PROFESSOR: OK, let's talk about the important thoughts for the week, important concepts. What do you have?

AUDIENCE: Energy method.

PROFESSOR: Energy method. I'm going to say this work energy theorem. And from that, these turn out to be our potential energies. And this minus the work done by the conservative forces we call V2 minus V1. That's the change in the potential energy of the system. So that's the work energy theorem. That's an important one for the week. What else?

AUDIENCE: Different types of problems.

PROFESSOR: Yeah, we did several example problem particularly emphasizing what?

AUDIENCE: With respect to moving points?

PROFESSOR: Yeah, or points that look like they're moving. But you can use special considerations in calling them instantaneous centers of rotation and things like that, so summation of the torques with respect to points sometimes moving defined with respect to ICRs, Instantaneous Centers of Rotation, and also moving points. We definitely did some of that.

How about a third? I'd say one more thing that we got started on is expressions-- 1/2 i, 1/2 omega dot h plus 1/2 mv, v dot v, those kind of terms for this. And for potential energy, if it's purely mechanical devices, we have two kinds of potential energy that we'll be dealing with.

And they are associated with springs and gravity. So that's the two we'll deal with.

Are there others? What's an example of a different kind of potential energy that you might run into?

AUDIENCE: Magnetic.

PROFESSOR: Magnetic fields, yeah. We can do work pulling something out of a magnetic field. What else?

AUDIENCE: Electric fields.

PROFESSOR: Electric fields, sure. But they do have their complications. So that's pretty good for the week. Now, we're going to talk about this problem. I'll give him a chance to shoot it. This is a good diagram of it. You guys had a homework problem that instead of a uniform rod, you had a concentrated mass on the end of a massless rod. So this is almost identical to that.

But we now have actual mass, distributed mass, in the stick. I've got a little demo here, which will work, works after a fashion. Put a little tension on these springs, so now I've got my cart on a spring. The damping is natural in the system, comes from the wheels and everything else. I've got to get this kind of close to the edge so it'll work.

This thing has two natural frequencies actually. They're kind of hard to isolate them. There's one-- no, can't do it. In one natural mode, the thing swings forward and moves forward together. And in the other natural mode, this'll go aft, that way, while the other one's going frontwards.

So let's see if I can-- that's higher frequency. That's the motion of what's called the second mode, opposite. And the first mode is lower frequency than that. It's hard to get a perfect start to it. So this is the kind of motion we're talking about.

And we're going to have you help me solve. We want to obtain equations of motion for this system. And let's quickly run through the exercise of-- you can do this intuitively first, it's so simple. Plus you've already done it. How many independent coordinates does it take to do it? Two, and how many rigid bodies are there in this problem? How many rigid bodies in the problem?

AUDIENCE: Two.

PROFESSOR: There's gotta be two, right? There's a block that moves back and forth, and there's this rod that swings. So the number of degrees of freedom, we'll call it d, is 6 times the number of bodies. n is the number bodies minus the number of constraints. So this is 6 times 2 minus the constraints. So we've got 12 minus c. And you know the answers. The answer, how many independent coordinates to completely describe this motion?

AUDIENCE: Two.

PROFESSOR: Just the two. So you know the answer is two. So that means you've got to come up with 10, a list of 10 of these things. So let's see if you can write down quickly your own list. How would you eliminate through coming up with constraints? Write down the 10 constraints. And I recommend you do it by taking one body at a time. So constraints on mass one, the first block, what do you got?

AUDIENCE: y, y dot, and y double dot equal 0.

PROFESSOR: OK, so that says no motion in the y direction. Because we're on rollers. It's fixed that way, so no motion in the y. What else?

AUDIENCE: No motion in z.

PROFESSOR: No motion in z. We're telling it it can't come in and out of the board. OK, any other translational constraints on that one? So no y or z translation. And so that gives us two constraints there. What else on that body?

AUDIENCE: No rotations.

PROFESSOR: No rotations in what direction?

AUDIENCE: x, y, or z.

