Part I Problems and Solutions

Problem 1: *A driven spring-mass-dashpot system is modeled by the DE*

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

with $m = 1$, $c = 6$, and $k = 45$. $F_0 = 50$. Find the amplitude $A(\omega)$ of the response as a function of the input frequency ω and find the frequency which gives the largest system response. Is this a system for which 'practical resonance' occurs?

Solution: Using the formulas derived in this session, we have

$$A(\omega) = F_0 \left(\left(k - m\omega^2\right)^2 + c^2\omega^2 \right)^{-\frac{1}{2}}$$

$$A(\omega) = 50 \left(\left(45 - \omega^2\right)^2 + 36\omega^2 \right)^{-\frac{1}{2}}$$

$\omega_{max} = \left(\frac{k}{m} - \frac{1}{2} \left(\frac{c}{m} \right)^2 \right)^{\frac{1}{2}}$ = the frequency which gives practical resonance if $c < \sqrt{4km}$. In this case, $c = 6 < \sqrt{4 \cdot 45} \cdot 1 = 6\sqrt{5}$. So the maximum system response occurs when $\omega_{max} = \sqrt{\frac{45}{1} - \frac{1}{2}\frac{36}{1}} = \sqrt{27} \approx 5.196(\frac{\text{rad}}{\text{sec}})$.

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