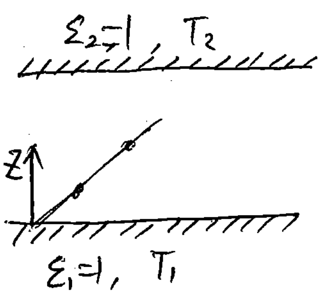
Last time



absorbing & emitting & isotropic scattering

$$\frac{dI_\eta}{d\tau_\eta} = -I_\eta + I_{b\eta}$$

$$\tau_\eta = \frac{z\,\kappa_{a\eta}}{\cos\theta} = \frac{\xi}{\mu}, \quad \mu = \cos\theta$$

$$I_\eta^+(\xi,\mu) = I_{\eta b1}\, e^{-\frac{\xi}{\mu}} + \int_0^{\xi} I_{b\eta}(\xi')\, e^{-\frac{\xi-\xi'}{\mu}} \frac{d\xi'}{\mu}$$

$0 \le \mu < 1$

$$I_\eta^-(\xi,\mu) = I_{\eta b2}\, e^{\frac{\xi_L-\xi}{\mu}} + \int_{\tau_L}^{\tau} I_{b\eta}(\xi')\, e^{\frac{\xi-\xi'}{\mu}} \frac{d\xi'}{\mu}$$

If isotropic scattering

$$\frac{dI_\eta}{d\tau_\eta} = -I_\eta + (1-\omega_\eta) I_{b\eta} + \frac{\omega_\eta}{4\pi}\int_{4\pi} I(\Omega')\,d\Omega'$$

$$\int_{4\pi} \mu \frac{dI_\eta}{d\xi_\eta} d\Omega = -\int I_\eta\, d\Omega + (1-\omega_\eta) I_{b\eta}\cdot 4\pi + \omega_\eta \int I(\Omega')\,d\Omega'$$

$$= (1-\omega_\eta)\left[4\pi I_{b\eta} - \int I_\eta\, d\Omega\right]$$

$\Uparrow$

heat flux at equilibrium $=0$

$$\Rightarrow I_{b\eta} = \frac{1}{4\pi}\int I_\eta\, d\Omega$$

$$\frac{dI_\eta}{d\tau_\eta} = -I_\eta + I_{b\eta}$$

$$q_z = \int_0^\infty d\lambda \int_{4\pi} I_\lambda \cos\theta \, d\Omega$$

$$= \int_0^\infty d\lambda \int_0^{2\pi} d\varphi \int_0^\pi I_\lambda \cos\theta \sin\theta \, d\theta$$

$$= \int_0^\infty d\lambda \int_0^{2\pi} d\varphi \Big[\int_0^1 I_\lambda \, \mu \, d(-\mu) + \int_0^{-1} I_\lambda(\mu, z) \, d(-\mu) \Big]$$

$$= \int_0^\infty \int_0^{2\pi} d\varphi \Big[\int_0^1 I_\lambda^+ \mu \, d\mu - \int_0^1 I_\lambda^-(-\mu, z) \mu \, d\mu \Big]$$

$$\int_0^1 I_\lambda^+ \mu \, d\mu$$

$$= I_{\lambda b1} \int_0^1 \mu e^{-\frac{\tau}{\mu}} d\mu$$

$$+ \int_0^\tau I_{b\lambda}(z') \, d\eta' \int_0^1 e^{-\frac{\tau - \tau'}{\mu}} d\mu$$

$$E_n(t) = \int_0^1 \mu^{t-2} e^{-\frac{t}{\mu}} d\mu \quad \text{— Exponential Integral funct.}$$

$$= I_{\lambda b1} E_3(\tau) + \int_0^\tau I_{b\lambda}(\eta') E_2(\tau - \tau') \, d\tau'$$

$$\int_0^1 I_\lambda^-(-\mu) \mu \, d\mu$$

$$= I_{\lambda b2} \int_0^1 \mu e^{-\frac{\tau_L - \tau}{\mu}} d\mu + \int_\tau^{\tau_L} I_{b\lambda}(\eta') \int_0^1 e^{-\frac{\tau' - \tau}{\mu}} d\mu'$$

$$= I_{\lambda b2} E_3(\tau_L - \tau) + \int_\tau^{\tau_L} I_{b\lambda}(\tau') E_2(\tau_L - \tau') \, d\tau'$$