PROFESSOR: In x, y, or z. The thing can only translate, so no x, y, z rotations. That's three. So that's a total of five. But we have one left. It's got to be able to move in the x. And how about M2? What are the constraints on the second one?

AUDIENCE: [INAUDIBLE]

PROFESSOR: So one at a time.

AUDIENCE: [INAUDIBLE]

PROFESSOR: So omega which?

AUDIENCE: x and y.

PROFESSOR: No rotations about which axes?

AUDIENCE: x and y.

PROFESSOR: So you're saying no rotations of this about the x. So it can't roll over. About the y, it can't turn. And about the z. But we're doing this one. So it's no spinning about-- let's do it in its body coordinates. x1-- no spinning about x1. No spinning about y1. But can it rotate around z1?

OK, so for the second mass, we have no x1, y1 rotations. That's two. And what about translations on the second body? Hmm?

AUDIENCE: No translations x, y, and z.

PROFESSOR: No translations x, y, and z. So z, pretty obvious-- we're not letting it do this. But why can you argue there's no translation in x and y?

AUDIENCE: Because it's strictly rotation.

PROFESSOR: You say it's strictly rotation.

AUDIENCE: For it to translate [INAUDIBLE].

PROFESSOR: OK, so you're saying translation is strictly described as parallel motion of all points on it. But where is it constrained in x and y?

AUDIENCE: [INAUDIBLE]

PROFESSOR: It's pinned to the cart. And if you fix x, which you're allowed to do, a test you're allowed to do, you have set this as an independent coordinate. If you fix it, this point is fixed. And you've constrained it in x and y. And if any point in the object is constrained in x and y, then all points are constrained in x and y pure translation. So we have in this one no x, y, and z translations.

So that's three. You add them up, you get 10. OK, good. Now, the next thing I want you to do, another key step in doing problems like this, is our free body diagrams. And how many do you need? As many as you have rigid bodies. You need one for every rigid body.

So we got two rigid bodies. Draw for me your free body diagram for the cart and for the rod. OK, what group wants to volunteer here, tell me what I ought to do? How about you guys in the back? You're quite today. So pick this first one.

AUDIENCE: You have the spring force kx to the left.

PROFESSOR: kx.

AUDIENCE: Also bx dot.

PROFESSOR: bx dot.

AUDIENCE: Normal force up.

PROFESSOR: N.

AUDIENCE: You have M1g down, and then the force of the rod is down and to the right.

PROFESSOR: Oh, so you've made-- ahh, colinear with the rod?

AUDIENCE: Yeah.

PROFESSOR: OK, so you're saying we've got a vector force like that, some F, right? OK, how about a little help on this. Anybody have a different result here for this one?

AUDIENCE: I broke up my normal into two [INAUDIBLE].

PROFESSOR: So you have two pieces here? So in what direction do you draw them?

AUDIENCE: Reaction force of the first failed reaction?

PROFESSOR: I guess which coordinate system? We need two coordinate systems to define this problem.

AUDIENCE: Just in the regular x, y.

PROFESSOR: So I've got an inertial frame here. And that gives me my X, capital X, motion for this guy. But we need, because if we want to be able to moments of inertia and things like that, a coordinate system attached to this rigid body, right? So I've called it little x1, y1, and it moves with the body. OK, so you've picked your two reaction forces aligned with these two or aligned in this system?

AUDIENCE: With the inertial.

PROFESSOR: OK, you've done it this way. So you're coming up with a pair of forces that have a piece like this and a piece like that?

AUDIENCE: No, I'm talking about for the wheels.

PROFESSOR: Oh, for the wheels. Oh, OK. You came up with individual ones here? Yeah, OK. I'll call that N1 and N2. And that's probably an improvement. Because that's what keeps it from rolling over. OK, that's true. And what else? So you've got a reaction force here like that. Can we improve on that?

AUDIENCE: You can break it up into tangential and radial.

PROFESSOR: Yeah, but I understood. The intention here was it was drawn colinear with the axis of the thing. And that suggested just that one part. You could break that into a horizontal in the master frame, yeah.

