

## Session 19 (In preparation for Class 19, students are asked to view Lecture 19.)

### Topics for Class 19

**Polyhedron refolding:** Fractal unfolding, three boxes, flat boxes.

**Kinetic sculpture:** Theo Jansen's Strandbeests, Arthur Ganson.

### Detailed Description of Class 19

This class starts with some updates on common unfoldings:

- The fractal unfolding of a cube and a regular tetrahedron is still a conjecture, but it is known to work for a tetrahedron that is within 1796 orders of magnitude of regular. [[Shirakawa, Horiyama, Uehara 2011](#)]
- We now have common unfoldings of three boxes. In fact, we have infinitely many examples. [[Shirakawa & Uehara 2012](#)]
- For flat boxes (doubly covered rectangles), we can even get common unfoldings of  $n$  boxes for any  $n$ . [[Abel, Demaine, Demaine, Matsui, Rote, Uehara 2011](#)]

Then, as a transition back to linkage folding (the next two lectures), we cover the beautiful leg mechanism of [Theo Jansen's Strandbeests](#), and briefly review some of Arthur Ganson's kinetic sculptures.

### Topics for Lecture 19

**Polyhedron refolding:** Dissection-like open problem, regular tetrahedron to box, Platonic solids to tetrahedra, box to box, polycubes, orthogonal unfoldings with nonorthogonal foldings.

**Smooth polyhedron folding:** Smooth Alexandrov, D-forms, ribbon curves.

**Smooth polyhedron unfolding:** Smooth prisms.

**Smooth origami:** wrapping smooth surfaces with flat paper, Mozartkugel, contractive mapping, Burago-Zalgaller Theorem (crinkling/crumpling), stretched path, stretched wrapping, source wrapping, strip wrapping, petal wrapping, comb wrapping, Pareto curve.

### Detailed Description of Lecture 19

This lecture is a big collection of fun results related to unfolding, refolding, and smooth folding:

- common unfolding of a regular tetrahedron and near-cube
- common unfoldings of Platonic solids and near-regular tetrahedra
- common unfoldings of boxes of different sizes
- common unfoldings of many polycubes
- orthogonal unfoldings with nonorthogonal foldings
- smooth Alexandrov's theorem, applied to smooth convex shapes (D-forms)
- unfolding smooth convex polyhedra (prisms)
- wrapping (paper folding) smooth surfaces like spheres, using a new definition of origami, where distances can shrink instead of necessarily staying the same

The last section has practical applications to computational confectionery, reducing the material usage in wrappings of spherical chocolates such as Mozartkugel. Yum!

MIT OpenCourseWare  
<http://ocw.mit.edu>

6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra  
Fall 2012

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.