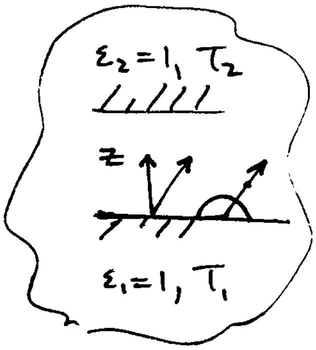
## ISOTROPIC SCATTERING

$$\frac{dI_\eta}{d\tau_\eta} = -I_\eta + (1-\omega_\eta) I_{b\eta} + \underbrace{\frac{\omega_\eta}{4\pi}\int_{4\pi} I'(\Omega')\, d\Omega'}$$

CAN $\int$ BOTH SIDES OVER SOLID ANGLE

IS A CONSTANT OVER THIS PROCESS, SINCE ALREADY $\int$ OVER SOLID $\measuredangle$

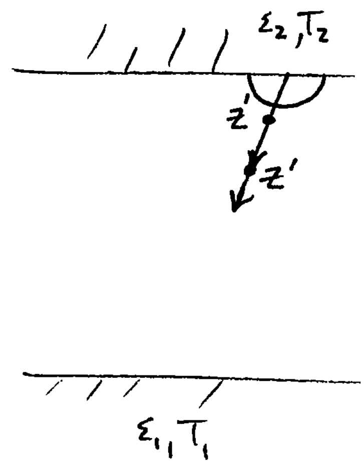



** HE'S DROPPING THE PRIME $I' \Rightarrow I$

$$\int_{4\pi} \mu \cancelto{0}{\frac{dI_\eta}{dz}}\, d\Omega = -\int_{4\pi} I\, d\Omega + (1-\omega_\eta) 4\pi I_{b\eta} + \omega_\eta \int_{4\pi} I\, d\Omega$$

$$q'' = \int \mu I_\eta\, d\Omega \qquad I_{b\eta} = \frac{1}{4\pi}\int_{4\pi} I\, d\Omega$$

$$\frac{dI_\eta}{d\tau_\eta} = -I_\eta + I_{b\eta}$$




$$\tau_\eta = \kappa_{e\eta}\frac{z}{\mu}, \quad \mu \equiv \cos\Theta$$

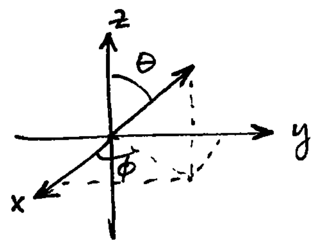
$$\xi \equiv \frac{z}{1/\kappa_e} = \frac{z}{\Lambda}$$

* SCATTERING TERM DEPENDS ON LOCAL TEMP., BUT WE ONLY KNOW BOUNDARY TEMP.

$$I_\eta^+(\xi,\mu) = I_{b\eta_1} e^{-\xi/\mu} + \int_0^\xi I_{b\eta} e^{-\frac{\xi-\xi'}{\mu}} \frac{d\xi'}{\mu}$$

$$I_\eta^-(\xi,\mu) = I_{b\eta_2} e^{\frac{\xi_L-\xi}{\mu}} + \int_{\xi_1}^{\xi} I_{b\eta} e^{-\frac{\xi-\xi'}{\mu}} \frac{d\xi'}{\mu}$$

$$q_z'' = \int_0^\infty d\eta \int_{4\pi} I_\eta \cos\theta \underbrace{d\Omega}_{\sin\theta\, d\theta\, d\phi}$$




$$= \int_0^\infty d\eta \left[ \int_0^1 I_\eta^+(\xi,\mu)\,\mu\, d\mu - \int_0^1 I^-(\xi,-\mu)\,\mu\, d\mu \right]$$

$$\left( \int_0^{180} d\theta \rightarrow \int_0^{90} + \int_{90}^{180} \right)$$

** NOTE

COULD HAVE OTHER COORDS e.g.,


$$= \int_0^1 I^+(\xi,\mu)\,\mu\, d\mu = I_{b\eta_1} \underbrace{\int_0^1 e^{-\xi/\mu} \mu\, d\mu} + \int_0^\xi I_{b\eta}(\xi')\, d\xi' \left( \underbrace{\int_0^1 e^{-\frac{\xi-\xi'}{\mu}} d\mu} \right.$$

