a big audience, you may be reluctant. Because you say well, gee, what if my question is not good, or what if I embarrass myself?

So there is no such thing, OK? If a question pops up in your mind, the probability is very high that someone else in the class has the exact same question. And this person is equally shy as you are to ask that question. So you do yourself a favor, and you do many of your other classmates a favor, if you just interrupt me and ask a question so please do that. And also, on my side, I will treat all questions equally, and I will do my best to answer every possible question.

Sometimes I might not answer a question if I don't know the answer myself. This happens very often in my classes and, I think, in everybody's classes. Always, someone can come up with a question that I don't know the answer. If that happens I'll tell you, sorry, I'll get back to you in the next lecture, right?

Anyway I think, it will take a few lectures. Usually, in my classes, it takes a few lectures for the students to overcome the threshold. But I think it becomes very productive, actually, when you engage in discussions in the class. And yeah, don't worry about falling behind on the syllabus or anything. It is much more important that we learn well what we do learn than that we cover a lot of material and, in the end, nothing has been left in your minds.

So one of the benefit of discussions, especially arguments, if we get into an argument about something, then everybody will remember, right? Because it's kind of fun to watch people argue a topic. So yeah, by all means, please interrupt and ask.

And we don't have recitations, but as you noticed, the class has a slightly unusual schedule. You have a two-hour lecture on Wednesday and one hour on Monday. So we'll structure the syllabus so that most of the material is covered on Wednesday and most of the examples, and practice, and so on, they happen on Mondays. So this is how we deal with the recitation issue.

And, of course, there's also the office hours. And-- oh, yeah, and sometimes, we cover some mathematical topics that some of you may have forgotten or may not be very up to speed with, especially Fourier transforms. So when the time comes, we might do a special lecture in the evening, MIT time. And if we need to do that in Singapore, we'll do it, actually, individually with me, sort of in my office or something like that.

Those of you at MIT, you'll probably do it with Se Baek or Pepe, come back one evening after 7:00 PM, so we obey Institute policies, and do a math review. So this may happen once or twice in the semester, especially when it comes to Fourier transforms. Because my experience. You need the logistics of Fourier transforms in order to follow a significant fraction of the class. So if you are not really up to speed with your basic Fourier from wherever you learned things 18.03 or 2.671, and so on, we'll do that for you.

OK, this is a list of topics that we'll cover in the class. You can browse them in the website. I will not go through each topic now, because I don't want to give it away. I want to leave an element of surprise. But basically, the class is divided into geometrical optics and wave optics. And we'll start with geometrical optics for about four weeks, and then we'll move to wave optics.

So the difference, it really has to do with approximations of when we deal with light phenomena, it's actually very complicated how light interacts with matter, and how it propagates, and so. It is really a horrendous problem. But over the years-- over the centuries, actually-- people have come up with different approximations of progressive accuracy. So geometrical optics is the simplest approximation that gives you very simple formulas, very simple math, and actually describes light quite well up to a point.

So we'll do that first, and then we'll graduate to wave optics, which is a little bit more involved mathematically, but it also gives better approximations about the propagation of light. And then, finally, in the very last few-- probably in the last lecture, and only if we have time-- we'll cover a topic called sub-wavelength optics, which is an even better approximation. But that is actually pretty much impossible to do analytically. So one has to go onto a computer and do numerical solution of a nasty set of coupled differential equations.

So we'll do that not in the computational way, but we'll cover some of the related phenomena and what this approximation gives. But roughly, anyway, the class is structured according to these approximations. Any questions so far?

And if think of a question, you can also interrupt me, right? You're always welcome. Well, and one more thing. Yes, I'll tell you the one more thing, perhaps, later.

Let me start with a little bit of history. This, of course, is not the subject of the class, but it's kind of fun to do. I used to start my classes with a joke that not everybody got. I used to start by saying that optics is the most ancient science. Not the most ancient profession-- that's something else-- but is the most ancient science.

And I think the reason is because humans are very visual animals. Our vision, for those of us who are lucky enough to have it, our vision is one of them most dominant census. So people got interested in phenomena involving light relatively early on in the early ages. And this probably happened across civilizations-- Chinese, Egyptian, Western Greek, and so on.

But as it happens, the Greeks were the only ones to publish. They were open about what they were discovering. The Egyptians and the Chinese, the priests kept everything under control. So we don't know much about what they did. But from what we've discovered, archaeologists have discovered in ancient tombs and so on, they also knew quite a bit about light.

Of course, the Greeks made the major mistake-- so Greek science was very interesting because it was the flip side of modern science. The Greeks had the attitude that you can understand nature just by thinking. So the Greeks actually discouraged experiment. Strangely enough, this tradition has followed the Greek psyche, because if you look at the faculty in the mechanical engineering department, there's a lot of Greeks, or professors of Greek origin, and lots of them are theoreticians. Anyway, I'm just joking.

But anyway, the ancient Greeks, they have this attitude that you don't need to do experiment. In fact, you must not do experiment. You have to understand everything by thought.

So because of that, they came up with some strange ideas. So for example, the Greeks, they thought that when you look at something, your eyes emit some substance, which they called simulacra. And this absence is their thought of what light is. So I look at you, I transmit a substance. The substance comes back to me, and that's how I see you. Which is, of course, a very bizarre, bizarre way of thinking. But anyway, that's what they thought.

And it was the Arabs, much later in the 10th century or so, who read the Greek script. And they said, well, that doesn't make any sense at all. So the Arabs, for the first time, thought, well, it must be the other way around. There must be light sources, like the sun, or a fire, or-- that's about it at the time. They didn't have light bulbs yet. So it must be that they emit something that is called light, and that's how we see.

So it took us about 1,000 years, I guess, to resolve that question. And also, the Arabs did a lot of the very first basic work in optics. For example, Snell's Law, the law of refraction that I will cover in a little bit, the Arabs discovered it first.

And then, much later, another 400 years later, Descartes, he did two things. He put the basic foundations of science-- Descartes, you may know this, Descartes was the first philosopher, I should say, who flipped the Greek point of view. And he said that it's actually the other way around. Science must follow experiment. Instead of just thinking about nature and explaining it, he said that it's the other way around-- science must be driven by observation. So we observe something, we create experimental conditions to test it, and then we come up with a theory that tries to explain the observation-- not the other way around.

And of course, modern science still follows that principle. At least hopefully, right? Because there have been occasions of cheating and so on. You probably see those in newspapers. I think there was a guy who invented, who sort of created data at HP Labs. Was it HP Labs?