Gray medium

$$q(\tau) = 2\pi I_{b1} E_3(\tau) - 2\pi I_{b2} E_3(\tau_L - \tau)$$

$$+ 2\pi \int_0^{\tau} I_{b}(\tau') E_2(\tau - \tau') d\tau' - 2\pi \int_{\tau}^{\tau_L} I_{b}(\tau') E_2(\tau' - \tau) d\tau'$$

($I_{b1} = \frac{\sigma T_1^4}{\pi}$, $I_b(\tau') = \frac{\sigma T^4(\tau')}{\pi}$)

If no conduction

$q(\tau) = \text{constant}$

$$\frac{dq}{d\tau} = 0 \Rightarrow \qquad \left(\frac{dE_n}{d\tau} = -E_{n-1} \right)$$

$$0 = -E_{b1} E_2(\tau) - E_{b2} E_2(\tau_L - \tau)$$

$$\cancel{+ E_b(\tau) + E_b(\tau) + \int_0^{\tau} E_b E_1(\tau - \tau') d\tau' + \int_{\tau}^{\tau_L} E_b E_1(\tau' - \tau) d\tau}$$

$$T^4(\tau) = \frac{1}{2}\left[T_1^4 E_2(\tau) + T_2^4 (\tau_L - \tau) + \int_0^{\tau_L} T^4(\tau') E_1(|\tau - \tau'|) d\tau' \right]$$

↳ Temperature distribution can be determined.

~~We have eliminated Ω in the equation.~~

same as what you did before discretize numerically.

~~Solution~~

Non-dimensionalize

$$\Phi_b(\tau) = \frac{T^4(\tau) - T_2^4}{T_1^4 - T_2^4}, \quad \Psi_b = \frac{q}{\sigma(T_1^4 - T_2^4)}$$

$$\Rightarrow \quad \Phi_b(\tau) = \frac{1}{2}\left[E_2(\tau) + \int_0^{\tau_L} \Phi_b(\tau') E_1(|\tau - \tau'|) d\tau' \right]$$

$$\Phi_b = 1 - 2 \int_0^{\tau_L} \Phi_b(\tau') E_2(\tau') d\tau'$$

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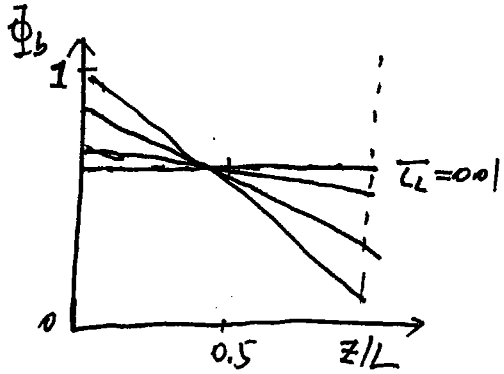
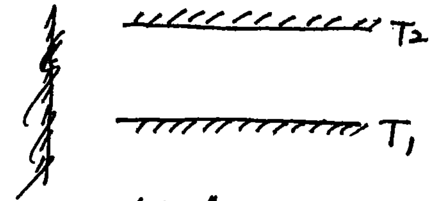


discussion

① when $\tau_L \to 0$ ballistic



$$T^4 = \frac{T_1^4 + T_2^4}{2}$$

we assumed molecules absorb, re-emit.
really nonequilibri

② $\tau_L \to \infty$
diffusion.

③ temperature slip.

④ non black surfaces

$\tau = 0:$ $\quad q = \frac{\varepsilon_1}{1-\varepsilon_1}\left(\sigma T_1^4 - J_1\right)$

$\tau = \tau_L:$ $\quad -q = \frac{\varepsilon_2}{1-\varepsilon_2}\left(\sigma T_2^4 - J_2\right)$

$$\Rightarrow \quad \Phi(\tau) = \frac{T^4(\tau) - T_2^4}{T_1^4 - T_2^4} = \frac{\Phi_b(\tau) + \left(\frac{1}{\varepsilon_2} - 1\right)\Psi_b}{1 + \Psi_b\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 2\right)}$$

$$\Psi = \frac{q}{\sigma\left(T_1^4 - T_2^4\right)} = \frac{\Psi_b}{1 + \Psi_b\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 2\right)}$$

⑤ existence of molecules:
absorbing & emitting assumption.
Comment: conduction can.