* INTEGRAL EXPONENTIAL FUNCTION $\quad E_n(t) = \int_0^1 \mu^{n-2} e^{-t/\mu} d\mu$

$$q_z'' = I_{b\eta_1} E_3(\xi) + \int_0^{\xi} I_{b\eta}(\xi') E_2(\xi - \xi')\, d\xi'$$

$$\int_0^1 I^-(-\mu, \xi)\mu\, d\mu = I_{b\eta_2} E_3(\xi_L - \xi) - \int_{\xi_L}^{\xi} I_{b\eta}(\xi') E_2(\xi' - \xi)\, d\xi'$$

* IF $E_n$ HAS SPECTRAL BEHAVIOR, ONE MUST $\int$ OVER WAVELENGTH, MAKING AN ANALYTIC SOL'N VIRTUALLY IMPOSSIBLE. i.e., YOU'RE INTEGRATING AN INTEGRAL FUNCTION

$\therefore$ ASSUME GRAY

TEMPERATURE LIVES HERE $\frac{\sigma T^4}{\pi}$

GRAY MEDIUM:

$$q''(\xi) = I_{b_1} E_3(\xi) - I_{b_2} E_3(\xi_L - \xi) + \int_0^{\xi} I_b(\xi') E_2(\xi - \xi')\, d\xi' + \ldots$$
$$+ \int_{\xi_L}^{\xi} I_b(\xi') E_2(\xi' - \xi)\, d\xi'$$

IF WE HAD CONDUCTION, 1st LAW

$$\frac{d}{dz}\left(\cancel{q''_{cond}} + q''_{rad}\right) = 0$$

WE'LL ONLY FOCUS ON RADIATION FOR NOW

FROM FIRST LAW

$$0 = -I_{b_1}E_2(\xi) - I_{b_2}E_2(\xi_L - \xi) + \underline{I_b(\xi)E_2(0)} - \int_0^{\xi} I_b(\xi')E_1(\xi - \xi')\,d\xi' + \cdots$$

$$+ \underline{I_b(\xi)}E_2(0) + \int_{\xi_L}^{\xi} I_b(\xi')E_1(\xi' - \xi)\,d\xi' + \underbrace{k\frac{d^2T}{dz^2}}$$

$$\Uparrow \quad -\int_{\xi}^{\xi_L} \quad \sigma T^4$$

IF COND., BUT MAKES EQN. VERY NASTY

REWRITE AS,

$$0 = e_{b_1}E_2(\xi) + e_{b_2}E_2(\xi_L - \xi) + 2\underbrace{e_b}_{\sigma T^4(\xi)} - \int_0^{\xi_L} e_b(\xi')E_1(|\xi - \xi'|)\,d\xi'$$

$$\xi_L = K_e L = \frac{L}{\Lambda}$$

$$\Phi_b(\xi) = \frac{e_b(\xi) - e_{b_1}}{e_{b_2} - e_{b_1}} = \frac{T^4 - T_1^4}{T_2^4 - T_1^4}$$

$$\Longrightarrow \Phi_b = \frac{1}{2}\left[E_2(\xi) + \int_0^{\xi_L} \Phi_b(\xi')\,E_1\underbrace{(|\xi - \xi'|)}\,d\xi'\right]$$

FREDHOLM INT. EQN OF SECOND KIND

HERE, KERNEL HAS SINGULARITY

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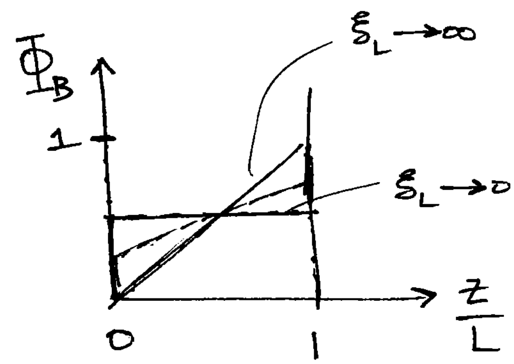
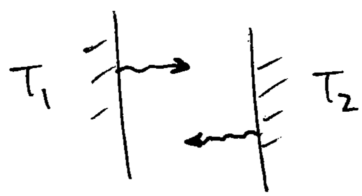
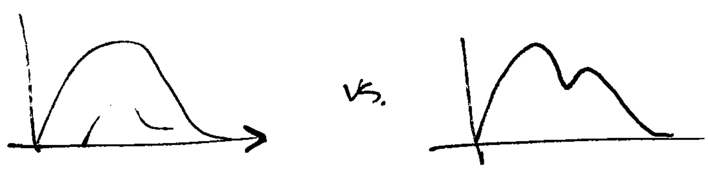
NON-DIM. FORM

$$\psi_b = \frac{q''}{e_{b_1} - e_{b_2}} = \frac{q''}{\sigma(T_1^4 - T_2^4)} = 1 - 2\int_0^{\xi_L} \Phi_b E_2(\xi')\,d\xi'$$

