# 6.837 Introduction to Computer Graphics
# Quiz 1

Tuesday, October 18, 2011 2:40-4pm
One hand-written sheet of notes (2 pages) allowed

## NAME:

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| 1 | / 20 |
| 2 | / 18 |
| 3 | / 18 |
| 4 | / 24 |
| Total | / 80 |

The answer are in blue and extra information that was not required is in *green*.

## 1 Coordinate systems [ /20]

Given that basis $\vec{\mathbf{a}}^t$ can be expressed in basis $\vec{\mathbf{b}}^t$ as $\vec{\mathbf{a}}^t = \vec{\mathbf{b}}^t M$, what are the coordinates of the vector $\vec{\mathbf{b}}^t N \mathbf{c}$ with respect to the basis $\vec{\mathbf{a}}^t$, where $N$ and $M$ are matrices and $\mathbf{c}$ is a column vector of coordinates?
[ **/2**]

$M^{-1} N \mathbf{c}$

Let $\vec{0}$ be the zero vector. For any linear transformation $\mathcal{L}$, what is $\mathcal{L}(\vec{0})$? [ **/2**]

$\mathcal{L}(\vec{0}) = \vec{0}$

Let $\mathcal{T}(\vec{v})$ be the transformation that adds a specific non-zero constant vector $\vec{k}$ to $\vec{v}$: $\mathcal{T}(\vec{v}) = \vec{v} + \vec{k}$. Is $\mathcal{T}$ a linear transformation? [ **/2**]

No
*(because $\mathcal{T}(\vec{0})$ is not $\vec{0}$)*

Suppose $\vec{\mathbf{f}}^t$ is a 2D orthonormal frame, and we apply the transform $\vec{\mathbf{f}}^t \Rightarrow \vec{\mathbf{f}}^t ST$, where $S$ is a matrix that applies a uniform scale by a factor of 2, and $T$ translates by 1 along the x axis.

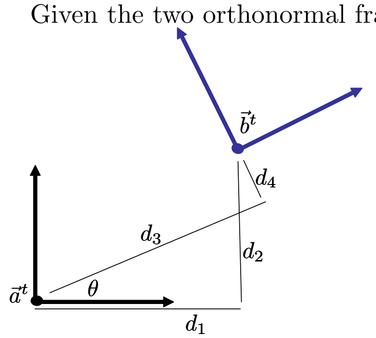
Write these two $3 \times 3$ matrices. [ **/4**]

$$S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

How far does the frame's origin move, measured in the original units of $\vec{\mathbf{f}}^t$ ? [ **/4**]

2

Given the two orthonormal frames $\vec{\mathbf{a}}^t$ and $\vec{\mathbf{b}}^t$ shown below:

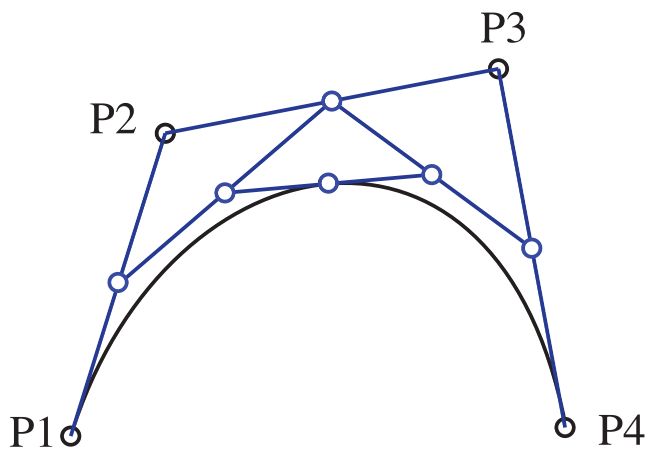



with distances given by the positive quantities $d_i$. What are the matrices $R$ and $T$ such that $\vec{\mathbf{b}}^t = \vec{\mathbf{a}}^t TR$? Note: do this without using trigonometric terms in the matrix $T$. [ **/6**]

$$T = \begin{bmatrix} 1 & 0 & d_3 \\ 0 & 1 & d_4 \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## 2 Splines [ /18]

Draw the DeCasteljau construction for the evaluation of the point t=0.5 of the Bézier spline defined by the 4 points below. [ /6]




How can you choose the control points so that a cubic Bézier curve is a closed loop? [ /4]

$P_1 = P_4$

What is the order of continuity that you get, in general, with such closed-loop cubic Bézier spline? [ /4]

$C^0$

*but it has a $C^1$ discontinuity*

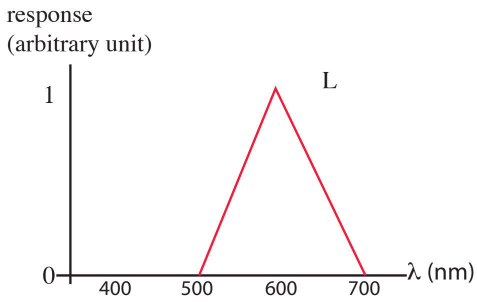
What additional constraint on the control points is needed to get one more order of continuity (while keeping it a closed-loop cubic spline defined by four points), and what would the consequence be? [ /4]

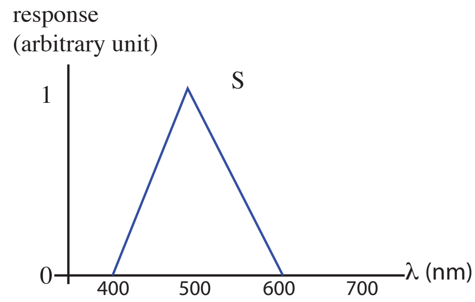
$P_1P_2$ and $P_3P_4$ should be aligned. Since $P_1 = P4$, would mean that all four points are aligned and the spline reduces to a segment.

