13.49 Homework #7

The parameters for the linearized sway/yaw motions of a swimmer delivery vehicle are given below.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Parameters, all nondimensional except [U,L]

U =             4.0 ; % m/s
L =             5.3 ; % m

Izz =           0.006326 ;
m =             0.1415 ;
xg =            0. ;
Yv =           -0.1 ;
Yr =            0.03 ;
Nv =           -0.0074 ;
Nr =           -0.016 ;
Ydelta =        0.027 ;
Yvdot =        -0.055 ;
Yrdot =         0. ;
Nvdot =         0. ;
Nrdot =        -0.0034 ;
Ndelta =       -0.013 ;
```

You are asked to develop an LQG/LTR controller for this plant, and it is suggested that you compose a single Matlab script to perform the steps in sequence. Please make sure you answer all the questions, and include a listing of your code. This entire design is made in nondimensional coordinates.

1. **Plant Modeling and Characteristics**
   (a) Construct a state-space plant model, to take rudder angle δ as an input and give heading angle ϕ as an output. Please provide the *numerical* values for the A, B, C matrices. There should be three states in your model, with one input channel and one output channel.
   (b) Compute and list the controllability and observability matrices; is the plant state-controllable and state-observable?
   (c) Where are the poles of your plant model? Is this model stable?
   (d) Show a plot of your plant's step response.

2. **LQR and KF Designs**
   (a) Using the Matlab command `lqr()`, you can compute the LQR feedback gain K, for given A, B, Q, and R matrices. With the choices $Q = C^T C$, and $R = \rho$, list K and plot the closed-loop step responses for the choices $\rho = [0.1, 0.001, 0.00001]$. How do the gains and step responses change as you make ρ smaller and smaller?
   Note that the fundamental closed-loop LQR system is

$$\begin{aligned} \dot{\vec{x}} &= (A - BK)\vec{x} + BK\vec{x}_{desired} \\ y &= \vec{x}, \end{aligned}$$

   i.e., the input to the closed-loop system is $\vec{x}_{desired}$ and the output is $\vec{x}$. Your plot should show specifically the output ϕ, for an input of $\vec{x}_{desired} = [v_{desired} = 0, r_{desired} = 0, \phi_{desired} = 1]$. This compression can be achieved in one step by premultiplying the system by C^T, and post-multiplying it by C:

$$\begin{aligned} \dot{\vec{x}} &= (A - BK)\vec{x} + BKC^T y_{desired} \\ y &= C\vec{x}, \end{aligned}$$

(b) The Matlab command `lqe()` can be used to generate the Kalman filter gain H, given design matrices A, C, V_1, and V_2. For the choices $V_1 = I_{3\times3}$ and $V_2 = 0.01$, compute H, and make a plot of the closed-loop step response. Be sure to give the numerical values of H.

Note that the `lqe()` command asks for a disturbance gain matrix G; you should set this to $I_{3\times3}$. The closed-loop KF system is as follows:

$$\begin{aligned} \dot{\hat{x}} &= (A - HC)\hat{x} + Hy \\ \hat{y} &= C\hat{x}, \end{aligned}$$

i.e., the input is the measurement y and the output is an estimated version of it, $\hat{y}$.

3. **Loop Transfer Recovery**

The LQG compensator is a combination of the KF and LQR designs above. With normal negative feedback, the compensator $C(s)$ has the following state space representation:

$$\begin{aligned} \dot{\vec{z}} &= (A - BK - HC)\vec{z} + He \\ u &= K\vec{z}, \end{aligned}$$

so that the input to the compensator is the tracking error $e = r - y$, and its output u is the control action to be applied as input to the plant. The total open-loop transfer function is the $P(s)C(s)$; in Matlab, you may simply multiply the systems, e.g., `sysPC = sysP * sysC ;`.

(a) Make a log(magnitude) plot of the KF open-loop transfer function $L(s) = C(sI - A)^{-1}H$, versus log(frequency). You may find the Matlab command `freqresp()` helpful. $|L(s)|$ should be large at low frequencies, and small at high frequencies, consistent with the rules of loopshaping.

(b) As $\rho \to 0$, the product $P(s)C(s) \to L(s)$. Demonstrate this by computing $P(s)C(s)$ for the three different values of ρ above, and overlaying the respective $|P(s)C(s)|$ over the plot of part 3a).

(c) Make a closed-loop step response plot for the smallest value of ρ. How does it compare with the KF step response of part 2b)?

In real LTR applications, the particular values of V_1 and V_2 can be picked to control the low-frequency gain, and crossover frequency of the open-loop KF system $L(s) = C(sI - A)^{-1}H$.