But does anybody have something different about that, another way to do this? Do you agree with this? Is this correct? Pardon? You're treating it like a string. Is it a string? Can that shaft transmit shear forces? Hmm?

If I grab this thing and go left and right on it, it's putting forces in that direction perpendicular to this. So is it conceivable that there are forces on this axle caused by that that are in that direction as well as that direction? What do you think?

Because I can't make an argument for sure that would eliminate either one of those. Pretty sure there's certainly a radial one, because that's centripetal, and there's got to be some of that. But might there be some in the other direction?

OK, so an improvement on this, a necessary improvement, I think, is that rather than just one, we'd better have two. And we're going to use my system of naming so that the answers I've got here will work out. I'm going to call that one F2 and this one F1. So now the rod is placing on the cart two reaction forces which you don't know.

Now, is that the complete thing for the cart? OK, now let's do this one. Another group, tell me how to do the second one, free body diagram.

AUDIENCE: So you have F1 and F2, but opposite.

PROFESSOR: Ahh, good point. You've got an F2 this way and an F1 like that, is what you're saying?

AUDIENCE: Yeah, and an Mg comes down.

PROFESSOR: And an Mg, yeah, M2g. OK, good. Real important point-- Newton's third law. If you draw F1 and F2 here, those on that side have to be equal and opposite in order for all this to work out without problems. OK, those look pretty good to me.

Now let's move on. This is a complicated problem. And to get a lot with it in an hour is a challenge. And you've done a bunch of this problem already. You did the one with the particle on it. So there's not a great deal of difference here. So I'm emphasizing a few of the nuances. What I now want to do is to talk about, how do we get equations of motion for this problem? How many will there be to start with?

AUDIENCE: Two.

PROFESSOR: Got to get two, two rigid bodies and two degrees of freedom. And you're going to end up-- it's the two degrees of freedom that tell you you're going to get two equations of motion at the end. OK, how many-- and I've written three up there, but it's not necessarily true. You've been taught to this point two different methods for getting at these equations of motion.

Somebody describe one for me. How would you personally go about this problem? Just pick a method and in words describe it. And think of what you did in the last homework.

AUDIENCE: Sum of external force equals [INAUDIBLE].

PROFESSOR: So sum of external forces is--

AUDIENCE: [INAUDIBLE]

PROFESSOR: OK, so on method one here, I'll call it, you're going to say sum of the forces on what?

AUDIENCE: The rigid body.

PROFESSOR: So you've got two bodies here. So pick one. Huh? All right, so I'm going to put this on M1 for sure. That one is translating. And you're going to get an equation that involves its x double dot. It's got to be able to move back and forth.

You're going to need an equation like this. So the sum of the external forces on that mass has got to be equal to M1 times-- right? And we can write an equation for that in terms of this free body diagram. In the i direction, you've got this. You've got this.

You have the normal forces gravity don't contribute. And components of this and this have to be added in. You have to do an F2, probably cosine theta, and an F1 sine theta, and add them in. Because they're the external forces. So you've got all these summation of forces on that side.

That gives you one equation. This will give you an equation for the motion of the first mass. And it'll have some unknowns in it. And what are they? Describe them. So you're going to get a function over here of a couple unknowns. This is going to be a function of what that you don't know?

AUDIENCE: [INAUDIBLE]

PROFESSOR: F1 and F2, and then also x double dot and thetas and theta dots and so forth, all the variables in the problem. But these are two unknowns you've got to ultimately get rid of. OK, what else are you going to do in method one? You've got to deal with the second mass, right? What would you do then?

AUDIENCE: [INAUDIBLE]

PROFESSOR: So some of the force is now on M2, for sure. And that's going to give you a couple equations. What else do you do in your method?

AUDIENCE: [INAUDIBLE]

PROFESSOR: [INAUDIBLE] torques about what?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Have I named them? Look at that diagram there. So you're going to do your torques, sum of the torques, about A, OK? And if you sum torques about A, what are they? I'll let you guys pursue that a little bit, figure out, what are the torques about this point in this problem?