Anyway, so there are some people who violated principle. But hopefully, 99.9999% of us will actually follow Descartes, right? We make an observation, we report it faithfully, and then we try to explain it to the best we can and in the simplest way that we can. That's the scientific method.

But also, Descartes, as it turns out, worked on optics. And he also derived Snell's Law in his own way. And there's something called the Descartes Sphere, which was invented independently by the Arabs in the 11th century or 12th, I forget-- Ninth, actually-- and then Descartes reinvented it about 400 or 500 years later.

Then, the next major advance in optics came from Newton-- actually, Newton and Huygens, who tried to explain light in two different ways. Newton was of the opinion that light is a bunch of particles that travel in air. And he tried to explain various phenomena like refraction from a prism, and so on, based on this idea. Huygens thought that light is a wave very similar to water waves. They did not know about-- I believe Newton also postulated that sound is a wave. But for some reason, Newton thought that light is not a wave. In fact, the two of them fought. I don't know if the fought over it, but they disagreed over it.

But I guess because Newton was more famous-- he was a professor, a Lucasian professor at Cambridge-- so Newton's view actually prevailed for several years, prevailed. And the particle theory of light was dominant until about a century later when people experimentally-- again, here comes the scientific method-- people experimentally observed phenomena like interference that would only be described if light were a wave. So the particle theory got a big hit then, because it could not explain diffraction, interference, and so on, and so forth. But yet, people were observing them in the laboratory.

Of course, now, after one more century, people discovered that, guess what? Both theories are correct. With quantum mechanics, light can be thought of as both as particle and as a wave. And in fact, Einstein and some other scientists-- Schrodinger, Planck, and so on-- they reconciled the two points of view. Not perfectly-- there still some puzzling aspects of the quantum theory of light. But I think nowadays, most people are comfortable with the idea that you can use both approaches to describe light phenomena. You simply select the ones that best suits your approximations and your conditions at any given moment.

So for example, if a particle approach is sufficient to describe the phenomenon, then you use it. And of course, there's also, typically-- or, not typically, always-- let me restate. There had better be an equivalent wave description of whatever phenomenon you're describing, but it may be more complicated, right? So in that case, you use particle theory. Or the other way around. If it is easier to describe something as a wave, you opt for the wave description.

So of course, in this class, we don't cover quantum optics at all. Professor Shapiro in the electrical engineering department offers a class in quantum optics. And I suppose that there is also a quantum optics glass for those of you who are interested. It's a very elegant topic.

So the other major advance in optics came in the middle of the last century, when the laser was invented and the technique called holography called holographic was invented. Not because holography is somehow dominant in practice. I mean, you see holograms typically in museums. It's no big deal. But holography turned out to be a very interesting mathematical way of looking at optics.

Now, that really had a major impact in the subsequent development of optical science. So for this reason, both of these inventions led to Nobel prizes. They were very major advances in the field of light. And especially after the invention of the laser, optical science had a huge impact on everyday applications.

If you think about devices you use in your everyday life, every time you pick up a telephone or you use the internet, there's typically-- especially if you use it long-distance-- there's some optics involved. Because the signals propagate through optical fibers, at least in part of the telecommunications network.

If you're unlucky enough to have surgery, there's many kinds of laser surgery. Lots of clinical medical diagnosis is done using high-end microscopes, including confocal microscopes, optical coherence tomographers, and so on, and so forth. These are all commercial instruments.

There's applications in industry-- for example, laser cutting, laser welding, laser metrology that is used in high-end precision engineering applications. And of course, finally, every time you pick up a computer, the chips that are all made using optical lithography, which is a really highly sophisticated form of optical imagine.

And I use the term "optical" here in a very general way. Most of it is really optical. We use light. In some really extreme, high-end applications, they use electron lithography. But still electrons, behave like light when it comes to this, to this scale. So basically, the same equations that we use to describe light, they use them to describe imaging by electrons. So that is really a huge, huge, huge domain of application.

It still remains a very active scientific field. So if you look at the list of Nobel Prizes-- this is a very incomplete list that I compiled from the Nobel website-- even that latest Nobel Prize that was awarded-- in chemistry, actually; it was in the field of optics-- these fellows, they invented something, green fluorescent protein, which is-- I may embarrass myself now, because I don't understand the biology of it,

But my very simple understanding is that they can genetically program this protein to get into some animals' DNA. So they basically create animals that have this protein embedded in their genes, and then these proteins can also be designed to turn itself on or off depending on happens with the animal. For example, if the animal is exposed to a disease, or if it is exposed to a certain chemical agent, or anyway, whatever is of interest to the particular biological experiment, it sounds kind of funny, but the animal becomes fluorescent. Or more interestingly, certain parts of the tissue of the animal-- for example, the liver, or some tissue of interest-- becomes fluorescent. So then you can pump the animal with a laser beam, you can measure the fluorescence that is coming out of the animal's tissue, and then you can derive conclusions about what happened to the animal.

So this is a fantastic way of studying genetics, studying diseases, studying a number of different and very important biological phenomena. So for that reason, these fellows were awarded the Nobel Prize. And actually, many people, including myself-- at least not yet, but I'm sure in Colin's lab, already, we'll use animals that are genetically modified with this-- It's called GFC, green fluorescent protein-- to study various biological phenomena. Now it is a very commonly used technique in microscopy. So this was the most recent optics-related Nobel Prize.

There's a bunch of others. My favorite is actually-- where is it? This one. In 1997, this was a given for optical traps. So an optical trap is actually a way to move particles by using light. It's a very surprising thing, because none of us in everyday life will experience mechanical force from a light beam. But in actuality, there is one. If you are sitting in the back of the light, you are feeling the force. This once is very weak. It's in the range of femtonewtons, typically-- a really tiny, tiny force. But we're also very big, so the force is not enough to move us.

But if you're really tiny-- like a cell, for example-- the force, especially if you design the optical right with a very highly focused beam, you can boost that force to the range of, perhaps, a few piconewtons. Not really that's a very high force, but anyway, in that order of magnitude, it can be enough to actually move a particle. So you can make, you can apply mechanical forces using light. So this was another Nobel Prize.

So anyway, the reason I'm bringing this up is because this is a very exciting field. At least-- well, I'm partial, because I work on it, but it's a very exciting field. People come up with clever, crazy inventions all the time. And many, many of these inventions have-- usually, they have a very high impact.

