2.710

Final examination

3 hours (9am–12 noon)

Total pages: 7 (seven)

PLEASE DO NOT TURN OVER

UNTIL EXAM STARTS

Name: ______________________________

PLEASE RETURN THIS BOOKLET

WITH YOUR SOLUTION SHEET(S)

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.710 Optics **Spring '09**

FINAL EXAMINATION Tuesday, May 19^{th}, 2009

Duration: 3 hours

Open books, open notes

Instructions

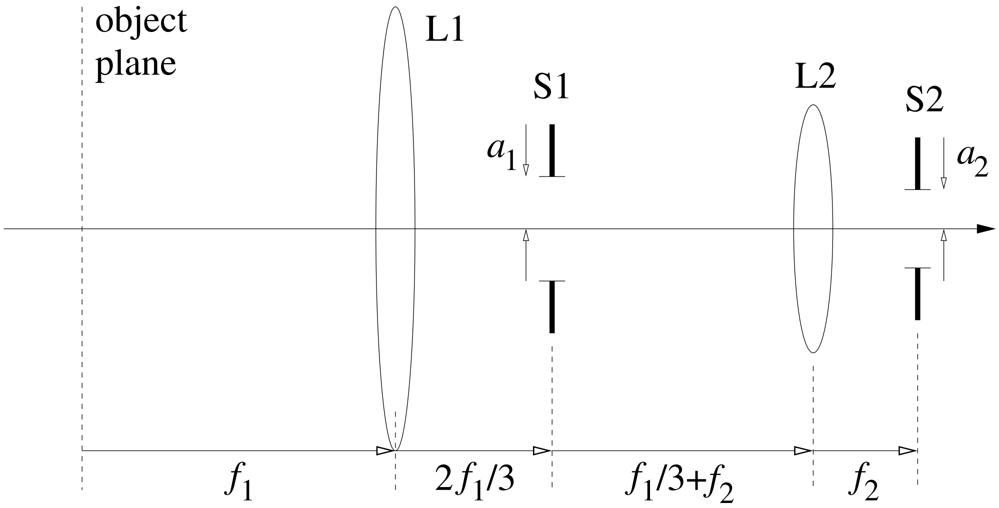
Treat all problems in one spatial dimension, and neglect apertures unless they are explicitly given in the problem statement. Multiplicative amplitude constants may be neglected when not required by the problem statement.

When a problem seeks a numerical result, you may leave it to a reduced fractional form without doing the numerical division, e.g. 4/7 instead of 0.57143.

When asked to "sketch" an optical system or a function, label your sketch clearly with as much quantitative detail as you can. If you are sketching a complex function, make sure to indicate the real and imaginary parts, or the magnitude and phase.

If a problem appears to be given insufficient data, make assumptions as necessary and state them clearly. When in doubt, make ample use of Occam's razor: "Reasons shall not be multiplied beyond necessity;" *i.e.*, among all possible and adequate explanations the simplest one is the most likely to be correct.

1. **(20%)** For the telescope configuration shown below, where lenses L1 and L2 have focal lengths f_1, f_2, respectively, the object plane and two stops S1 and S2 of half–sizes a_1, a_2, respectively, are at the locations shown,



a) identify the Aperture Stop and the Field Stop, and trace the Chief Ray and Marginal Ray for a sample off-axis point object of your choice;

b) locate the Entrance Pupil, Exit Pupil, Entrance Window, and Exit Window;

c) calculate the Numerical Aperture and Field of View; and

d) critique whether stops S1, S2 are optimally located and, if not, suggest better location(s).

PLEASE TURN OVER

2. (15%) The index of refraction in a GRadient INdex (GRIN) medium is given by

$$n(r) = \begin{cases} \sqrt{2 - r^2}, & \text{if } 0 < r < 1; \\ 1, & r \geq 1, \end{cases}$$

where $r = \sqrt{x^2 + z^2}$ is the cylindrical polar coordinate.