OK, what do you get? Sum of the torques about A? z is in which-- which positive z? x, y, positive z coming out of the board. What produces torque about this point in this problem? Just what?

AUDIENCE: Gravity.

PROFESSOR: Just gravity. And is it positive or negative? And it's a component of gravity perpendicular to this. So it's probably sine theta M2g sine theta in the minus. So it's minus M2g L over 2-- you need the moment arm-- sine theta in the k hat direction. And that's going to be equal to the angular dH, dT about A.

Sum of the torques, time rate of change of the angular momentum about A-- because that's what you've chosen to work about. And you always have this term you have to check on. OK, so that's the method. This is method one. I'll call this the tau with respect to A method. You have to write this equation. You have two unknowns that pop up in it.

You go to the second mass. You write the F equals ma for that, and you're going to get two equations. If you do it in the local coordinate system, in the rotating one, you get mass times acceleration in the i direction. You get F1 and a component of gravity. And in the j direction, m2 acceleration in j, you get F2 and a piece that's acceleration of gravity.

You don't know this yet. But you can find it. You know how to find those velocities now. You've done it before. So the velocity of G with respect to O, probably going to have an X double dot I hat plus something that will be like L/2 theta dot, and in what direction? Hmm?

It's rotating like this-- j hat, little j. So there's your velocity. You can take the time derivative of that to get an acceleration, X double dot-- that's not true. Down here, you get an X double dot.

In this one, you get two terms. Because that's a rotating piece. You get a centripetal term and an Euler term that come out of that. So then now you know velocities and accelerations. You can put the accelerations on this side. You can solve this first equation for F1, second equation for F2. You take those and put them in here. And you finally have one of your equations of motion.

Down here, the external torque only involves gravity. So do your F1 and F2 appear-- do you have to mess with those in this final equation or not? This is the reason you use point A. This equation does not involve F1 or F2. There's no moments created by F1 and F2.

The stuff on the right hand side is only kinematics and mass moments of inertia. This thing will give you an i with respect-- you can think of this as it's going to give you an i with respect to G theta double dot plus some other terms.

And I want to talk about this. Does this term go to 0? Is the velocity of A 0 in this problem? No way. And in what direction is it? Horizontal, right, capital I hat.

P, the momentum of the second thing, has these velocity components. The P is equal to m times that. And you have velocity components in the I, which is the same as vA. But you have another velocity component that's not the same. So is that cross product going to be 0? No way. You have to deal with that term.

It happens that you can express H with respect to A as H with respect to G plus rG A cross P. And you know P, because you just found v. It's mv. And r, rG A, is just L over 2 j, little j, which is from here to here. It's L over 2 little-- no i, excuse me. So that's r, L over 2 i. So you can crank out this cross product.

Then you have to take the time derivative of this right here, this whole thing. Every time when you take the time derivative of this part, you will get a P. So you get multiple pieces. But one of them will be minus that.

This thing creates a piece that's exactly minus that, and they'll cancel. But you've got to go through the cranking it out till you get to that point. OK, that's one method of doing it. Let's talk briefly about a second method. What is the other way to go about doing this problem?

AUDIENCE: [INAUDIBLE]

PROFESSOR: That's method three. We're going to hurry so I can get to it. What's a second way to do this problem, a close relative of the first way? But I want you to have all these different ways of doing it in your tool kit. Because some problems are easy, more easily done.

There's another way than this. This is perfectly appropriate for this type of problem. If the problem we were doing involved this, would you do this method? What would you do?

AUDIENCE: [INAUDIBLE]

PROFESSOR: No, can't use energy yet. That's next week.

AUDIENCE: [INAUDIBLE]

PROFESSOR: Ahh, right. So the second method is basically this is the same, this piece. But the second part-- and this is the same, sum of the forces about M2. You do both of those forces. You do sum forces for sure, M1 and M2. But the third step is you do sum of torques about g.

If you sum torques about g, which is here, what appears in the sum torques? Does Mg contribute? No, what are the external torques about g?