And so it is interesting to see both sides of the coin, both the science side of the coin that is purely curiosity-driven, and very often people in government question it, because they say, well, why are you guys doing all this crazy stuff? Who cares about optical forces?

But of course, these people are so excited, because history shows that most of the time, these curiosity-driven discoveries, they end up having a huge impact in everyday life. Some crazy persons-- they were French, American, and Chinese-American, right? Three crazy persons thought about focusing light to move particles. Then, all of a sudden, this is used in biological research to try to understand diseases like malaria.

I don't know if any of Professor Subra Suresh's students are here, but one of my colleagues, Subra Suresh, studied malaria using this technique that won the Nobel Prize in 1997. So it is our duty as engineers, or scientists, or whatever the case may be to emphasize to the people in government and politics that yes, there is value in fundamental science when they bash it and they say that, what, you guys are playing in laboratories, and so on.

OK, so I went onto my tirade. And any questions?

AUDIENCE: I've got one.

GEORGE BARBASTATHIS: Yes.

AUDIENCE: So I think I'm on.

GEORGE BARBASTATHIS: Yes, can you hear him? OK.

AUDIENCE: It says that you can describe how an electron beam images in a similar way as optics. But electrons interact with matter. So how does light interact with matter?

GEORGE BARBASTATHIS: Yes. So we will cover that. Of course, they interact in very different ways, right? The fundamental difference is that electrons are fermions, so they cannot really be in the same state. Photons are bosons, so they can actually be in the same state.

So what I really should have said-- and thank you for pointing it out-- is that in free space, they're described by the same equations. Of course, when they get inside matter, their behavior is quite different. But again, for example, if you look at the electrons that go ballistically through matter, they experience an effect that is very similar to refraction. So you can describe-- you still see Snell's Law, and so on. But of course, you see these additional phenomena, like all the electrons, which you don't see in light beams.

So you're absolutely right. There's some significant differences which are very important. But there's also some very, very dominant and prominent commonalities. The same goes for light and sound. And our department recently merged with ocean engineering. And in ocean engineering, there's a lot of professors who do acoustics.

So as a result, I started sitting in-- at the beginning, out of curiosity, I started sitting in a couple of the acoustic classes. And also, this year, I sat in the doctoral exam in acoustics. And I was surprised to see the same terms-- diffraction, refraction, Snell's law, waveguiding, all of these things, they happen in acoustics as well.

So you could say the same thing about sound. To some approximation, sound effects are identical to optics, identical to light diffraction. But there's also cases of interaction between sound and matter that is radically different than interaction between light and matter. For example, I think it is impossible for sound to ionize matter. Light can ionize matter. I think sound has to be pretty darn strong to ionize, right?

So there are significant differences, but also some very convenient commonalities. So all of a sudden, by studying one field, all of a sudden, you discover that you can understand quite a bit about a different field. So that's kind of useful. Any other questions?

Let's start by saying a few things about what is light. So light is actually a form of energy. Really that's the simplest-- that's the only correct way to describe it. It is a form of energy that is transmitted as an electromagnetic wave. That's a quite correct description.

But as I said before, you can think of it either as particles or as waves. So the particles are officially called photons. And what is a photon is actually not an easy thing to describe. Various scientists over the centuries fought over the definition of a photon. And we certainly don't want to go into quantum optics in this class.

So we'll think of photons in a very simple-minded way, as bullets that carry energy-- a very small amount of energy, as we'll see in a second-- and they follow certain trajectories. So the trajectories we'll call rays. And I will describe these rays a little bit later.

Now, the photon, one thing that the photons do have in common is their speed. It is, of course, the speed of light, which, in a vacuum, is the familiar 3 times 10 to the 8 meters per second. How much energy they carry? Well, the amount of energy is given by Planck's constant, which is a very small amount of 6.6 10 to the minus 34 joules times second, and then multiplied by a frequency.

OK, so the frequency is, of course, hertz. So the units work out. The product over there is energy. What is the frequency? Where does it come about? Well, to really justify the presence of a frequency there, I have to see it. I have to go, actually, to the other way of describing light, which is as an electromagnetic wave.

And of course, the name "wave" implies some sort of oscillatory motion. So the horizontal axis here is the direction of the propagation of the light. So the light is propagating from the left to the right. What is the vertical axis? The vertical axis is an electric field, actually. It is the same spot that you have in a capacitor where you charge it.

So it is convenient to describe it as an electric field. You can also describe it is a magnetic field-- the same stuff that you see when you have a refrigerator magnet. You can put either quantity over here in the vertical axis because there are a couple of electromagnetic fields by, as the name suggests, it is a coupled oscillation of electric and magnetic fields. For now, let's stick to electric fields. In this class, I will say very little about magnetic fields. When I describe light as a wave, I will by default refer to an electric field, OK?

So light is an electric field that oscillates as a function of position. And of course, a wave is not static. You, all of you, have seen waves in the Singapore Harbor, the Singapore River. You cannot see waves on the Charles right now because it is frozen. But during more normal times, you can see waves on the Charles. So you know a wave implies both a spatial structure-- if you look at the picture of a wave, you see oscillatory in the picture-- but also time, because a wave travels in time.

So in the context here, the time valuable-- well, OK, I'll [INAUDIBLE] that back. But after some time lapse, the wave will actually move a little bit further, OK? So this is the sense of the wave propagation.

So since you have an oscillatory quantity here, the period is called the wavelength-- the period in the space domain. So the distance between two peaks of the electric field oscillator there would define it as a wavelength. And it is related to the frequency, this quantity that enters in the particle description of light uses this equation here. The speed of light equals the product of the wavelength times the frequency.

So we'll do the calculation here. Well, before we go calculation, I need to say something about what is-- I think I did something I was not supposed to do here. But anyway, the blackboard went up by itself. But this classroom is highly automated. So I guess there's some things that cannot be done.

OK, so what are the typical wavelengths? So the electromagnetic spectrum actually spans all wavelengths from sort of infinitely long, or kilometers long, to very, very short, down to nanometers. The visible light, the light that we see with our own eyes, is in this range over here between approximately 650 nanometers or so and 450 nanometers or so.

So what's a nanometer? It's 10 to the minus 9 meters. So let's pick a convenient number here. Let's say lambda equals-- how about I do this here? OK, this is the wavelength. So our other question is c equals lambda nu. So it means that nu equals 3 times 10 to the 8 over five times 10 to the minus 7. So this is something of the order of 6 times 10 to the 14-- what? Hertz, right? It's a temporal frequency.

