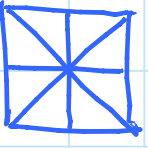


Universal hinge patterns: (for origami transformers)

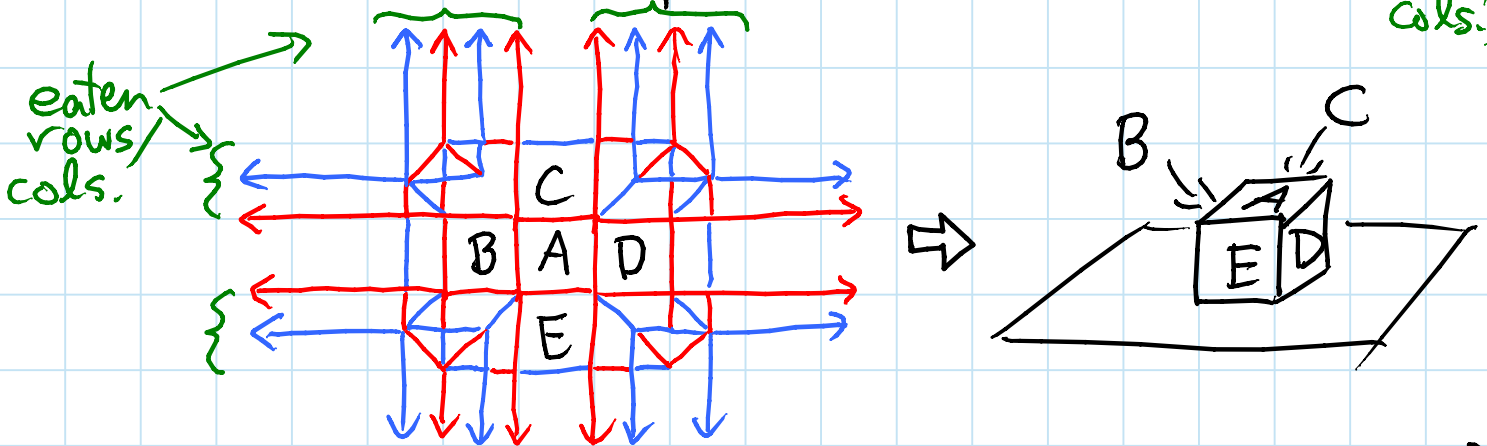
[Benbernou, Demaine, Demaine, Oradya 2010]

- suppose crease pattern required, to be subset of fixed "hinge pattern"

(e.g. Origamizer uses completely different creases for every model)


- $n \times n$ box-pleat pattern can make any polycube of $O(n)$ cubes, seamless: 

- cube gadget turns $O(1)$ rows & columns into a cube sticking out of sheet ~ even if bumps elsewhere (not in eaten rows/cols.)



- to make a tree of cubes: (= any polycube)
 - make a leaf
 - conceptually remove it } "postorder traversal"
 - repeat

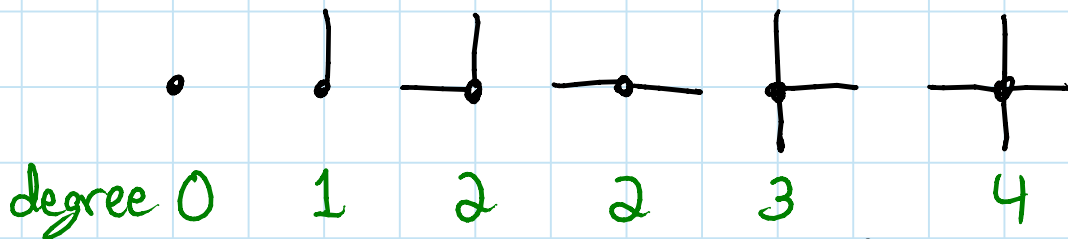
- actually need to reserve space ahead of time for all the cube gadgets

- $\Theta(n)$ cubes is optimal in worst case:
 $1 \times 1 \times n$ needs diameter $\Omega(n)$
- but sometimes can do better:

Maze folding: [Demaine, Demaine, Ku 2010]

any $n \times n$ orthogonal maze extruded from square can be folded from $\Theta(n) \times \Theta(n)$ square

- constant scale factor! (3 for unit extrusion)
- gadget for each possible vertex:



degree 0 1 2 2 3 4

- designed to have compatible interfaces:
 - ridge for maze edges
 - flat "double pleat" for nonedges
- cut & paste

try it out: <http://erikdemaine.org/maze/>

Origami design is hard ~ how to formalize?