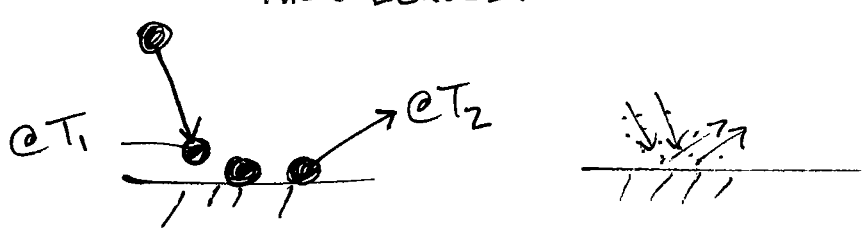
$$u_e = 4\pi \frac{I}{c} = 4\pi \frac{\sigma T^4}{\pi c}$$




PARTICLE COMES IN, STICKS, AND THEN LEAVES.




BUT WHAT IS LOCAL

IS IT THE MEDIUM TEMP. OR PHOTON TEMP.

CANNOT PLOT A REAL LOCAL TEMP.

HOW TO THINK OF TEMP. BTN. PLATES.

EQUIL. TEMP. VS. NON EQUIL. TEMP.

IN BETWEEN PLATES
THINK OF LOCAL PHOTON NRG DENSITY RATHER THAN LOCAL PHOTON TEMP.

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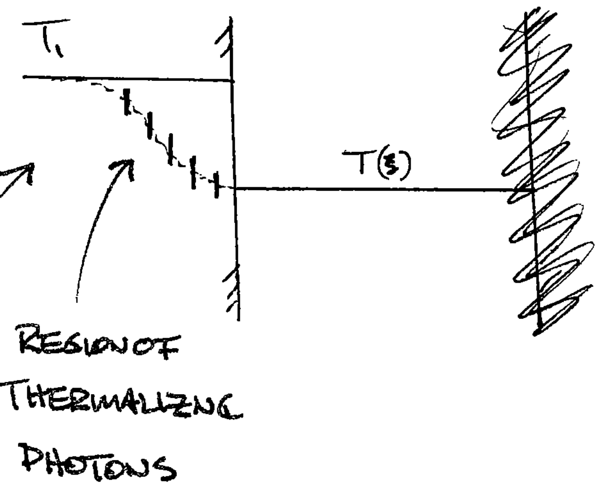
WHAT DOES DISCONTINUITY MEAN PHYSICALLY

→ IGNORE PHONONS (CONDUCTION) AND FOCUS ON PHOTONS ONLY




THIS EFFECT IS ALWAYS PRESENT, EVEN IN THE CONTINUUM, IT'S JUST THAT THE JUMP IS "SMALL" IN CONTINUUM AND NOT NOTICEABLE OR NOT IMPORTANT.

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APPROXIMATIONS:

$\xi_L \to 0$ OPTICALLY THIN $\Rightarrow q'' = (J_1 - J_2)(1 - \xi_L)$

$\xi_L \to \infty$ OPTICALLY THICK $\mu \frac{dI}{d\xi} = -I + I_b$

$\xi_L$ LARGE $I = I_b + c\frac{dI_b}{d\xi} + \cdots$

$= I_b - \mu \frac{dI_b}{d\xi}$

$$q'' = \int_{4\pi} I \cos\theta \, d\Omega$$

$$= \int_0^{2\pi} d\phi \int_0^{180} \left[ I_b - \cos\theta \frac{dI_b}{d\xi} \right] \cos\theta \sin\theta \, d\theta$$

$$= -\frac{4\pi}{3} \frac{dI_b}{d\xi}$$

$$= -\frac{4}{3K_e} \frac{de_b}{dz}$$

"LOOKS" LIKE FOURIER LAW