1.053/2.003 Dynamics and Control I
Fall 2007

Problem Set 9

Out: Tuesday, November 27th, 2007
Due: Monday, December 3rd, 2007

Equilibria, linearization, and stability

1. Problem 1

A disc of mass m and radius r is sandwiched between a horizontal surface and a slender rod of mass m which can slide in and out of a horizontal groove without friction as shown in Figure 1. The disc *rolls without slippage* with respect to both the horizontal bottom as well as the rod. The rod is connected to a spring of a spring constant k as shown in the figure, and the other end of the spring is attached to the inertial frame. Assume some length L_0 of the un-stretched spring (although that will turn out to be irrelevant to the problem). The rod is pulled out of the groove such that the spring is stretched by some length *and* then released.

a. Derive the equation of motion of the system using the Lagrangian approach.
b. Find the equilibria of the system.
c. Linearize the equation of motion about the equilibrium points and examine the stability of the equilibria.

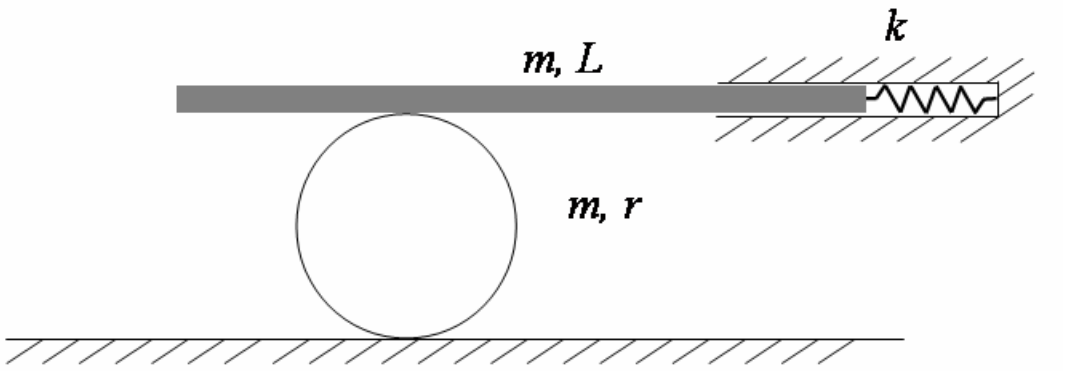

Figure 1

2. Problem 2

A disc of mass m and radius r rolls without slipping on an incline which is at angle θ with respect to the horizontal as shown in Figure 2 below. A spring of a spring constant k is connected to a fixed support and a pivot at the center of the disc as shown in the figure. Note that gravity acts. Initially the disc is held such that the spring is un-stretched. The disc is then suddenly released. It starts rolling without slipping on the incline.

a. Derive the equation of motion of the system.
b. Find the equilibria of the system.
c. Linearize the equation of motion about the equilibrium points and examine the stability of the equilibria.

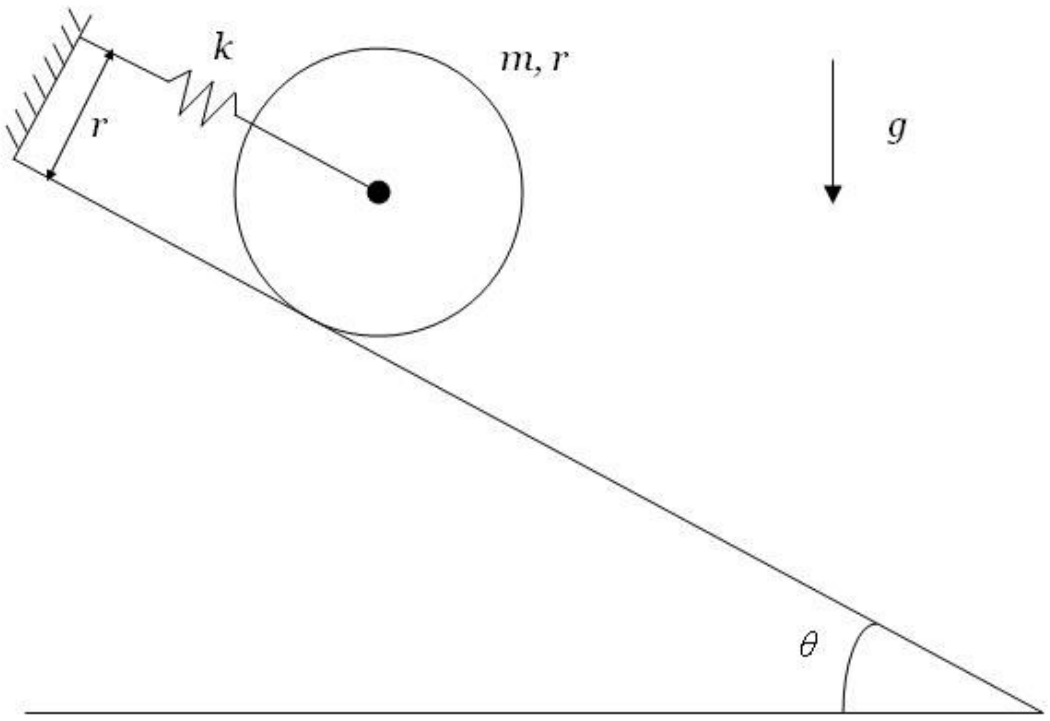

Figure 2

3. Problem 3

Consider the system shown in the following figure as in Problem 2 of Exam 2. You may use the equation of motion from the solutions posted online.

