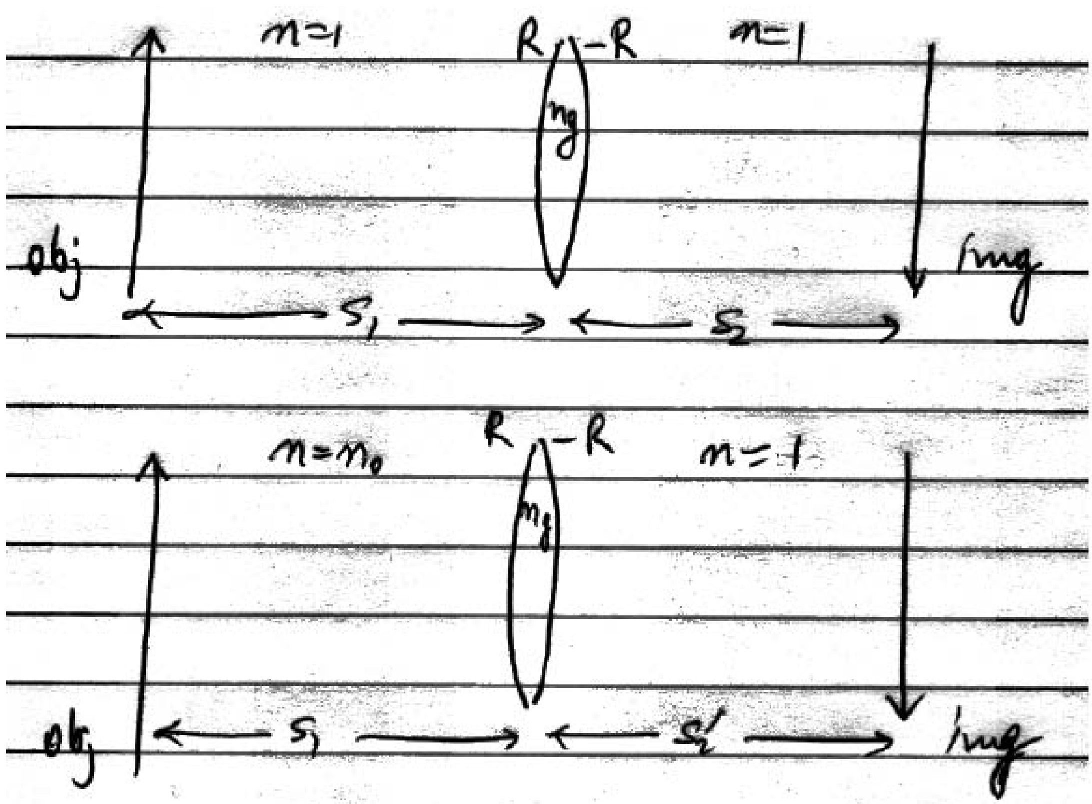
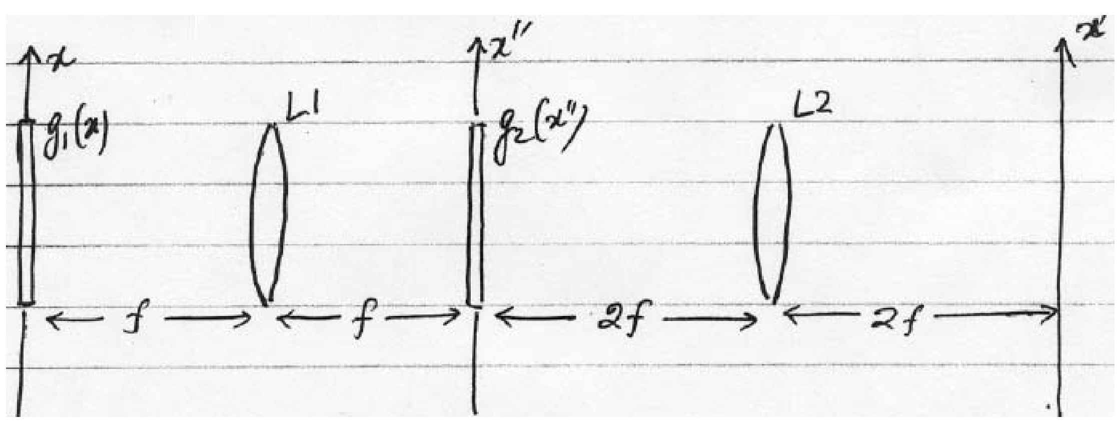
1. A thin bi-convex lens with the same absolute curvature on both faces is used in the two imaging systems shown below. In the first, both object and image are in air, whereas in the second the object is "immersed" in a material of index $n_0 < n_g$, where n_g is the index of the glass used to make the lens. Compare the two imaging systems in terms of imaging condition and magnification.