So this oscillation that I showed before, it's a very high-frequency oscillation. is in there in the order of 10 to the 14 had now we don't listen to the radio anymore we listen to you know podcasts or our satellite and so on. So we're not very familiar with radio frequencies. But I'm old enough to remember when you tuned your radio to 104.3 Megahertz. That happens to be Boston's WBCN station.

So OK, Boston's WBCN station emits at 100 megahertz. It is actually the same stuff-- I'm going to force the machine to do what I want it to do, so I'm going to keep this down. OK, so let's say nu equals 100 megahertz. That is 10 to the 8, correct? So therefore, the wavelength now is what? c over the frequency so it is 3 times 10 to the 8 meters per second over 10 to the 8 hertz. So this is now 3 meters.

So it is still the same stuff. It is still light, if you wish, but of a much, much longer wavelength at the radio frequencies. So you can see that electromagnetic waves can span a very broad range of scales. In this class, the wavelengths of interest are in this range between the dashed lines and the infrared, which nominally ends at about 10 micrometers, and the ultraviolet, which nominally ends at about 30 nanometers.

Where do the names come from? "Infra," in Latin, means "below"-- below red. So therefore, the term "infra" refers to what, the frequency or the wavelength? The frequency, right? Infrared has longer wavelengths than visible, and therefore it has smaller frequencies. So it is below the red in frequency. "Ultra," of course, means higher, also in Latin, so "ultraviolet" means higher frequencies than violet light. Not "violent" light, but "violet" light.

OK, and the major difference, if you look at light propagation in free space it doesn't really matter which wavelength you are considering. But of course, the interaction with matter is radically different as you change wavelengths. So it is similar to the question you asked about electrons. It is also true for microwaves, or even for electromagnetic waves themselves. The way visible light interacts with matter is very different than microwaves and RF waves, and it is also very different than X-rays.

So X-rays are actually the next highest in frequency after ultraviolet, and even higher in frequency are gamma rays. And I guess we stop there. But actually, we don't stop. The frequency can go, in principle, all the way to infinity. But the gamma rays is the highest that we can observe.

Now, I promised to do a calculation of the energy-- how much energy is carried by a photon. So remember the formula, equals h nu, where h is 6.6 times 10 to the-- remember how much it was, the exponent? 34, right? So let's pick one here-- let's say this one-- which is visible wavelength. So it is six times 10 to the 14 times inverse second.

OK, so these conveniently cancel. And 6 times 6 is-- let's call it 10 for convenience. So it is 10, and this is 14, of course. So this is 10 to the minus 20 joules. OK, my arithmetic is, obviously, wrong. OK, what is the pedagogical message here? That when we do order of magnitude calculations-- 6.6 times 6.6 times equals 10, let's-- actually, I got it wrong. I should have put "100." So minus 19.

OK, so just the order of magnitude, right? I'm not looking for the exact answer here. Actually, it's very interesting. I did my graduate in Greece. And over there, the professors were very careful. So here, they would have written 39.6, or whatever it is-- the actual number. Then, I went to graduate school at Caltech. And I took my first class in quantum electronics by a fellow called Amnom Yariv, who is pretty well-known in the field of lasers.

So we walked into classes, and he started doing things like that-- that pi equals 3, pi squared equals 10. And at first, I was horrified. But then I realized he had a point-- that very often, it is pointless to do exact calculations if you're looking for an order of magnitude result.

For example, is the bullet going to crash into a wall, or is it going to go through? Is it going to go back, or what? For that, you don't need the exact numbers. Of course, exact numbers are valuable in some other cases. It's kind of an interesting skill, to know when to do an half-calculation and when to do an exact calculation, and what level of accuracy you need, depending on the resources that you have, the time that you have, the nature of the answer you're looking for, and on, and so forth.

OK, so for our purposes here for now, 6 times 6 equals 100. So we'll get a very small energy, right? It's 10 to the minus 19 joules. If you go to higher frequencies, in the range of X-rays, the energy would go up by a factor of maybe a couple of orders of magnitude.

So in these kinds of frequencies, of energies, if you go up to 10 to the minus 16 joules or so, it becomes comparable with the ionizing radiation. So you see now why light frequency is very important when it comes to interaction with matter. Because visible light, the photon, each photon that impinges on an atom in a material has a relatively low energy. It can do something to the material-- we'll talk about it later-- but it cannot ionize it.

If you increase the frequency, you increase the energy of the photon, and all of a sudden, you can get ionizing effects. So these sort of calculations give you an idea of what's going on. And of course, if you conclude the energy carried by a microwave photon, it will be several orders of magnitude lower-- I think something like five or six orders of magnitude smaller.

OK, any questions about that? About photons or-- Let me say a few things about wave propagation. At the beginning of the class, we'll do geometrical optics. But I want to say a few things about waves that we need before geometrical optics begins to make sense.

So the first things that we learned is wavelength and frequency, right? These two things are important even in geometrical optics. The wavelength is a very important concept.

The thing I want to talk about is, a little bit, to show you what the wave looks like. So this is very simple, one-dimensional wave. And what I've done is I have plotted it at different snapshots in time. So the horizontal axis, again, is the propagation distance that the wave is propagating, and the vertical axis is-- well, this part of the small axis is time, and then I have captured different snapshots.

So as you can see, there's a sense of motion here. If you latch on a peak of the wave, you will see that at different instances, the peak is moving from the left to the right. And of course, the symbol uppercase T here is the frequency. I think it was defined in an earlier slide. It is simply the inverse-- I'm sorry, that is the period, the temporal period, the inverse of the temporal frequency.

And of course, after one full time period, the wave replicates itself. So if you look carefully at this wave from over here, it is identical to the wavefront at t equals 0 that I'm not tall enough to reach. So you can pick arbitrarily. You can pick any point in the wave. I happened to pick a peak. OK, I happened to pick a peak, and I tracked that peak as the wave propagates. I could equally well have picked a point over here and tracked it. It would also have propagated the same way.

And so this concept of a point in the wave, if you wish, or more generally a surface of a wave that propagates with a wave as a function of time is called a wavefront. And the term is very suggestive. It implies motion. It's like a battlefront.

