18.440 PROBLEM SET TWO, FEBRUARY 24

A. FROM ROSS 8TH EDITION CHAPTER TWO:

1. **Problem 25:** A pair of dice is rolled until a sum of either 5 or 7 appears. Find the probability that a 5 occurs first. *Hint.* Let E_n denote the event that a 5 occurs on the nth roll and no 5 or 7 occurs on the first $(n-1)$ rolls. Compute $P(E_n)$ and argue that $\sum_{i=1}^{\infty} P(E_n)$ is the desired probability.
2. **Problem 48:** Given 20 people, what is the probability that, among the 12 months in the year, there are 4 months containing exactly 2 birthdays and 4 containing exactly 3 birthdays?
3. **Problem 49:** A group of 6 men and 6 women is randomly divided into 2 groups of size 6 each. What is the probability that both groups will have the same number of men?
4. **Theoretical Exercise 10:** Prove that $P(E \cup F \cup G) = P(E)+P(F)+P(G)-P(E^cFG)-P(EF^cG)-P(EFG^c)-2P(EFG)$.
5. **Theoretical Exercise 15:** An urn contains M white and N black balls. If a random sample of size r is chosen, what is the probability that it contains exactly k white balls?
6. **Theoretical Exercise 20:** Consider an experiment whose sample space consists of a countably infinite number of points. Show that not all points can be equally likely. Can all points have a positive probability of occurring?

B. A deck of cards contains 30 cards with labels $1, 2, \ldots, 30$. Suppose that somebody is randomly dealt a set of 7 cards of these cards (numbered with seven distinct numbers).

1. Find the probability that 3 of the cards contain odd numbers and 4 contain even numbers.
2. Find the probability each of the numbers on the seven cards ends with a different digit. (For example, the cards could be 3, 5, 14, 16, 22, 29, 30.)

C. (Just for fun – not to hand in.) The following is a popular and rather instructive puzzle. A standard deck of 52 cards (26 red and 26 black) is shuffled so that all orderings are equally likely. We then play the following game: I begin turning the cards over one at a time so that you can see them. At some point (before I have turned over all 52 cards) you say “I’m ready!” At this point I turn over the next card and if the card is red, you receive one dollar; otherwise you receive nothing. You would like to design a strategy to maximize the probability that you will receive the dollar. How should you decide when to say “I’m ready”?

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