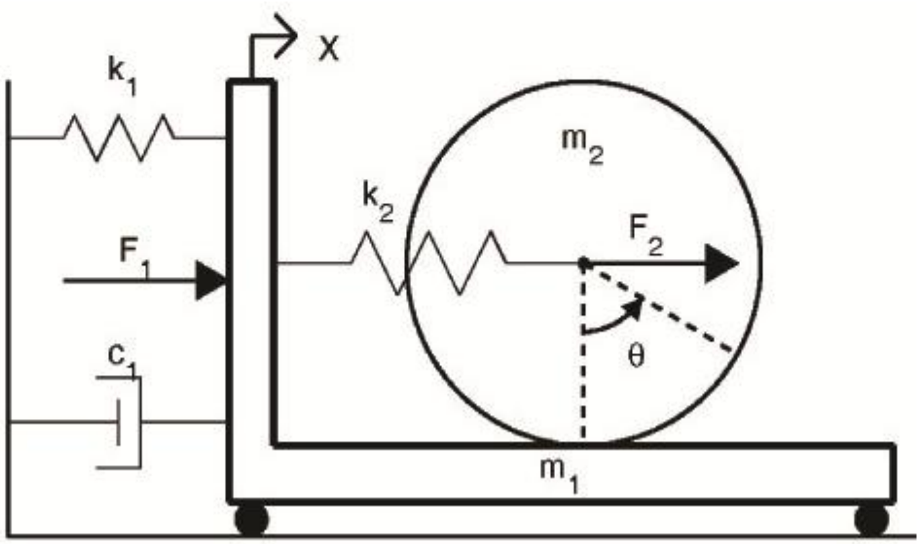
2.003SC Engineering Dynamics
Quiz 3

This exam is closed book. You are allowed **three sheets of notes, front and back**. No laptops or electronic communication devices are allowed in the exam. This includes cell phones. Calculators ARE allowed (but not on cell phones).

Unless otherwise specified, feel free to express vector answers in terms of any unit coordinate vectors defined in the problem. It is strongly recommended that you show your work in symbolic terms first before you substitute in numbers. This makes it more likely that we can award partial credit when numerical errors crop up.

There is a total of 100 points on this quiz. You have 120 minutes to complete this quiz.

Problem 1 (25%)



The figure shows a cylinder of mass m_2 and radius r, which rolls without slipping on a cart of mass m_1. Springs with constants k_1 and k_2, a dashpot of constant c_1, and external forces $F_1(t)$ and $F_2(t)$ complete the system. A damper not shown on the figure accounts for damping on the axle of the wheel, which can be modeled as a term $c_2\dot{\theta}$, which appears only in the second equation of motion shown below. The position of m_1 is given by $x(t)$ and the angle of rotation of the cylinder is given by θ. Note that θ is positive in the counter-clockwise direction. This is a 2 DOF system. The equations of motion are shown below.

$$\begin{bmatrix} m_1+m_2 & -m_2R \\ -m_2R & \frac{3}{2}m_2R^2 \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix}+\begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix}+\begin{bmatrix} k_1 & 0 \\ 0 & k_2R^2 \end{bmatrix}\begin{Bmatrix} x \\ \theta \end{Bmatrix}=\begin{Bmatrix} F_1+F_2 \\ -F_2R \end{Bmatrix}$$

Let m_1 = 6 kg, m_2 = 10 kg, $k_1 = k_2$ = 100 N/m, R = 0.1 m, and m_2 is a uniform cylinder. The damping constants are c_1 = 5 N-s/m and c_2 = 0.05 N-m-s. When these values are substituted into the EOMs, they appear as shown below.

$$\begin{bmatrix} 16 & -1 \\ -1 & .15 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & .05 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} F_1 + F_2 \\ -F_2 R \end{Bmatrix}$$

By setting the damping and excitation temporarily to zero, the undamped natural frequencies and mode shapes may be found. The mode shape matrix is:

$$[U] = \begin{bmatrix} 1.0 & 1.0 \\ -9.5125 & 10.5125 \end{bmatrix}$$

The inverse of the mode shape matrix is $U^{-1} = \begin{bmatrix} 0.525 & -.0499 \\ .4750 & .0499 \end{bmatrix}$