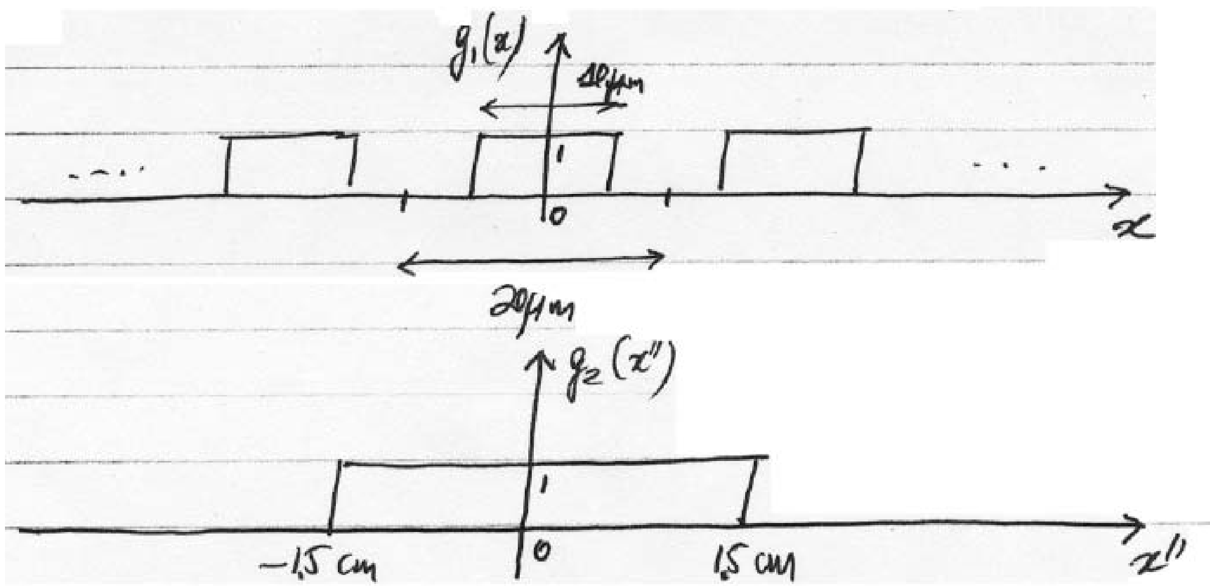


2. In the configuration below, lenses L1 and L2 are identical with focal length f, and we consider them to have infinite aperture. The system is illuminated coherently by an on-axis plane wave.



(a) Write an expression for the field at x' in terms of the thin complex transparencies g_1, g_2.

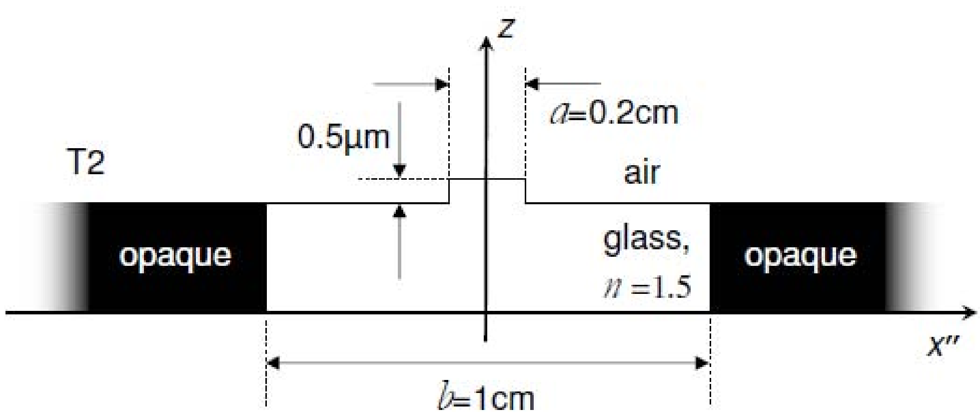
(b) Consider the specific case with $f = 10\text{cm}$ and g_1, g_2 defined as:



If $\lambda = 1\mu\text{m}$, derive and sketch the intensity at the output plane x'.