What is a battlefront? It's the people who are unlucky enough to have been picked to be at the frontline of the battle. So if you've seen all these old, horrible movies about battles in the Middle Ages, you see all these guys with their shields going ahead. And that's a battlefront, moving. So a wavefront is a similar concept. You have a front that is moving as the energy of the wave is propagating.

So it's perhaps not a very interesting wavefront because it is one-dimensional, right? The wave is propagating along a sort of linear axis. In a second, I will show you more interesting wavefronts that are of relevance in this class. The other thing I want to point out here is, again, I want to bring up the concept of the wavelength.

And we defined the wavelength already as the distance between two peaks. So if you look at two peaks over here, the distance is, by definition, of a wavelength. But also, the wavelength, it has a different meaning. If you look at it from the point of view of propagation of the wave, it is also the distance that the wave propagated in-- how long? Well, one period, right? And in fact, this is where the equation c equals lambda nu comes from.

I'll let you do that as a homework. But if you think about it, if you treat the wave as a particle that took 3 seconds to move lambda units of distance, then its velocity c must obey this equation over here. It's a one-line derivation kind of thing. I'll let you do it as a homework.

And also, I want to emphasize that this equation that we called the spatial relation in the previous slide, it is only true if light propagates in free space or in uniform media. If you put light, for example, in a confined space, this equation changes, and it becomes a little bit more complicated. But in this class, we don't deal with this phenomenon.

If you want to learn about more complicated dispersion relations, you will have to take Professor Fujimoto's class in electrical engineering. I might mention something like this in passing, but not in great detail. For this, we will be happy enough to take this as a dispersion equation of the light. But again, I want to alert you that there's also other dispersion relations that may occur.

OK, the other thing that I will not spend too much time here, but it will come up later with great force and great detail is the concept of phase delay. So you probably remember this from your trigonometry class. If you have a periodic phenomenon or a periodic function, you can pick an arbitrary point in time and call it your reference. And then, as the phenomenon evolves, you can refer to this initial point as if with a phase delay.

So the phase delay is relative to 2pi, which measures one full cycle. So then you can interpret these snapshots over here as phase-delayed versions of the original wave. So for example, between 0 and 1/8 of a period, your phase delay equals pi over 4, because 2pi over 8 equals pi over 4. So this is a concept that will come up again, as I said, in great detail. I just wanted you to be a little bit aware of it right now.

What I really wanted to emphasize over here is that light does not usually propagate along an exact one line, as I showed in a previous line, but it propagates in 3D space. So it can expand. It can contract. It can do weird things, right? So to do that, we need a slightly more elaborate description.

So this is an attempt to plot 3D space. And if you think about the wavefront in this case, it is a surface that moves from left to right as a function of time. So as the wavefront propagates, the surface is moving. By the way, this is a fictitious surface. I'm not thinking of a physical surface or anything. But if you think about the energy that the light carries, the energy is actually moving as the wave propagates. This is how you can connect the two.

If you go out to the Charles-- OK, you have to wait until April when it unfreezes-- but if you look at the waves on the water-- of course, you can do it at home on your bucket or something like that-- but if you look at the waves, there's also a physical sense, because the water wave, there's a crest. So there wavefront of the water wave is the crest that moves as the wave propagates.

So these surfaces, the weapons, they can have different shapes. They cannot have arbitrary shapes because they're governed by the laws of light propagation. But certain allowable shapes turn out to be very simple, and we will be dealing with them a lot.

So the simplest is a planar wavefront that, as the name implies, is a plane. And the next simple is a spherical wavefront, which, again, as the name implies, is a sphere. So you can think those as our two major wavefronts that we'll be dealing with throughout the class-- the planar wavefront and the spherical. And we'll-- yes.

AUDIENCE: So in the last slide, you showed the electric field as being the y-axis. But here, it's space, not field.

GEORGE BARBASTATHIS: Yes, so that is correct, and thank you. And I should relabel the slide. Here, the two axes, they correspond to x and y, the space coordinates, yeah. And the electric field is not found here because I cannot plot a fourth dimension.

So what is happening here is the electric field is maximum at t equal to 0. So at t equals 0, the electric field is maximum on this plane. If you wait 1/8 of a period, the field will be maximum at this plane. If you wait another 1/8 of a period, the field is maximum on this plane. So the wavefront, in this case, is the maximum of the electric field-- the crest, if you wish, of the electric field-- as it propagates through space.

And the same is here. Again, these surfaces mean that the field is maximum on the surface at t equals 0. And then, 1/8 of a period later, the field is maximum on the surface, and so on, and so forth. Thanks, yeah. So that's a very important clarification.

Now, what do you think should happen to the energy density in the two cases? Suppose I have a fixed amount of energy that is entered in on the left. Are they different in some way? Yeah.

AUDIENCE: [INAUDIBLE]

GEORGE BARBASTATHIS: That's right. Can I can ask you to push the button and repeat that? Yeah.

AUDIENCE: The plane wave's energy is constant, energy density is constant, and the spherical wave's energy density is smaller.

GEORGE BARBASTATHIS: Correct.

AUDIENCE: As it expands.

GEORGE BARBASTATHIS: Correct, yes, because energy has to be concerned. So in this case, the wavefront is invariant, so the energy must remain constant. In this case, the wavefront is expanding, so the energy density will decrease as you go away. We will make this more precise later. We'll define what we mean by energy density. In fact, it is called intensity. So we'll define that.

So basically if you measure the energy in watts per area, watts per centimeter square, in this case, it has to decrease in order to make sure that the energy you started with at the center of the sphere is preserved throughout.

The next concept I would like to define is the ray. And the reason we introduced wavefronts is primarily because I wanted to define relatively precisely, now, what I mean by a ray. For the next 4 weeks, we will be talking about rays exclusively, OK? So a ray is basically a normal to the wavefront. That is the correct, accurate definition of a ray. You take this surfaces, you plug the normals, and these lines are the rays.

So in this case, the rays are parallel because all the surfaces are parallel planes. In this case, the rays form a fan, like a divergent fan, that attaches at each point on the spheres. The fan components are normal to the surfaces. And you can also think of them as trajectories over which the particles of light propagate. It is not, perhaps, very accurate to think of the particles as photons in this case. So think of them as some sort of fictitious light bullets that propagate down the ray trajectories.

And the rays have several properties, which I will ratify later in the class. The rays have to be continuous in piecewise differential. A ray cannot jump, for example. You cannot have a ray that looks like this. That cannot happen-- forbidden. A ray can have a continuous but, perhaps, not differentiable band like this. So that's allowed. And it can also have a smooth, continuous path. That's also allowed.