AUDIENCE: [INAUDIBLE]

PROFESSOR: F2, L/2, positive F2 L/2 k. So that equation, summing torques about g, will give you an F2 L/2 k. And now you've got that unknown popping up in your torque equation. And you know that it also pops up in these two equations. Because we solved them. So now why does this method have a slight advantage over this method?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Well, you don't get F2 or F1 popping up in your torque equation. You immediately get to the answer for that one. So it's a little more efficient. So is there a way to get directly at the two equations we're after?

The second equation we get from this torque thing. The first equation we get from resolving this. This is a mass times-- M1 X double dot. And you get this function of F1 and F2. And you have to substitute in for these F1's and F2's until you finally get that equation.

So that's kind of tedious. So is there a way to get directly at this without having to mess with F1 and F2 at all? And that's what is the key to what you're talking about.

Let's look at something here. Let's do the vector sum of the forces on M1. We get an F. Let's look at our free body diagram, F2j plus F1i minus kx minus bx dot and a minus M1g.

Well, those are-- well, we'll put them in there, at least throw them in there. This is in my capital J plus N1 plus N2 capital J hat here. Is that all the forces, all the vector forces in that problem? Did I miss anything? I've got spring, the dashpot, the forces caused by the rod, gravity, and the normal forces.

OK, and we could figure out the component of this, the sum of the forces on M1 in the capital X direction. We could figure that out from-- this has component in that direction. It just involves thetas. These go away, then. You don't have to deal with this stuff in the x equation.

But there's our sum of the forces on the first body. Let's do the sum of the forces on the second body. What are they?

AUDIENCE: Gravity.

PROFESSOR: OK, so you've got a minus M2g on it. And what else?

AUDIENCE: [INAUDIBLE]

PROFESSOR: But with opposite sign, right? F2 little j minus F1 little i, and anything else? Nothing, right? Let's add the two together. Oh, and this one has got to be equal to the mass, M1, times the acceleration, the x component I plus M1 acceleration in the-- I can't write over here-- ay in the capital J hat direction. That gives us mass times acceleration, is what all the sums of the forces is equal to.

The same thing here-- this is equal to the little mass, M2, times its acceleration. And it'll have components in the-- this is mass 2. And it'll have components we can put in the I hat direction, and plus M2 acceleration 2-- maybe I'll do it like this. a2y, and this is in the j direction.

And those two things we can add together. And so if I add them together, and I put M1 plus M2 times-- I'll just call it the total acceleration. Well, I don't want to do that. I don't want to say this.

I'm hurrying because we're running out of time. M1 times its acceleration times M2 plus its acceleration is equal to-- and now we sum up everything on the right hand side. And the key thing that happens here is what happens at F1 and F2?

AUDIENCE: Drop out.

PROFESSOR: They completely drop out. So on the right hand side, it's not a function of F1 and F2. The equating of motion that we were trying to get here was just in the I direction. That's all we need for that big cart. So if we go down here and extract the I component of this total sum, it'll be an equation.

It'll involve M1 and M2. And it'll have cosines and sines and thetas and things like that, but no F1's, no F2's. You will get directly to this, the final result that you got here where you had to go in and substitute in for F1's and F2's. So it'll get it to you directly.

What do you need to solve it, though? You need to know the acceleration of mass 1. You know that. That's trivial. You need to know the acceleration of mass 2. And that you did by taking the time derivatives of these terms. You know the acceleration of that mass.

So you need that expression. And you plug that in here. And everything else in the right hand side you know. It's kx dot, bx dot, minus Mg, M1g minus M2g, and so forth. All the rest of the stuff is known. Yeah.

AUDIENCE: Does this give you three equations of motion?

PROFESSOR: Well, it gives you two equations. At this point, you'd break this into capital I and capital J. The one in the capital J direction is equal to 0. There's no motion in that direction for the main mass, the cart. There's only motion in the inertial frame x direction.

So that's all. This is a vector equation. It has two pieces to it, I and J. The J is a static equation. The I is your dynamic one. And it's this one that you would have found in this problem, but without ever having to solve for F1 and F2.