3. **Example: OTF of the Zernicke phase mask**

The thin phase transparency whose schematic is given below is placed at the Fourier plane of a unit magnification telescope with focal length $f = 10\text{cm}$. What is the optical transfer function for quasi-monochromatic illumination at wavelength $\lambda = 1\mu\text{m}$?



4. Goodman, Problem 6-10

5. Calculate and sketch the Fourier transform $F(u)$ of the function

$$f(x) = \text{sinc}\left(\frac{x}{b}\right)\left[cos\left(\frac{2\pi x}{\Lambda_1}\right) + \cos\left(\frac{2\pi x}{\Lambda_2}\right)\right]$$

Assume that the following condition holds:

$$\frac{1}{b} \ll \frac{1}{\Lambda_1}, \frac{1}{\Lambda_2}, \left| \frac{1}{\Lambda_1} - \frac{1}{\Lambda_2} \right|$$

6. A very large observation screen (e.g., a blank piece of paper) is placed in the path of a monochromatic light beam (wavelength λ). A sinusoidal interference pattern of the form:

$$I(x) = I_0 \left(1 + \cos \frac{2\pi x}{\Lambda} \right)$$

is observed on the screen, where I_0 is a constant with units of optical intensity, Λ is a constant with units of distance, and x is a distance coordinate measured on the observation screen.

   (a) Describe quantitatively two alternative optical fields that could have led to the same measurement on the observation screen.

   (b) Describe an experimental procedure by which we can determine which one of the two alternative fields is illuminating the observation screen.

7. Figure 3 below shows the schematic diagram of a simple grating spectrometer. It consists of a sinusoidal amplitude grating of period Λ and lateral size (aperture) a followed by a lens of focal length f and sufficiently large aperture. To analyze this spectrometer, we will assume that it is illuminated from the left in spatially coherent fashion by two plane waves on-axis. One of the plane waves is at wavelength λ and the other is at wavelength $\lambda + \Delta\lambda$, where $|\Delta\lambda| \ll \lambda$. (The two plane waves at different wavelengths are mutually incoherent.) Since the two colors are diffracted by the grating to slightly different angles, the goal of this system is to produce two adjacent but sufficiently well separated bright spots at the output plane, one for each color.

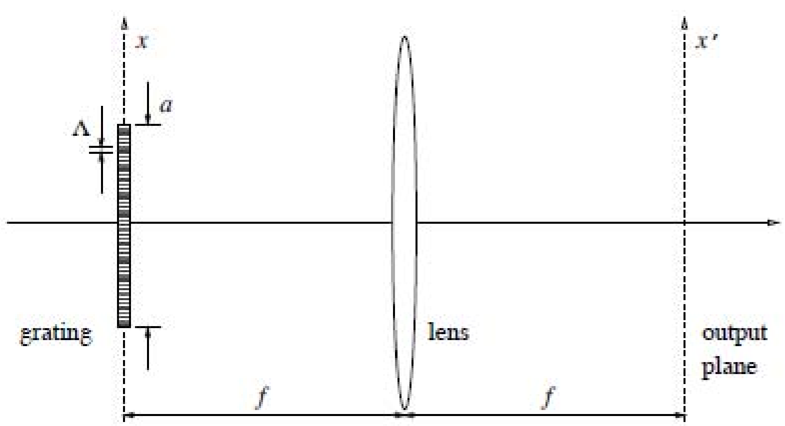


Figure 3

   (a) Estimate the minimum aperture size of the lens so that it does not impair the operation of the spectrometer.

(b) What is the maximum power efficiency that this spectrometer can achieve?

(c) Show that a condition for the two color spots to be "sufficiently well" separated is:

$$\frac{\lambda}{|\Delta\lambda|} < \frac{a}{\Lambda}$$

This result is often stated in spectroscopy books as follows: *The resolving power of a grating spectrometer, defined as the ratio of the mean wavelength λ to the spectral resolution $|\Delta\lambda|$, equals the number of periods in the grating.*

8. Consider the optical system shown in Figure 1, where lenses L1, L2 are identical with focal length f and half-aperture a. A thin-transparency object is placed $2f$ to the left of L1.