OK, that is, obviously, why it is forbidden. It is kind of strange to imagine the photon disappearing and appearing again someplace else. The others we will see later in action. And from our experience, rays are straight lines. And why do we say that, from our experience? Where have you seen rays in everyday life? Button.

AUDIENCE: Lasers.

GEORGE BARBASTATHIS: Lasers is one. But usually, if you have the beam coming out of a laser, it looks kind of like a straight line. Another example, perhaps, from even more everyday life.

AUDIENCE: Shadows.

GEORGE BARBASTATHIS: Shadows, that's right. If you look at shadows, the light appears to come out straight out of the shadows. So it's a little bit strange that I drew a ray as a curved trajectory. We will see a bit later, maybe even today if we don't run out of time, that light rays can actually follow curved paths under certain conditions.

But what is for sure, that in free space or uniform space like air-- we observe shadows in air. So air is pretty much uniform, and for that reason, rays propagate in straight lines. By the way, you don't have to go too far to see examples of curved ray propagation. The best example is flicker.

If you go up in the mountains at night and you look down at the city beneath-- I don't think you can do that in Boston. We don't have any mountains high enough. But in Los Angeles, for example, it's very pronounced. If you go to the Hollywood Hills and look down, you see flicker. The lights in the city, they are not steady. They kind of flicker.

So the result of that is because the atmosphere, it is not an exactly uniform medium. You have temperature changes, air currents, and so on. And because of that, the rays between the city lights and your eye, the rays follow a sort of curved path. Very slightly curved, but because they propagate a long distance, it is enough to result in this flicker phenomenon.

OK, so from that-- it is not quite obvious as in the shadow-- but from that, we have kind of experienced, all of us, that the rays might actually not propagate along a straight path. And in a second, I will define when that happens-- when a ray can propagate in a curved trajectory.

Before I do that, I already implied that the reason rays might do strange things, like deviate from the straight and narrow-- I'm sorry, might deviate from the straight path-- is because of interaction with matter, right? It is the non-uniformity in air, in my earlier example, that caused the rays to bend.

So then, the next topic is a very simple description of how light interacts with matter. And for now, I will say it is a fairy tale, because we don't know enough electromagnetics yet. I will come back to this topic after we define light as an electromagnetic wave. I will come back to this topic and tell you more rigorously how light interacts with matter.

For now, very briefly, I will tell you that there is three types of interaction that we'll discuss in this class-- absorption, refraction, and scattering. So today, I will define absorption and refraction in very simple phenomenological terms. Anybody knows what it means, "phenomenological"? I am Greek, so I have a benefit of the language. "Phenomenological" means based on observation. It means we define this phenomenon based on what we observe, but we don't try to dig any deeper into the basic principles behind this phenomenon. You will see in a second what I mean.

Very well, we'll define absorption and refraction. And what I want to emphasize, also, is that this is not the only three types of interactions. Light can do a lot of other things. There's fluorescence, that you are familiar if you go to nightclubs where they use ultraviolet, right? People look kind of funny. That is fluorescence. There is non-linear phenomena. There's ionization that can happen. So a lot of different things that can happen, but I will not cover them in this class. So as I say, they're outside the scope of our interest here.

Going on with this, let me define absorption first. So from experience, we know that anything that travels through a medium suffers losses. Many of your mechanical engineers, you know that if you have a mechanical disturbance like sound propagating down the medium, at the exit, you see less of the energy that you put in. Some of you are electrical engineers. You know that if you run current through a device, typically, at the exit of the device, you see less current or less electrical energy. Typically, you see a voltage drop than the energy that you put in.

So why does this happen? Well, because of-- Why? Why does it happen? Yes. [LAUGHS]

AUDIENCE: [INAUDIBLE].

GEORGE BARBASTATHIS: That's right. It is conversion of energy to heat, right? And that generally is undesirable unless-- anyway, we seldom heat ourselves with-- well, unless it's the sunlight on a day like this, we appreciate the heating. But in Singapore, generally, we don't appreciate the heating, because the sunlight is too intense, typically, to tolerate.

But yeah, the fact of the matter is that there is ohmic losses, or dissipation, like you very correctly said, that cause a decrease in power. So the phenomenological law that describes this effect is exponential in the length of propagation in matter. So by "phenomenological," what I mean is that I throw this equation at you, but I haven't told you why, OK? I will tell you why later. When we do electromagnetics, I will justify why this law comes about.

And strangely enough, it is called Beer's Law. No, it's not pronounced "beer." It's pronounced "bear." This fellow was German. But anyway-- and light does get absorbed by beer, actually. And strangely enough, I did see a paper at a conference once where someone was measuring the optical properties of beer. And I'm not kidding, actually. They had their project funded by I don't know whom, to shoot laser beams through big containers of beer. And then I don't know exactly what they were measuring, but I thought it was a very cleverly conceived project. Because after you finish the experiment, what do you do? You drink the beer, right? OK.

So I said that the output energy decays exponentially as a function of the length of the medium. The coefficient that goes in the exponent is, again, very highly dependent on the material that the light propagates. Conductors-- like metals-- they tend to have very high dissipation. If you might propagate a few microns inside the metal, the light is all lost. It is all converted to heat.

So on the other hand, the electrics like glass, they can have very low dissipation. In fact, some materials, they use some special glasses in optical fibers that they usually transmit light over very long distances. In these occasions, the dissipation is in the order of a fraction of a dB per kilometer. So it is I don't know how many orders of magnitude there, around eight or nine orders of magnitude in the dissipation coefficient. And again, that depends on the way light interacts with matter.

I will say a little bit more about that later. But for now, again, take me to my word. Never take anybody to their word, by the way. But I think for reasons of organizing the presentation of the material, sometimes I will ask you to take my word for granted. And usually, when I ask you to do that, I will come back and justify myself perhaps a few lectures later, or something like that.

OK, and the other thing I want to emphasize is that dissipation or absorption depends strongly on the wavelength. And that, again, goes back to what we were saying before. Different wavelengths carry different energy, and different energies of the photons, they will interact with the matter in different ways. They might [INAUDIBLE] due to oscillation. They might set it into dipole polarization. They might set matter, they might ionize it-- whatever. So depending on who happens, you get different behavior.

So this is the atmosphere. The percent of transmission-- not exactly the alpha coefficient, but a transmission, percent transmission throughout a nominal length-- I believe it is in the order of over a few meters-- as a function of wavelength.