NP-hard \approx "computationally intractable"

- if a problem is NP-hard, then there's no efficient algorithm to solve it unless $P=NP$

(famous unsolved problem, worth \$1M+)

- $P \neq NP \approx$ "computers can't simulate lucky guessing, say heads vs. tails, without trying both options"

\hookrightarrow almost everyone believes it

Examples of NP-hard problems:

- Partition: given n integers, can you split them into two halves of equal sum?

(e.g. equalizing teams for a game)

- actually only hard for exp. large integers:
"weakly NP-hard"

- SAT: given Boolean formula $(x \text{ AND NOT } y) \text{ OR } z$
can you set the variables x, y, z true/false so that formula is true?

Approach: show e.g. Partition is easier / a special case of your problem: any Partition problem can be converted into a problem of your type
 \Rightarrow your problem is NP-hard too

Simple example: (from Problem Session 1)

given single-vertex hinge pattern,
is some subset of ($> \emptyset$) creases
flat foldable?

(posed by student after class)

is NP-hard:

- given Partition problem, scale integers
uniformly so that their sum = 360°

→ angles of single-vertex crease pattern

- looking at angular travel (Kawasaki-Justin),
at each hinge can crease → change direction
or not → same direction

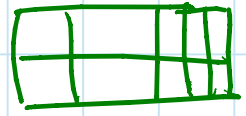
⇒ can choose $+\theta_i$ or $-\theta_i$ for each i

- must have $\sum_i \pm \theta_i = 0$

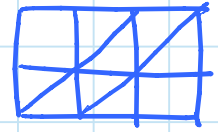
ie. $\sum_i +\theta_i$'s = $\sum_i -\theta_i$'s

- YES to Partition \Leftrightarrow YES to flat-foldable CP

Simple folds: can given crease pattern be folded flat by sequence of simple folds?
 - saw how to solve for 1D patterns & 2D orthogonal maps:

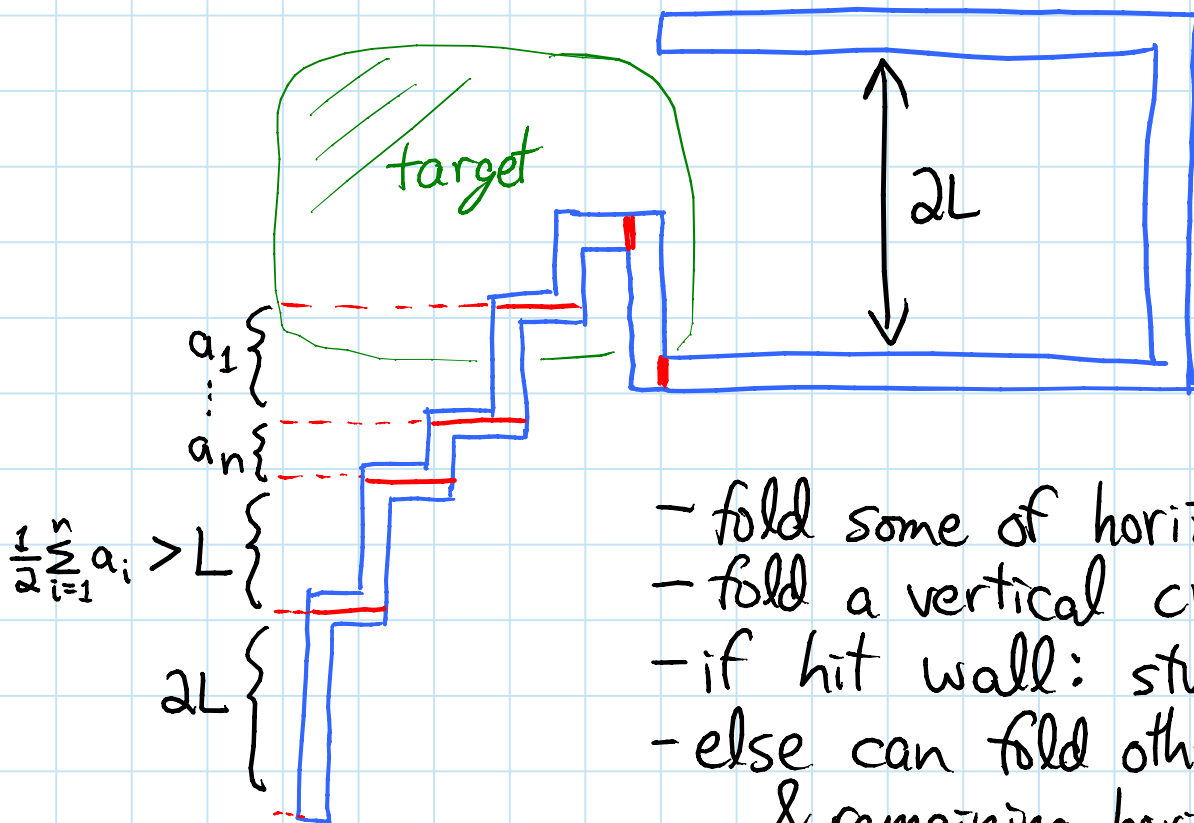


NP-hard if we add 45° diagonal creases or allow orthogonal paper



[Arkin, Bender, Demaine, Demaine, Mitchell, Sethia, Skiena 2000]

- reduction from Partition (weakly NP-complete)



- fold some of horiz. creases
- fold a vertical crease
- if hit wall: stuck
- else can fold other vertical & remaining horiz. creases

□

Global flat foldability: [Bern & Hayes 1996]

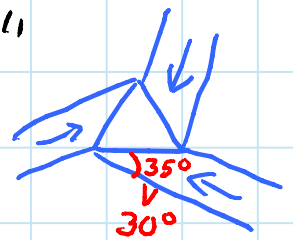
- ① deciding flat foldability of given crease pattern is strongly NP-hard
- ② constructing valid layer ordering for given flat-foldable mountain-valley pattern is strongly NP-hard

Proof: (①) reduce from all-positive not-all-equal 3-satisfiability: given triples (x_i, x_j, x_k) , is there a Boolean assignment to x_1, x_2, \dots, x_n such that no triple is all-true or all-false? (strongly NP-hard, like SAT)

Wire = "pleat" = two close parallel creases
false \Leftrightarrow left mountain



NAE clause = triangular "overtwist"
- can't all fold same way
(twist is borderline)

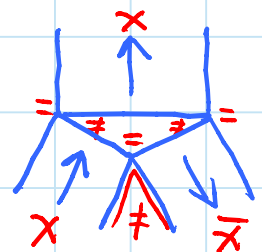


Reflector splits wire x into two copies, one negated

\Rightarrow split gadget

& turn gadget (with noise)

\Rightarrow can connect variable wires to desired clauses



Also need crossover gadgets.

□

Disk packing: [Demaine, Fekete, Lang 2010]

can you place n given disks nonoverlapping with centers in given square?

= can you make uniaxial base from given square?



is NP-hard

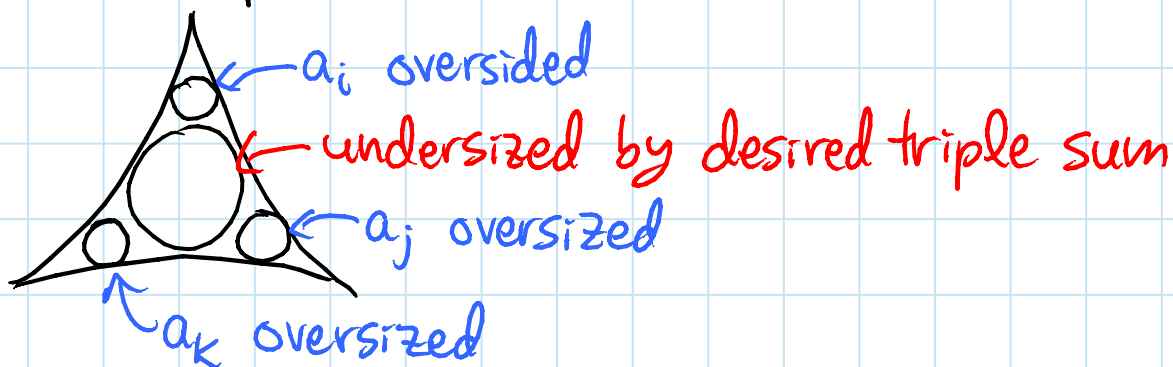
- reduction from 3-Partition:

given n integers, can you split them into $n/3$ triples of equal sum?

- strongly NP-hard \sim integers $= O(n)$, not exp.

- lots of disks to force identical pockets & make all other pockets too small

- within one pocket:



\Rightarrow just fits if 3-partition

□

MIT OpenCourseWare
<http://ocw.mit.edu>

6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
Fall 2012

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