2.a) Write down the set of Hamiltonian ray–tracing differential equations for the ray trajectories $\mathrm{d}x/\mathrm{d}s$, $\mathrm{d}z/\mathrm{d}s$ and moments $\mathrm{d}p_x/\mathrm{d}s$, $\mathrm{d}p_z/\mathrm{d}s$, where s is the indexing variable along the rays. Do not attempt to solve the 4×4 set of Hamiltonian equations.

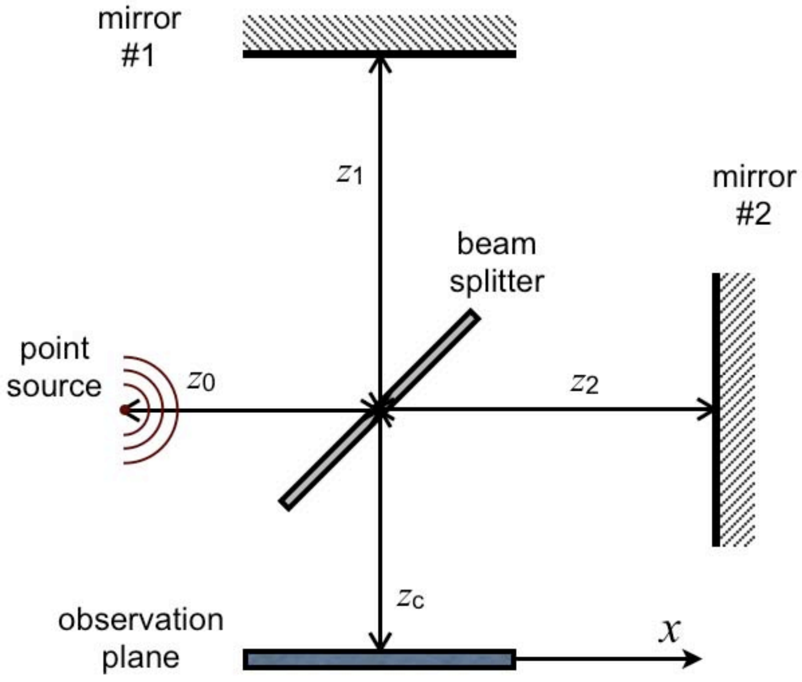
2.b) Prove that, within the disk $r < 1$,

$$\left(\frac{\mathrm{d}p_x}{\mathrm{d}s}\right)^2 + \left(\frac{\mathrm{d}p_z}{\mathrm{d}s}\right)^2 = \frac{2}{p_x^2 + p_z^2} - 1.$$

2.c) Is the Screen Hamiltonian preserved in this system?

PLEASE TURN OVER

3. **(20%)** A Michelson interferometer is illuminated by a perfect spherical wave, as shown below. The beam splitter is an infinitely thin partially reflecting mirror of 50% reflectivity. The wavelength is λ. The origin of the spherical wave is distance z_0 to the left of the beam splitter. The lengths of the two interferometer arms are z_1, z_2, respectively. The recombined fields propagate a common distance z_c before reaching an observation plane.