So we can see that it varies quite a bit even within the visible range. The atmosphere is not completely transparent. It is a little bit less transparent at blue wavelengths. It becomes, then, almost transparent at the longer wavelengths. And then-- this is infrared now-- in the infrared, you see you have some very strong absorption. The transmission coefficient goes down.

That means strong absorption here. That actually has to do with molecular resonances. And you can think of molecules as little mass spring damper systems. And the photon comes in and kicks them so it sets them into oscillation.

When that happens, the energy of the photon gets transformed resonantly into the molecule, and then you get loss. So in this case, it is a little bit more complicated than heating, than simple heating. But the net effect is still the same. You still get heating from the motion of these molecules. But anyway, that's the reason why you get these strong absorption maxima over here.

What I really wanted to get to today so that we can progress with geometrical optics is the phenomenon of refraction. So refraction, is-- is refers to, actually, a rather strange thing that, again, I will ask you to take for granted until we talk in detail about polarization. And that is the fact that the speed of light changes when light enters the material.

In this case, we're talking primarily about dielectrics, but it's also true for metals. But of course, in metals, the light doesn't go very far. So OK, the speed changes, but it doesn't go very far.

In dielectrics, the light can go quite far, but its speed is different. And again, phenomenologically-- without describing the physical origins of why-- the speed is, of course, reduced, and it's used by a constant that is known as index of refraction, or refractive index. And most books use the symbol n to denote it.

And the value of n can value a lot. In a vacuum, n equals exactly 1. So the speed of light in vacuum equals exactly c. In air, it is close to 1, within two significant digits. It's maybe 1.005 or something like that. And it depends also on the temperature, the pressure, a number of different properties over the air, as we'll see again later.

And then, typical dielectrics that we see are water, of course. So then, why is it the water interesting? Well, because-- well, water. But also because our bodies are composed mostly of water. Our tissue is approximately 70% or 75% water. So light index of refraction in a body is also equal to the same quantity, 1.3. Actually, this would be 1.33, if you really want to be more accurate. And glass-- so glass is used in pretty much every optical instrument for visible wavelengths. So in glass, the index of refraction is approximately 1.5.

OK what does this mean now, the speed of light changes? Another way to say it is that the wavelength of the light changes. And this is actually a more proper way to think of the phenomenon. When we'll do electromagnetics, we'll see that the change in speed is actually derived from that observation.

And the way the wavelength changes is it becomes shorter. So if you have a light happily propagating in free space, and then, all of a sudden, there's an abrupt interface and light enters a dielectric, the wavelengths becomes shorter. Now, what does that mean? Does this mean that, for example, if I have red light, and the red light goes into glass, the light becomes green? That's actually not what we observe, right? if you observe, many of you have seen, if you put a yellow pencil in glass, the pencil remains yellow. It does not turn blue, right? So what is a possible-- does anybody know the explanation? Yeah.

AUDIENCE: The frequency stays the same. So if you see the frequency as it comes out, it's just going to be affecting pretty much [INAUDIBLE].

GEORGE BARBASTATHIS: That's right. So very correctly, he said that the frequency of the light remains the same. Anybody want to guess why the frequency of the light might remain the same but the wavelength can change?

AUDIENCE: The speed changes.

GEORGE BARBASTATHIS: That's right. The speed, yes. That's right. So both of you are right. So first of all, they can change simultaneously, the wavelength and the speed, because of this equation. We said that the speed of light-- I mean the speed of light in vacuum, or in general-- well, in vacuum, it is related to the frequency of the wavelength by this equation over here.

Well, you can then divide the two sides of the equation by n, and the question remains the same. But that's not very physical. I did a mathematical trick. What the hell does that mean? I divided both sides by n.

That doesn't have to be correct, and this is, indeed, the dispersion relation in a dielectrical material. But you don't really know, which one should you divide? Should you divide the wavelength by n, or should you divide the frequency by n? Or maybe divide both by square root n? All of these are possibilities.

So the physical argument that allows you to decide whether to divide is what-- I'm sorry, your name?

AUDIENCE: Uh, Liz.

GEORGE BARBASTATHIS: What Alice-- Alice, right?

AUDIENCE: Just Liz.

GEORGE BARBASTATHIS: Liz, I'm sorry. So the physical argument is what Liz just said, which is that the energy of the photon cannot change. Because the photon-- well, I haven't said it yet, but the photon is a fundamental quantum mechanical particle. It cannot be divided-- well, it can be divided in some special cases.

But in the approximation that we deal with here, the photon must maintain its energy. So therefore, the frequency of the photon must remain the same. New, the temporal frequency is conserved. So if the wavelength changes because light entered into matter, then the velocity must change to compensate in the dispersion relationship.

OK, in our context here, the only thing that can happen to the photon is it can disappear. And it doesn't really disappear. It gets converted to heat, right? So when the light hits the material, it actually heats the material in discrete quanta.

Some of the photons that come from the light source, discrete quantities that are equal to-- you cannot see it over there because I erased it. But discrete quantities of approximately 10 to the minus 19 joules, one of them at a time can be converted to heat and heat the material. But you cannot get 60% of the photon energy to go to heat the material. That's not the correct way to think about it.

If 60% of the energy goes into heating the material, it means that 60% of individual photons died and gave up their energy to the molecules of the material. If you have a single photon arriving into a material, it will either survive and go through impact, or it will die and be converted to heat. You cannot have 60% of an individual photon heating the material.

OK, so because of this line of reasoning, that really requires quantum mechanics. So I cannot really justify it very well without spending a semester of quantum mechanics. But because of this line of reasoning, the energy of the photon is invalid. Therefore, nu is invariant.

So believe it or not, I have watched someone at a conference stand up and say, well, doesn't the light really become green? And that was an optics conference, so it was very embarrassing. But it's very useful to remember. No, light does not become green when it enters glass. And in any case, your eyes respond to the energy of the photon, right? Because-- well, I haven't said how your eyes perceive color yet, but they respond to the energy. So you still perceive it as red, because the energy of the photon is still red.

OK, so having said that now, having said that the index of refraction is a property of the material, I can conceive of materials where the index of refraction is a function of position. And if that was the case-- the bad thing about any measures is you have to wait for them to finish. OK, so you can conceive materials where the index of refraction is variable. So the example I gave before is the air, where, because of temperature, pressure, and so on, the index changes.