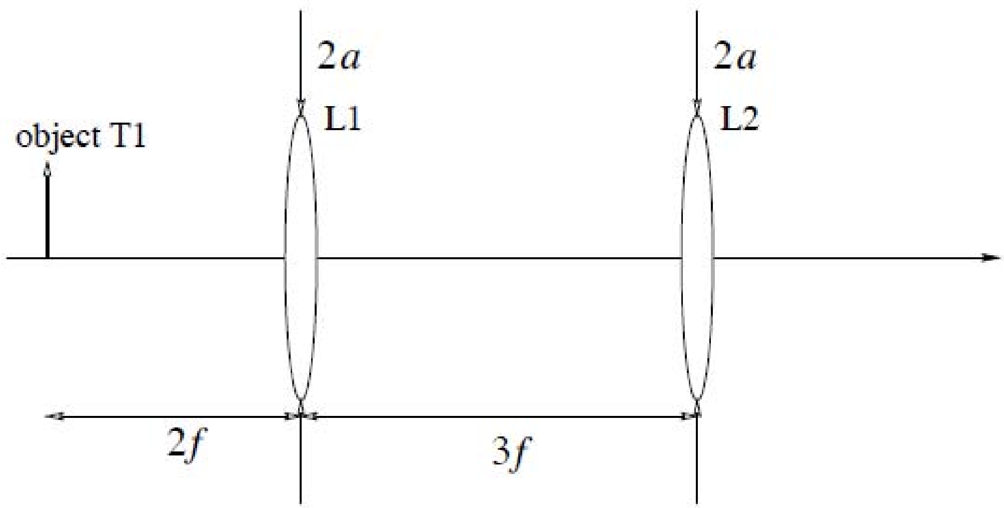


Figure 1

(a) Where is the image formed? Use geometrical optics, ignoring the lens apertures for the moment.

(b) If the object T1 is an on-axis point source, describe the Fraunhofer diffraction pattern of the field to the right of L2.

(c) How are your two previous answers consistent within the approximations of paraxial geometrical and wave optics?

(d) The point source object T1 is replaced by a clear aperture of full width w and a second thin transparency T2 is placed between the two lenses, at distance f to the left of L2. The system is illuminated coherently with a monochromatic on-axis plane wave at wavelength λ. Write an expression for the field at distance $2f$ to the right of L2 and interpret the expression that you found.

(e) Derive and approximately sketch, with as much quantitative detail as you can, the intensity observed at distance $2f$ to the right of L2 when T2 is an infinite sinusoidal amplitude grating of period Λ, such that $\Lambda \ll a$.

9. An infinite periodic square-wave grating with transmittivity as shown in Figure 3A is placed at the input of the optical system of Figure 3B. Both lenses are positive, $F/1$, and have focal length f. The grating is illuminated with monochromatic, spatially coherent light of wavelength λ and intensity I_0. The spatial period of the grating is $X = 4\lambda$. The element at the Fourier plane of the system is a nonlinear transparency with the intensity transmission function shown in Figure 3C, where the threshold and saturating intensities $I_{\text{thr}} = I_{\text{sat}} = 0.1 I_0$. To calculate the response of this system analytically, we need to make the paraxial approximation; strictly speaking, that is questionable for $F/1$ optics, but we will follow it nevertheless. An additional necessary assumption is discussed in the first question below.

   (a) To answer the second question, we need to neglect the Airy patterns forming at the Fourier plane and pretend they are uniform bright dots. Explain why this assumption is justified and what effects it might have.

   (b) Derive and plot the intensity distribution at the output plane using the above assumption.