By the method of modal analysis one may compute the modal mass, damping and stiffness matrices as follows:

$$U^T M U = \begin{bmatrix} 48.981 & 0 \\ 0 & 11.5519 \end{bmatrix}$$

$$U^T K U = \begin{bmatrix} 190.4875 & 0 \\ 0 & 210.5125 \end{bmatrix}$$

$$U^T C U = \begin{bmatrix} 9.5244 & 0 \\ 0 & 10.5256 \end{bmatrix}$$

a. 5% Compute the **undamped natural frequencies** of the system. (Hint: You do not need to solve the characteristic equation)

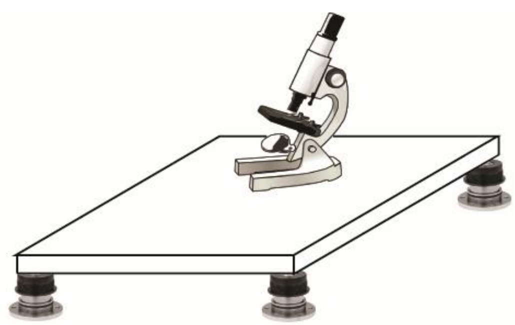
b. 5% Compute ζ_1, the **damping ratio** for mode 1 of the system.

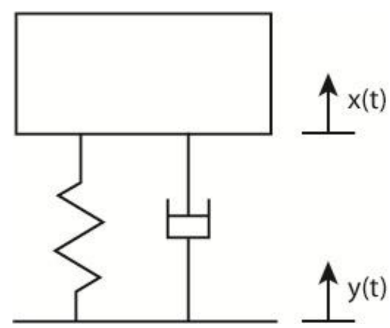
c. 5% For the case that the external exciting forces, F_1=0 and F_2=0, the system is given the following initial conditions: $\begin{Bmatrix} x_0 \\ \theta_0 \end{Bmatrix} = 0,$ $\begin{Bmatrix} \dot{x}_0 \\ \dot{\theta}_0 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix},$ find the corresponding **initial conditions in terms of the modal coordinates. Find** $\begin{Bmatrix} q_1(0) \\ q_2(0) \end{Bmatrix}$ *and* $\begin{Bmatrix} \dot{q}_1(0) \\ \dot{q}_2(0) \end{Bmatrix}$**.**

d. 10% Using the method of modal analysis, find the response for $\begin{Bmatrix} x(t) \\ \theta(t) \end{Bmatrix}$ to the initial conditions described in the part c, above. To reduce the total amount of work you need to do, compute only that part of the response $\begin{Bmatrix} x(t) \\ \theta(t) \end{Bmatrix}$ which is due to the **contribution from mode one only**.

Problem 2 (25%)

The figure above shows a commercial vibration isolation bearing. It combines a spring and damper in a single device. You use it by placing one or more of them under the device that you wish to isolate from vertical motion of the floor on which it sits. A diagram is provided showing the equivalent model of the microscope sitting on the bench with these bearings installed. The floor is vibrating in the vertical direction at a frequency of 10 Hz.





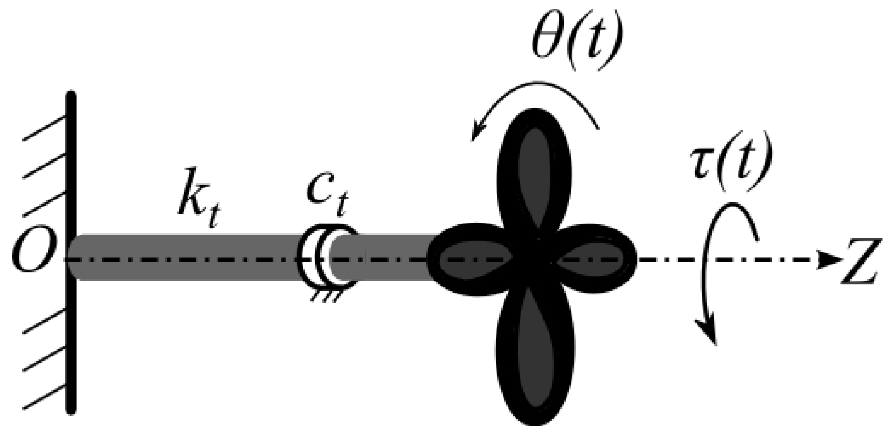
In this case four bearings support the lab bench as shown. One is attached to each corner of the bench. The microscope and bench sitting on the bearings is equivalent to the one dimensional mass-spring-damper system, as shown on the right. The mass of the bench plus microscope is such that the natural frequency of the bench sitting on the bearings is 3.0 Hz. The manufacturer claims that it has been designed to provide a damping ratio for vertical motion of 13%.