So if the index is functional position, you can define a quantity that is called the optical path length. So if you follow the trajectory of the ray-- so here is the ray, and that is its trajectory-- you can integrate the index of refraction in small, individual segments as the light propagates along the ray.

So the basic principle that I would like you to carry with you when you leave this class today is that the basic law that covers light propagation in the ray description is that this quantity, the optical path, must be preserved-- I'm sorry, it must be minimized. Of course, it has to be preserved. It also has to be minimized.

So if you have different paths-- for example, if you compare the path gamma and the path gamma prime-- the light will take the path where this quantity is minimal, OK? Now, that sounds very abstract, I know, but I will make it more specific with examples.

For those of you who are mechanical engineers-- or, more likely, applied mechanicians-- this is reminiscent of another principle that you may have learned in your Lagrangian mechanics. You can make the same, the exact same arguments about particles moving in a gravitational field. Particles moving in-- that's the reason why stars rotate around the sun, why they have elliptical trajectories, and so on, and so forth-- is because they also obey a minimum path principle.

And well, let's see it in action here. So we apply this principle-- we'll apply it in several occasions during the class, but today, I would like to apply to discover what happens to light when it arrives at an interface between two dialectics. So on the left, you have one dielectric-- say air-- and on the right-hand side of an interface, you have another dielectric-- say glass.

So what will happen to the light? Well, two things will happen. Some fraction of the light energy-- that is, some fraction of individual photons-- will be reflected. And some other fraction of the light will enter the interface, but it might enter at an angle that is different than the angle of arrival.

That portion of light that goes in, we call it refracted. So this is the refracted portion of the light. It's a little bit confusing, because the two terms are the same except for two letters. So I hope you can sort of recapture the difference. One is reflected, and the other is refracted.

So the question is, what is the direction that these rays propagate at the interface? So we will invoke the minimal path principle for that. So consider, first, reflection. And what I'm about to say applies to the electric interface just as well as a mirror, a metal or metallic mirror.

So the minimum path principle says that the light must be reflected symmetrically. So the reflected ray makes the same angle with the normal as the incident ray, because that's the minimum path, really. If you force the light to go to a different path-- for example, this way-- then you can see very easily, it is a very simple calculation to show that P, O, tilde p-prime is longer than P, O, P, OK? I will let you think about that on your selves. You can very easily convince yourselves that this is true. And therefore, the light must follow the symmetric path.

So this is rule number one. Let me skip the next slide. I'll skip a little bit, and then I'll come back to this later. But I want to first say something else.

OK. When we think about the law of-- let's say we think about the refracted light. The [INAUDIBLE] is the medium. OK, so the first thing we need to do in order to complete this calculation is to define two points from the ray. So let's say you have point p on the incident ray and then point p-prime on the refracted ray.

So let's compute this quantity of the optical path. So the optical part equals the index of refraction on the leg on the left, times the length of the day from p to the interface, and then the index of refraction in n prime on the right-hand side, times the length of the ray from the interface to p prime.

So this, I will repeat what I have on the slide. I will repeat it on the blackboard. So you have n times the hypotenuse. So my notation here is x for this distance, z for this distance. So that's the hypotenuse plus n prime, times the other hypotenuse.

So this other hypotenuse, if I call h the vertical distance between the two points, simple geometry solves that this other hypotenuse is something like that. OK, so the question here-- how do I pose the question? What is the unknown here? What have I left unspecified?

Let me start with what I have already specified. I specified theta, the angle of incidence. I specified the two points, p and p prime. And I specified the vertical distance h. And also, I should say that the z prime is also specified, because z and z prime are specified because, OK, I have specified the coordinates of p and p prime.

What is left? theta prime, or another quantity that is also left unspecified? x and x prime, right? Because the ray might go like this. All we know is that the line starts at p and ends at p prime. It can go like this. It can go like this. It can go like this. It can go like this, right? Which is the case?

OK, so therefore, each one of those has a different value of x. So how do we find x? Well, Fermat says that light must minimize this quantity. The quantity that I wrote on the blackboard and on the slide, Fermat says that it has to be minimized. And OPL, by the way, stands for Optical Path Length.

So to minimize it, I have to compute the derivative of this quantity with respect to my unknown. I manage to have only one unknown of this quantity. To find this unknown, I'd better take the derivative and set it to 0.

So if I take the derivative, I will get n times-- what is the derivative? Someone? I guess I've had more coffee than everyone, so I can still do the derivative. So it is nx divided by the square root, minus n prime h minus x, divided by the other square root, OK?

OK, give me, now, a simple observation that solves the problem right away. That's right. This quantity, x over the hypotenuse, is the sign of this angle. This angle is the same as this angle, right? Therefore, this quantity is its sine.

And the same for the other one. The other quantity over here is the sine of theta prime. So if you substitute these quantities into the derivation, then you find this relationship over here. n times sine theta must equal n prime times sine theta prime. And this is the law of refraction, also known as Snell's Law.

So officially, I think, we're out of time. But I can take a couple of very few-- I should say very quick questions, about that or about anything else. Yes?

AUDIENCE: So how is it that you can assume p and p prime, without knowing these other things?

GEORGE BARBASTATHIS: OK, so I was hoping that you would ask that. That's a very good question. So the way this problem is solved is as follows I take for granted that there is a light ray that goes between p and p prime. The reason I can take it for granted is another very basic principle that says that if I have a light source-- say it's p-- then light propagates in spherical bundles. So I will actually have many, many rays coming from p.

And if I have another point, any point out here, p prime, I will also have many, many rays arriving here from the left-hand side. One of these rays has come from p. So the question is, how can you connect these rays?

Well, the principle that allows you to connect them is the minimum path principle. The problem can become impossible. I will show you examples later where, in fact, you might have a case where no rays reach p prime. You can see that here. If I play with the numbers and make one of these-- if I play with the numbers so that one of the sines becomes bigger than 1, then the problem becomes impossible, right? So what it means is that light never makes it there.

So this way of thinking is very typical when we deal with minimization principles like Fermat and Lagrangian principles in mechanics. We assume that our particle, or our system, or whatever has followed a trajectory that connects an initial point in the space with the final point in the space. And then we try to find a trajectory that minimizes the part between the two points, whether it is optical path length, or Lagrangian in mechanics, or a lot of other different contexts where the same thinking applied.

Any other questions? OK, so I will see you-- I will see you all in Singapore next week, and I will see you all on video next week.