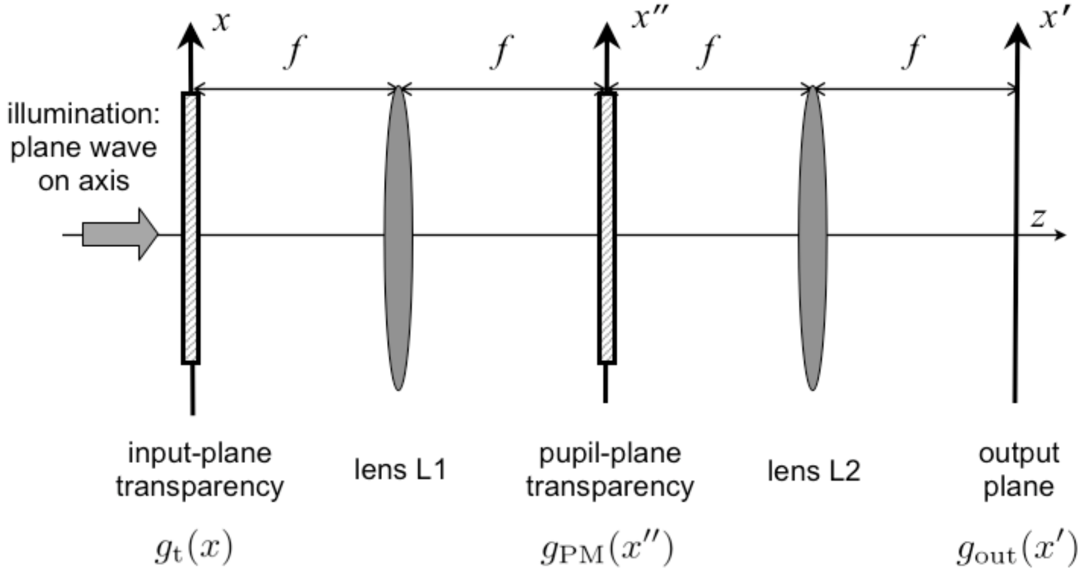


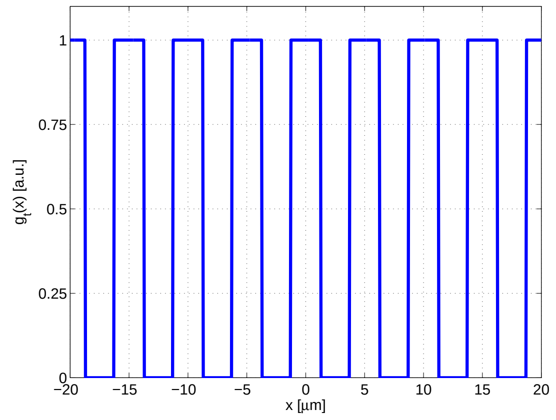
a) Assuming that the beam splitter is large enough to accommodate the entire lateral width of the expanding paraxial wave, write an analytical expression for and sketch the interference pattern as function of the coordinate x at the observation plane.

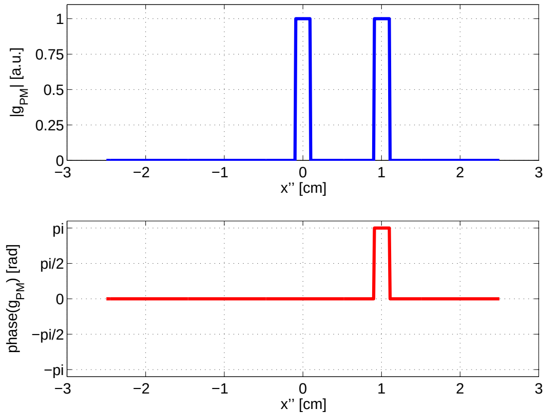
b) How does the interference pattern change if the flat mirror #2 is replaced by a convex spherical mirror of radius $2(z_0 + z_2)$?

PLEASE TURN OVER

4. **(20%)** Consider the imaging system shown below, consisting of two ideal thin lenses L1, L2 with the same focal length $f = 10\text{cm}$. Transparency g_t is real and its amplitude transmissivity is shown in the lower left diagram. The complex transmissivity of g_PM is shown in the lower right diagram. The illumination is monochromatic at wavelength $\lambda = 0.5\mu\text{m}$ and spatially *coherent.*







4.a) Propose a physical realization for the complex transparency g_PM.

4.b) Calculate the output field $g_\text{out}(x')$ and intensity $I_\text{out}(x') = |g_\text{out}(x')|^2$.

PLEASE TURN OVER

5. **(25%)** The same optical system of Problem 4 is illuminated by quasi–monochromatic spatially *incoherent* light at wavelength $\lambda = 0.5\mu\text{m}$. Calculate:

- **5.a)** the Optical Transfer Function (OTF) and the Modulation Transfer Function (MTF);
- **5.b)** the spatial frequency (or frequencies) that are visible at the output plane and the contrast of the output intensity pattern; and
- **5.c)** the Point Spread Function (PSF) of this spatially incoherent optical system.

GOOD LUCK!

2.71 / 2.710 Optics
Spring 2009