The vertical floor motion is given by $y(t) = y_0 \cos(2\pi f t)$, where $f = 10$ Hz,

a. 5% If the spring constant in each of the bearings individually is k, what is the **total effective spring constant** in the equivalent system diagram?
b. 5% What is the steady state response **frequency** of the microscope to the motion of the floor?
c. 15% What is the **magnitude** of the ratio x/y given steady state vertical motion of the floor?

Problem 3 (25%)

A ship propeller consists of four blades attached to a shaft. The mass moment of inertia of the propeller with respect to its axis of rotation is $I_{zz,G} = 100$ kg-m^2. As the propeller spins at constant rotation rate of 45 revolutions per minute, an irregularity in the flow results in an oscillatory torque on the propeller of $\tau(t) = \tau_0 \cos \omega t$, where $\omega = 6\pi$ rad/sec. The shaft has an equivalent torsional spring constant $k_t = 1600\pi^2$ N-m/rad and damping in the system can be modeled by a torsional dashpot of constant $c_t = 8\pi$ N-m-s/rad.

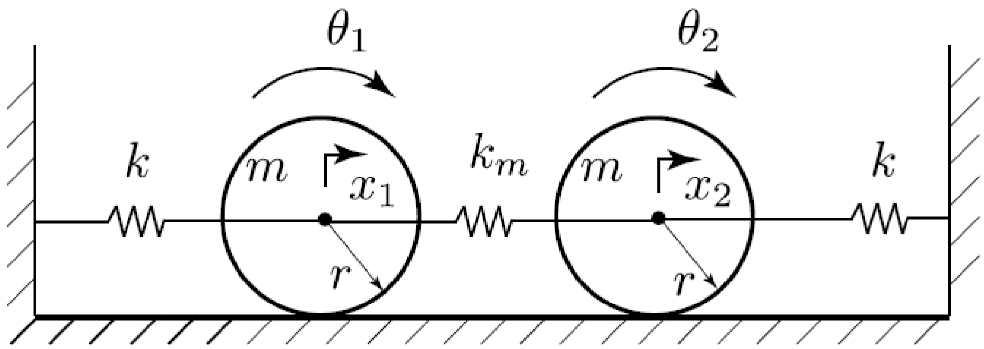


Since the system is linear, the constant rate rotation at 45 RPM may be separated from the oscillatory motion of the propeller, which results from the oscillatory torque. Thus the constant rate rotation may be neglected in the analysis of the response to the oscillatory torque.

a. 5% Write the **equation of motion** corresponding to the system described above.
b. 5% What are the **undamped** and **damped natural frequencies** of the system?
c. 5% What is the **steady state frequency** of oscillation of the system?
d. 10% A torque of magnitude τ_0 would result in a static rotation $\theta_s = 0.2$ rad. What is the **steady state amplitude of the response** due to the dynamic torque $\tau(t) = \tau_0 \cos \omega t$ at $\omega = 6\pi$ rad/sec? In providing the answer to this question, do it in two steps, as specified below:
   - Express **symbolically** the transfer function which you intend to use and
   - **Evaluate** the amplitude of the response at the frequency $\omega = 6\pi$ rad/sec.

Problem 4 (25%)

Consider the system below, consisting of two cylinders of mass m and radius r, and three springs of spring constant k and k_m, as shown. The mass in the cylinders is evenly distributed, therefore ${}^{C}I = \frac{1}{2} m r^2$.



Assume the cylinders <u>roll without slip.</u>

a. 3% How many **degrees of freedom** does the system have?

b. 2% Choose a set of **generalized coordinates** for the system from those shown in the figure. The coordinates shown are taken with respect to the static equilibrium position of the system.

c. 10% Write expressions for the system's **kinetic energy**, T, and its **potential energy**, V.

d. 10% Find the system's **equation(s) of motion**.