a. Find the equilibria of the system.
b. Linearize the equation of motion about the equilibrium $\mathbf{q} = \mathbf{0}$ and examine the stability of this equilibrium (**only**).

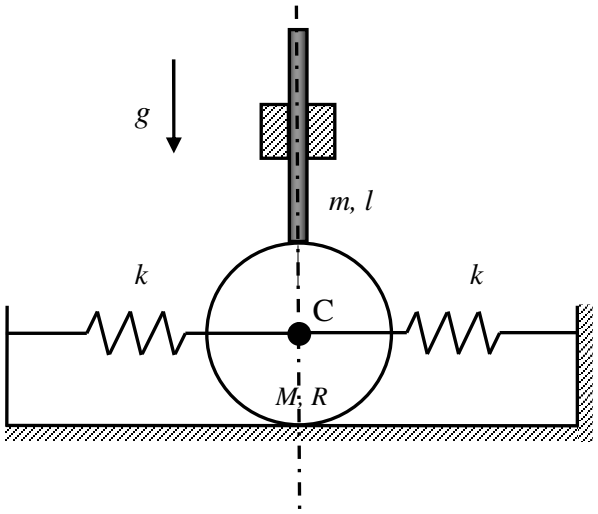

Figure 3

4. Problem 4

In the linkage shown in Figure 4 below, the disk B and slider C each have a mass m. The links each have a length l and are of negligible mass. The spring has a stiffness k and unstretched length 2l. The slider is pushed down by a small distance such that the spring is compressed, and the system is released. Note that gravity acts. Also notice that point A is pinned so it does not move.

a. Find the equation of motion.
b. Find the equilibria of the system.
c. Linearize the equation of motion about the equilibrium points and examine the stability of the equilibria.

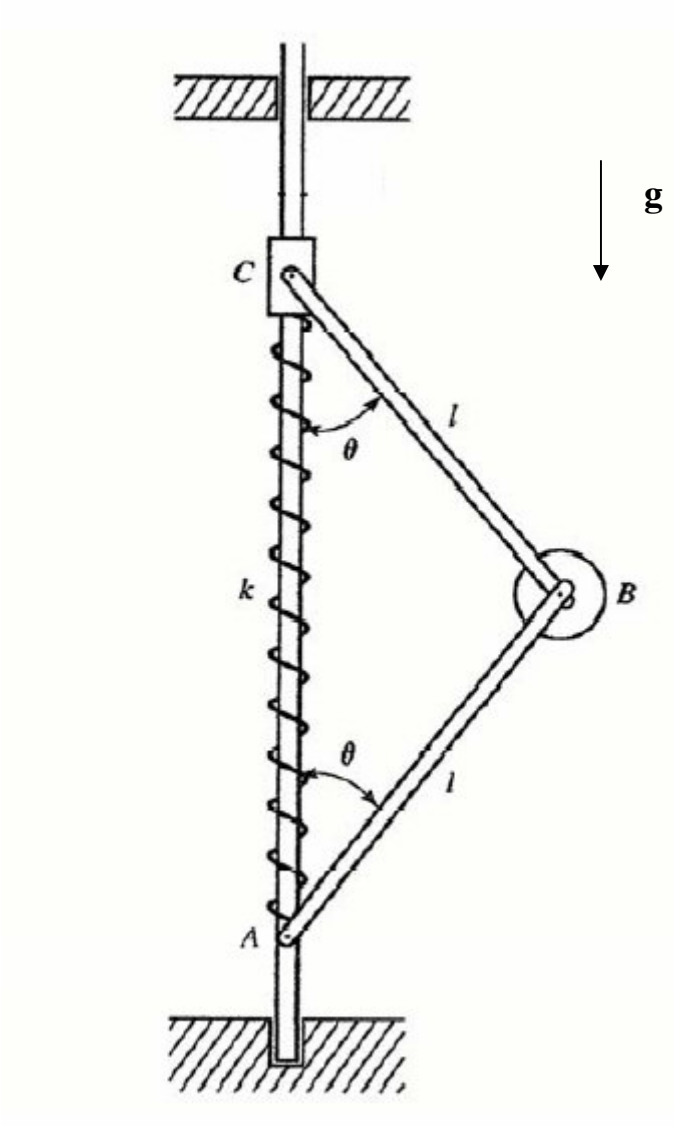

Figure 4