*$P_1 = P_4$ should also be between $P_2$ and $P_3$. Furthermore, if we want $C^1$ continuity and not just $G^1$, we need the length of $P_1P_2$ to be the same as $P_3P_4$*

## 3 Color [ /18]

Imagine that humans were intelligently designed with simpler color bi-chromat vision where the two types of cones have the following spectral responses:

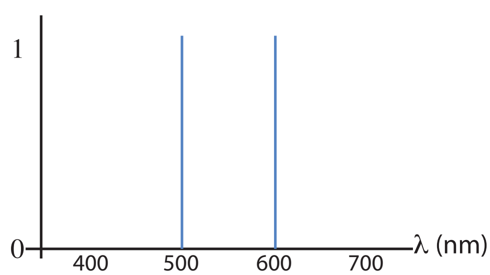


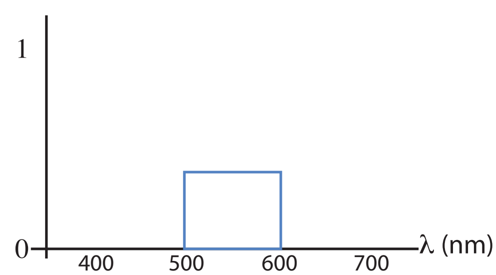




What would be the pair of cone responses LS for a stimulus of monochromatic light at 500nm with total power 2.73 in our arbitrary units? [ /**2**]

(0, 2.73)

Give two spectra that are metamers under this set of cones. [ /**4**]




*In fact any spectrum that is symmetric with respect to 550nm is a metamer of these two.*

*Another possible answer is to use parts of the spectrum that have zero response and add them. It's kind of correct but in my opinion it's a little bit cheating.*

We seek to reproduce the CIE color matching experiment with primary wavelengths 500nm and 600 nm. What is the required amount of these two primaries to match unit-power stimuli at the following wavelengths: [ **/6**]

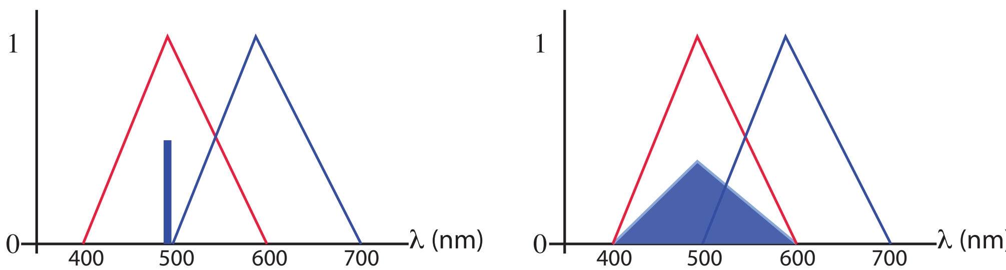
500nm (1 0)

550nm (0.5 0.5)

600nm (0 1)

Give an example that illustrates what happens if we seek to use the same spectra for analysis (e.g. the two color filters on a camera) and for synthesis (e.g. the two primaries of a video projector) with these 2-cone humans. [ **/6**]

Suppose we use the cone spectra both for analysis and synthesis and seek to reproduce a monochromatic stimulus at 500nm (left figure below). The response to our analysis is (1 0). We then scale the two spectra by these numbers for synthesis, as shown on the right, leading to a new stimulus with the shape of a tent. Unfortunately, this stimulus has a non-zero response for the S cone and the color is not accurately reproduced.



## 4 Physically-based animation [ /24]

We are given a single particle with 1D vertical position described by the scalar function $x(t)$ where the positive values of $x$ go up. It follows Newtonian mechanics and is affected by gravity described by the constant $g$ (in $ms^{-2}$) and a drag force $F = \text{-}dv$ where $v = \frac{dx}{dt}$ and $d$ is a constant. Initially, the particle is at $x_0$ and travels with velocity $v_0$. The particle has mass $m$.

Write the second-order ODE that describes this particle system. [ **/4**]

$$\frac{d^2x}{dt^2} = -g - \frac{1}{m}dv$$

Transform this second-order ODE into a first order ODE system . [ **/4**]

$$\frac{d}{dt}\begin{pmatrix} x \\ v \end{pmatrix} = \begin{pmatrix} v \\ -g - \frac{1}{m}dv \end{pmatrix}$$

Use the Euler method to estimate the state of the system after time step $h$. [ **/8**]

$$x(h) = x_0 + v_0 h$$

$$\begin{aligned} v(h) &= v_0 - gh - \frac{h}{m}dv_0 \\ &= v_0\left(1 - \frac{hd}{m}\right) - gh \end{aligned}$$

Use the trapezoid method to estimate the value of $x$ after time step $h$. Note: we do not ask you to compute the velocity after this full step. [ **/8**]

We perform a Euler step, read the right-hand side of the equation and use the average of the right-hand side at this location and at the original location.

$$\begin{aligned} x(h) &= x_0 + \frac{h}{2}\left[v_0 + v_0\left(1 - \frac{hd}{m}\right) - gh\right] \\ &= x_0 + hv_0\left(1 - \frac{hd}{2m}\right) - \frac{1}{2}gh^2 \end{aligned}$$