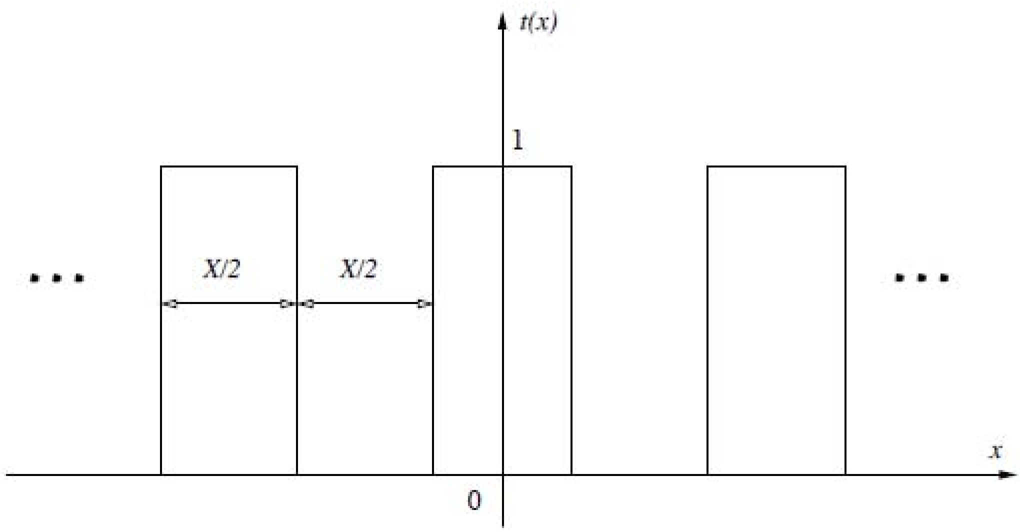


Figure 3A

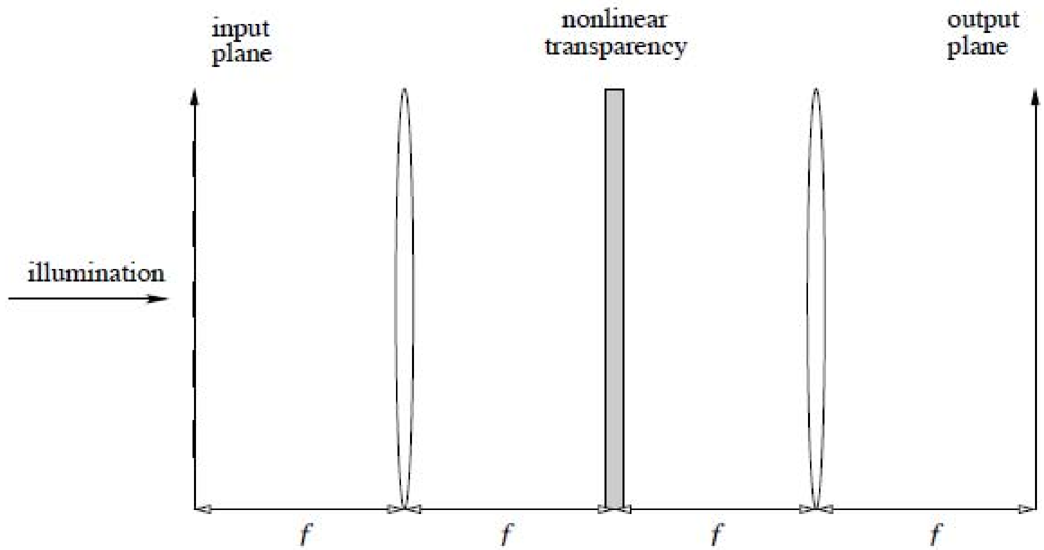


Figure 3B

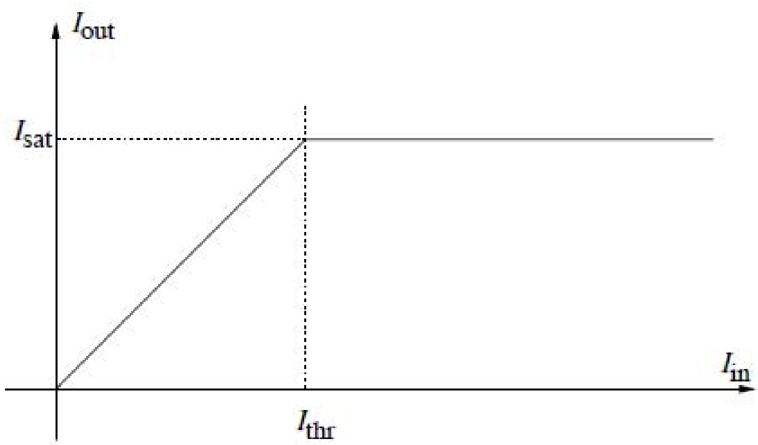


Figure 3C