18.01 Practice Questions for Exam 3 – Fall 2006

1. Evaluate *a*) $\displaystyle\int_0^1 \frac{x\,dx}{\sqrt{1+3x^2}}$ *b*) $\displaystyle\int_{\pi/3}^{\pi/2} \cos^3 x \sin 2x\,dx$

2. Evaluate $\displaystyle\int_0^1 x\,dx$ directly from its definition as the limit of a sum.

Use upper sums (circumscribed rectangles). You can use the formula $\displaystyle\sum_1^n i = \frac{1}{2}n(n+1)$.

3. A bank gives interest at the rate r, compounded continuously, so that an amount A_0 deposited grows after t years to an amount $A(t) = A_0 e^{rt}$.

You make a daily deposit at the constant annual rate k; in other words, over the time period Δt you deposit $k\Delta t$ dollars. Set up a definite integral (give reasoning) which tells how much is in your account at the end of one year. (Do not evaluate the integral.)

4. Consider the function defined by $F(x) = \displaystyle\int_0^x \sqrt{3+\sin t}\,dt$. Without attempting to find an explicit formula for $F(x)$,

a) (5) show that $F(1) \le 2$;

b) (5) determine whether $F(x)$ is convex ("concave up") or concave ("concave down") on the interval $0 < x < 1$; show work or give reasoning;

c) (10) give in terms of values of $F(x)$ the value of $\displaystyle\int_1^2 \sqrt{3+\sin 2t}\,dt$.

5. If $\displaystyle\int_0^x f(t)\,dt = e^{2x}\cos x + c$, find the value of the constant c and the function $f(t)$.

6. A glass vase has the shape of the solid obtained by rotating about the y-axis the area in the first quadrant lying over the x-interval $[0, a]$ and under the graph of $y = \sqrt{x}$. By slicing it horizontally, determine how much glass it contains.

7. A right circular cone has height 5 and base radius 1; it is over-filled with ice cream, in the usual way. Place the cone so its vertex is at the origin, and its axis lies along the positive y-axis, and take the cross-section containing the x-axis. The top of this cross-section is a piece of the parabola $y = 6 - x^2$. (The whole filled ice-cream cone is gotten by rotating this cross-section about the y-axis.)

What is the volume of the ice cream? (Suggestion: use cylindrical shells.)

8. Rectangles are inscribed as shown in the quarter-circle of radius a, with the point x being chosen randomly on the interval $[0, a]$. Find the average value of their area.

9. Find the approximate value given for the integral below by the trapezoidal rule and also by Simpson's rule, taking $n = 2$ (i.e., dividing the interval of integration into two equal subintervals):

$$\int_0^{\pi/2} \sin^6 x\,dx$$

Other possible problems: Volumes by vertical slicing 4B, Work problems (P.Set 5)