But you do have to go out and find the acceleration of both masses. Mass times acceleration, what this is really saying-- this is what you asked about the system. This is sort of the system approach. If you think of this whole thing as a system, draw a box around it, the sum of the external forces on the system is equal to the mass, the total mass, times actually the acceleration of the center of mass of the system.

But this acceleration times the center of mass of the system, you can break this into two pieces. It is this term plus this term. And then that's the total system. And all you have to put on this side is the summation of the external forces.

But the external forces are not-- the internal forces don't count. They're not part of the system. So you only have to put the external forces on it-- Mg's springs, dashpots, normal forces, outside forces, the only ones that go over here.

The reason we don't usually think of doing this is because when you think about the system, you write it as the acceleration of the center of mass. But you have to realize that you can get that expression as the sum of the parts, the sum of each bit times its acceleration, and sum them all up, is the same answer as the sum of the masses times the acceleration of the center of mass of the whole system.

So we've still got a couple of minutes. Last hour, I ran over, and we couldn't get to this. So there is, in a way, yet another thing you can do. And that's think of the thing as a whole system. And sometimes that'll give you an equation without having to use F1, without having to solve for internal forces-- pretty cool. All right, so questions, thoughts? Yeah.

AUDIENCE: Is it safe to say that if we have two more rigid bodies, we should probably consider it as a whole system [INAUDIBLE]?

PROFESSOR: Well, I can't generalize on that. I haven't done enough problems like that myself to even have enough experience to know just how often is that going to help you out.

This one, there's a pretty obvious equation that you're going to write in this direction on that mass. It's constrained to a single motion and a single coordinate you're assigning to it. And probably in cases like that where you can isolate one of the rigid bodies, there might be some advantage in doing something like this.

AUDIENCE: --more difficult to figure out what the accelerations for both rigid bodies was, then it would be better [INAUDIBLE].

PROFESSOR: Yeah, this one, all of these methods you're going to end up having to compute the velocities and accelerations of each piece. And if you've got that information, you may as well use it if there's something to be gained by doing a clever step like this. Yeah.

AUDIENCE: What confuses me is that method one, method three, there's no-- so you said there's going to be two things in motion. But why is it--

PROFESSOR: So this, this method three, this shows you a different way to get that equation. You still need this.

AUDIENCE: Oh, this is just for the first--

PROFESSOR: This is a clever way to get just this first equation down. You still need a second equation. And you still are going to have to-- and then you don't want to have to mess with F1 and F2. So we've just found a way to never have to mess with F1 and F2 in at least this problem. And that is get the first equation by using this trick, the system equation. And it's a sum forces expression. And then the second equation you need is this moment equation. And you have two choices, about G or about A.

And which one can you do it without having to find F1 and F2? I'm doing it about A. So there's a way that you can do this whole problem and never have to address what F1 and F2 are. And that's kind of a combination of these two pieces here.

The traditional way, the textbook way, you open it up, the textbook will teach you this one. And it'll teach you about A and teach you about G. But you don't often run into-- there's sometimes a direct way of doing it without finding the F1 and F2.

Unfortunately, there are thousands of problems in the world. And it's a little hard to say in advance what method to use and what's going to work the easiest. But there's some insight. There's some generalizations that you can get from looking at things in these different ways.

If you have forces at a pivot point, taking the moments of the pivot point, unknowns of the pivot point, taking the moments about that point will get rid of them for you. That's the one generalization that you can usually take to the bank. But beyond that-- and if the thing's unconstrained.

So if it's fixed axis rotation, then you want to use about the axis of fixed axis. This is a moving axis. But it is still rotating about a fixed point that's moving. There's still some advantage of doing this. If it's the eraser problem, this thing, where you have no fixed points of rotation-- if you take a body, and you do this with it, where is it spinning about? Where is its axis of rotation?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Always, right? That you can take to the bank. If there's no forced pivot, bodies on their own only rotate about their centers of gravity, centers of mass. And then when you have free bodies that don't have constraints that are forcing them to move with respect to something, if there's no constraints, then you want to take the torques about G for sure. Good, all right, see you guys next Tuesday.